Elements of Metric Gear Technology

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Staff of **SDP/SI**:

Dr. Frank Buchsbaum, Executive Vice President, Designatronics, Inc.
Dr. Hitoshi Tanaka, Senior Vice President, Designatronics, Inc.
Linda Shuett, Manager, Graphic Communications
John Chiaramonte, Senior Graphic Artist
Szymon Sondej, Graphic Artist
Melanie Hasranah, Graphic Artist
Luis C. Quinteros, Product Engineer
Milton Epstein, Assistant Editor

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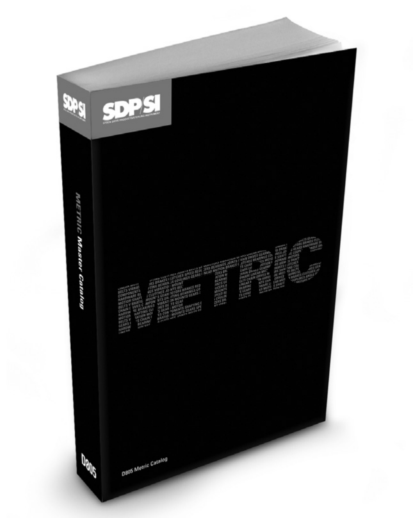

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Gears are some of the most important elements used in machinery. There are few mechanical devices that do not have the need to transmit power and motion between rotating shafts. Gears not only do this most satisfactorily, but can do so with uniform motion and reliability. In addition, they span the entire range of applications from large to small. To summarize:

1. Gears offer positive transmission of power.
2. Gears range in size from small miniature instrument installations, that measure in only several millimeters in diameter, to huge powerful gears in turbine drives that are several meters in diameter.
3. Gears can provide position transmission with very high angular or linear accuracy; such as used in servomechanisms and military equipment.
4. Gears can couple power and motion between shafts whose axes are parallel, intersecting or skew.
5. Gear designs are standardized in accordance with size and shape which provides for widespread interchangeability.

This technical manual is written as an aid for the designer who is a beginner or only superficially knowledgeable about gearing. It provides fundamental theoretical and practical information. Admittedly, it is not intended for experts.

Those who wish to obtain further information and special details should refer to the reference list at the end of this text and other literature on mechanical machinery and components.

SECTION 1 INTRODUCTION TO METRIC GEARS

This technical section is dedicated to details of metric gearing because of its increasing importance. Currently, much gearing in the United States is still based upon the inch system. However, with most of the world metricated, the use of metric gearing in the United States is definitely on the increase, and inevitably at some future date it will be the exclusive system.

It should be appreciated that in the United States there is a growing amount of metric gearing due to increasing machinery and other equipment imports. This is particularly true of manufacturing equipment, such as printing presses, paper machines and machine tools. Automobiles are another major example, and one that impacts tens of millions of individuals. Further spread of metric gearing is inevitable since the world that surrounds the United States is rapidly approaching complete conformance. England and Canada, once bastions of the inch system, are well down the road of metrication, leaving the United States as the only significant exception.

Thus, it becomes prudent for engineers and designers to not only become familiar with metric gears, but also to incorporate them in their designs. Certainly, for export products it is imperative; and for domestic products it is a serious consideration. The U.S. Government, and in particular the military, is increasingly insisting upon metric based equipment designs.

Recognizing that most engineers and designers have been reared in an environment of heavy use of the inch system and that the amount of literature about metric gears is limited, we are offering this technical gear section as an aid to understanding and use of metric gears. In the following pages, metric gear standards are introduced along with information about interchangeability and noninterchangeability. Although gear theory is the same for both the inch and metric systems, the formulae for metric gearing take on a different set of symbols. These equations are fully defined in the metric system. The coverage is thorough and complete with the intention that this be a source for all information about gearing with definition in a metric format.

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1.1 Comparison Of Metric Gears With American Inch Gears

1.1.1 Comparison of Basic Racks

In all modern gear systems, the rack is the basis for tooth design and manufacturing tooling. Thus, the similarities and differences between the two systems can be put into proper perspective with comparison of the metric and inch basic racks.

In both systems, the basic rack is normalized for a unit size. For the metric rack it is 1 module, and for the inch rack it is 1 diametral pitch.

1.1.2 Metric ISO Basic Rack

The standard ISO metric rack is detailed in **Figure 1-1**. It is now the accepted standard for the international community, it having eliminated a number of minor differences that existed between the earlier versions of Japanese, German and Russian modules. For comparison, the standard inch rack is detailed in **Figure 1-2**. Note that there are many similarities. The principal factors are the same for both racks. Both are normalized for unity; that is, the metric rack is specified in terms of 1 module, and the inch rack in terms of 1 diametral pitch.

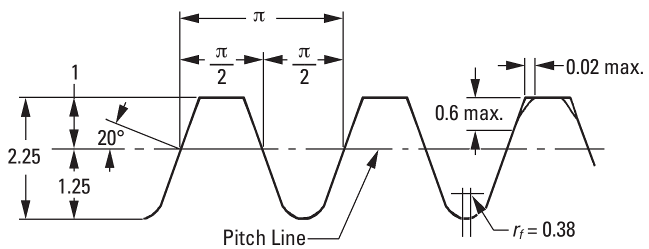

Fig. 1-1 The Basic Metric Rack From ISO 53 Normalized For Module 1

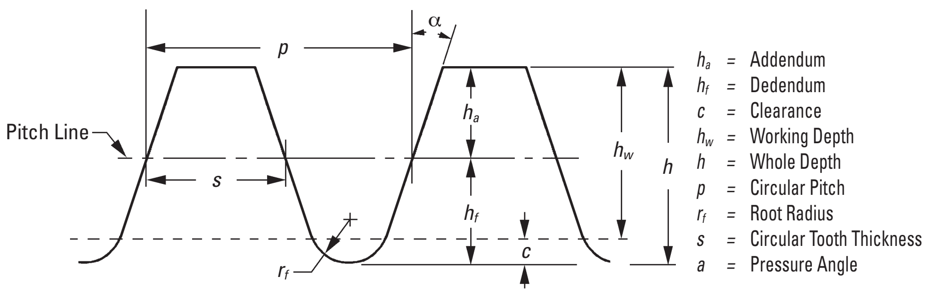

Fig. 1-2 The Basic Inch Diametral Pitch Rack Normalized For 1 Diametral Pitch

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Metric
0 10

From the normalized metric rack, corresponding dimensions for any module are obtained by multiplying each rack dimension by the value of the specific module *m*. The major tooth parameters are defined by the standard, as:

Tooth Form:	Straight-sided full depth, forming the basis of a family of full depth interchangeable gears.
Pressure Angle:	A 20° pressure angle, which conforms to worldwide acceptance of this as the most versatile pressure angle.
Addendum:	This is equal to the module *m*, which is similar to the inch value that becomes 1/*p*.
Dedendum:	This is 1.25 *m*; again similar to the inch rack value.
Root Radius:	The metric rack value is slightly greater than the American inch rack value.
Tip Radius:	A maximum value is specified. This is a deviation from the American inch rack which does not specify a rounding.

1.1.3 Comparison of Gear Calculation Equations

Most gear equations that are used for diametral pitch inch gears are equally applicable to metric gears if the module *m* is substituted for diametral pitch. However, there are exceptions when it is necessary to use dedicated metric equations. Thus, to avoid confusion and errors, it is most effective to work entirely with and within the metric system.

1.2 Metric Standards Worldwide

1.2.1 ISO Standards

Metric standards have been coordinated and standardized by the International Standards Organization (ISO). A listing of the most pertinent standards is given in **Table 1-1**.

1.2.2 Foreign Metric Standards

Most major industrialized countries have been using metric gears for a long time and consequently had developed their own standards prior to the establishment of ISO and SI units. In general, they are very similar to the ISO standards. The key foreign metric standards are listed in **Table 1-2** for reference.

1.3 Japanese Metric Standards In This Text

1.3.1 Application of JIS Standards

Japanese Industrial Standards (JIS) define numerous engineering subjects including gearing. The originals are generated in Japanese, but they are translated and published in English by the Japanese Standards Association.

Considering that many metric gears are produced in Japan, the JIS standards may apply. These essentially conform to all aspects of the ISO standards.

Table 1-1 ISO Metric Gearing Standards

ISO 53:1974	Cylindrical gears for general and heavy engineering – Basic rack
ISO 54:1977	Cylindrical gears for general and heavy engineering – Modules and diametral pitches
ISO 677:1976	Straight bevel gears for general and heavy engineering – Basic rack
ISO 678:1976	Straight bevel gears for general and heavy engineering – Modules and diametral pitches
ISO 701:1976	International gear notation – symbols for geometrical data
ISO 1122-1:1983	Glossary of gear terms – Part 1: Geometrical definitions
ISO 1328:1975	Parallel involute gears – ISO system of accuracy
ISO 1340:1976	Cylindrical gears – Information to be given to the manufacturer by the purchaser in order to obtain the gear required
ISO 1341:1976	Straight bevel gears – Information to be given to the manufacturer by the purchaser in order to obtain the gear required
ISO 2203:1973	Technical drawings – Conventional representation of gears
ISO 2490:1975	Single-start solid (monobloc) gear hobs with axial keyway, 1 to 20 module and 1 to 20 diametral pitch – Nominal dimensions
ISO/TR 4467:1982	Addendum modification of the teeth of cylindrical gears for speed-reducing and speed-increasing gear pairs
ISO 4468:1982	Gear hobs – Single-start – Accuracy requirements
ISO 8579-1:1993	Acceptance code for gears – Part 1: Determination of airborne sound power levels emitted by gear units
ISO 8579-2:1993	Acceptance code for gears – Part 2: Determination of mechanical vibrations of gear units during acceptance testing
ISO/TR 10064-1:1992	Cylindrical gears – Code of inspection practice – Part 1: Inspection of corresponding flanks of gear teeth

Table 1-1 FOREIGN Metric Gearing Standards

AUSTRALIA		
AS B 62	1965	Bevel gears
AS B 66	1969	Worm gears (inch series)
AS B 214	1966	Geometrical dimensions for worm gears – Units
AS B 217	1966	Glossary for gearing
AS 1637		International gear notation symbols for geometric data (similar to ISO 701)

FRANCE		
NF E 23-001	1972	Glossary of gears (similar to ISO 1122)
NF E 23-002	1972	Glossary of worm gears
NF E 23-005	1965	Gearing – Symbols (similar to ISO 701)
NF E 23-006	1967	Tolerances for spur gears with involute teeth (similar to ISO 1328)
NF E 23-011	1972	Cylindrical gears for general and heavy engineering – Basic rack and modules (similar to ISO 467 and ISO 53)
NF E 23-012	1972	Cylindrical gears – Information to be given to the manufacturer by the purchaser
NF L 32-611	1955	Calculating spur gears to NF L 32-610

Continued on the next page

Table 1-2 (Cont.) Foreign Metric Gearing Standards

GERMANY – DIN (Deutsches Institut für Normung)		
DIN 37	12.61	Conventional and simplified representation of gears and gear pairs [4]
DIN 780 Pt 1	05.77	Series of modules for gears – Modules for spur gears [4]
DIN 780 Pt 2	05.77	Series of modules for gears – Modules for cylindrical worm gear transmissions [4]
DIN 867	02.86	Basic rack tooth profiles for involute teeth of cylindrical gears for general and heavy engineering [5]
DIN 868	12.76	General definitions and specification factors for gears, gear pairs and gear trains [11]
DIN 3961	08.78	Tolerances for cylindrical gear teeth – Bases [8]
DIN 3962 Pt 1	08.78	Tolerances for cylindrical gear teeth – Tolerances for deviations of individual parameters [11]
DIN 3962 Pt 2	08.78	Tolerances for cylindrical gear teeth – Tolerances for tooth trace deviations [4]
DIN 3962 Pt 3	08.78	Tolerances for cylindrical gear teeth – Tolerances for pitch-span deviations [4]
DIN 3963	08.78	Tolerances for cylindrical gear teeth – Tolerances for working deviations [11]
DIN 3964	11.80	Deviations of shaft center distances and shaft position tolerances of casings for cylindrical gears [4]
DIN 3965 Pt 1	08.86	Tolerancing of bevel gears – Basic concepts [5]
DIN 3965 Pt 2	08.86	Tolerancing of bevel gears – Tolerances for individual parameters [11]
DIN 3965 Pt 3	08.86	Tolerancing of bevel gears – Tolerances for tangential composite errors [11]
DIN 3965 Pt 4	08.86	Tolerancing of bevel gears – Tolerances for shaft angle errors and axes intersection point deviations [5]
DIN 3966 Pt 1	08.78	Information on gear teeth in drawings – Information on involute teeth for cylindrical gears [7]
DIN 3966 Pt 2	08.78	Information on gear teeth in drawings – Information on straight bevel gear teeth [6]
DIN 3967	08.78	System of gear fits – Backlash, tooth thickness allowances, tooth thickness tolerances – Principles [12]
DIN 3970 Pt 1	11.74	Master gears for checking spur gears – Gear blank and tooth system [8]
DIN 3970 Pt 2	11.74	Master gears for checking spur gears – Receiving arbors [4]
DIN 3971	07.80	Definitions and parameters for bevel gears and bevel gear pairs [12]
DIN 3972	02.52	Reference profiles of gear-cutting tools for involute tooth systems according to DIN 867 [4]
DIN 3975	10.76	Terms and definitions for cylindrical worm gears with shaft angle 90° [9]
DIN 3976	11.80	Cylindrical worms – Dimensions, correlation of shaft center distances and gear ratios of worm gear drives [6]
DIN 3977	02.81	Measuring element diameters for the radial or diametral dimension for testing tooth thickness of cylindrical gears [8]
DIN 3978	08.76	Helix angles for cylindrical gear teeth [5]
DIN 3979	07.79	Tooth damage on gear trains – Designation, characteristics, causes [11]
DIN 3993 Pt 1	08.81	Geometrical design of cylindrical internal involute gear pairs – Basic rules [17]
DIN 3993 Pt 2	08.81	Geometrical design of cylindrical internal involute gear pairs – Diagrams for geometrical limits of internal gear-pinion matings [15]
DIN 3993 Pt 3	08.81	Geometrical design of cylindrical internal involute gear pairs – Diagrams for the determination of addendum modification coefficients [15]
DIN 3993 Pt 4	08.81	Geometrical design of cylindrical internal involute gear pairs – Diagrams for limits of internal gear-pinion type cutter matings [10]
DIN 3998 Suppl 1	09.76	Denominations on gear and gear pairs – Alphabetical index of equivalent terms [10]
DIN 3998 Pt 1	09.76	Denominations on gears and gear pairs – General definitions [11]
DIN 3998 Pt 2	09.76	Denominations on gears and gear pairs – Cylindrical gears and gear pairs [11]
DIN 3998 Pt 3	09.76	Denominations on gears and gear pairs – Bevel and hypoid gears and gear pairs [9]
DIN 3998 Pt 4	09.76	Denominations on gears and gear pairs – Worm gear pairs [8]
DIN 58405 Pt 1	05.72	Spur gear drives for fine mechanics –Scope, definitions, principal design data, classification [7]
DIN 58405 Pt 2	05.72	Spur gear drives for fine mechanics – Gear fit selection, tolerances, allowances [9]
DIN 58405 Pt 3	05.72	Spur gear drives for fine mechanics – Indication in drawings, examples for calculation [12]
DIN 58405 Pt 4	05.72	Spur gear drives for fine mechanics – Tables [15]
DIN ISO 2203	06.76	Technical Drawings – Conventional representation of gears

NOTES:

1. Standards available in English from: ANSI, 1430 Broadway, New York, NY 10018; or Beuth Verlag GmbH, Burggrafenstrasse 6, D-10772 Berlin, Germany; or Global Engineering Documents, Inverness Way East, Englewood, CO 80112-5704
2. Above data was taken from: DIN Catalogue of Technical Rules 1994, Supplement, Volume 3, Translations

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Table 1-2 (Cont.) Foreign Metric Gearing Standards

ITALY		
UNI 3521	1954	Gearing – Module series
UNI 3522	1954	Gearing – Basic rack
UNI 4430	1960	Spur gear – Order information for straight and bevel gear
UNI 4760	1961	Gearing – Glossary and geometrical definitions
UNI 6586	1969	Modules and diametral pitches of cylindrical and straight bevel gears for general and heavy engineering (corresponds to ISO 54 and 678)
UNI 6587	1969	Basic rack of cylindrical gears for standard engineering (corresponds to ISO 53)
UNI 6588	1969	Basic rack of straight bevel gears for general and heavy engineering (corresponds to ISO 677)
UNI 6773	1970	International gear notation – Symbols for geometrical data (corresponds to ISO 701)

JAPAN – JIS (Japanese Industrial Standards)		
B 0003	1989	Drawing office practice for gears
B 0102	1988	Glossary of gear terms
B 1701	1973	Involute gear tooth profile and dimensions
B 1702	1976	Accuracy for spur and helical gears
B 1703	1976	Backlash for spur and helical gears
B 1704	1978	Accuracy for bevel gears
B 1705	1973	Backlash for bevel gears
B 1721	1973	Shapes and dimensions of spur gears for general engineering
B 1722	1974	Shape and dimensions of helical gears for general use
B 1723	1977	Dimensions of cylindrical worm gears
B 1741	1977	Tooth contact marking of gears
B 1751	1976	Master cylindrical gears
B 1752	1989	Methods of measurement of spur and helical gears
B 1753	1976	Measuring method of noise of gears
B 4350	1991	Gear cutter tooth profile and dimensions
B 4351	1985	Straight bevel gear generating cutters
B 4354	1988	Single thread hobs
B 4355	1988	Single thread fine pitch hobs
B 4356	1985	Pinion type cutters
B 4357	1988	Rotary gear shaving cutters
B 4358	1991	Rack type cutters

NOTE:
Standards available in English from: ANSI, 1430 Broadway, New York, NY 10018; or International Standardization Cooperation Center, Japanese Standards Association, 4-1-24 Akasaka, Minato-ku, Tokyo 107

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Table 1-2 (Cont.) Foreign Metric Gearing Standards

UNITED KINGDOM – BSI (British Standards Institute)		
BS 235	1972	Specification of gears for electric traction
BS 436 Pt 1	1987	Spur and helical gears – Basic rack form, pitches and accuracy (diametral pitch series)
BS 436 Pt 2	1984	Spur and helical gears – Basic rack form, modules and accuracy (1 to 50 metric module)
BS 436 Pt 3	1986	(Parts 1 & 2 related but not equivalent with ISO 53, 54, 1328, 1340 & 1341) Spur gear and helical gears – Method for calculation of contact and root bending stresses, limitations for metallic involute gears (Related but not equivalent with ISO / DIS 6336 / 1, 2 & 3)
BS 721 Pt 1	1984	Specification for worm gearing – Imperial units
BS 721 Pt 2	1983	Specification for worm gearing – Metric units
BS 978 Pt 1	1984	Specification for fine pitch gears – Involute spur and helical gears
BS 978 Pt 2	1984	Specification for fine pitch gears – Cycloidal type gears
BS 978 Pt 3	1984	Specification for fine pitch gears – Bevel gears
BS 978 Pt 4	1965	Specification for fine pitch gears – Hobs and cutters
BS 1807	1981	Specification for marine propulsion gears and similar drives: metric module
BS 2007	1983	Specification for circular gear shaving cutters, 1 to 8 metric module, accuracy requirements
BS 2062 Pt 1	1985	Specification for gear hobs – Hobs for general purpose: 1 to 20 d.p., inclusive
BS 2062 Pt 2	1985	Specification for gear hobs – Hobs for gears for turbine reduction and similar drives
BS 2518 Pt 1	1983	Specification for rotary form relieved gear cutters – Diametral pitch
BS 2518 Pt 2	1983	Specification for rotary relieved gear cutters – Metric module
BS 2519 Pt 1	1976	Glossary for gears – Geometrical definitions
BS 2519 Pt 2	1976	Glossary for gears – Notation (symbols for geometrical data for use in gear rotation)
BS 2697	1976	Specification for rack type gear cutters
BS 3027	1968	Specification for dimensions of worm gear units
BS 3696 Pt 1	1984	Specification for master gears – Spur and helical gears (metric module)
BS 4517	1984	Dimensions of spur and helical geared motor units (metric series)
BS 4582 Pt 1	1984	Fine pitch gears (metric module) – Involute spur and helical gears
BS 4582 Pt 2	1986	Fine pitch gears (metric module) – Hobs and cutters
BS 5221	1987	Specifications for general purpose, metric module gear hobs
BS 5246	1984	Specifications for pinion type cutters for spur gears – 1 to 8 metric module
BS 6168	1987	Specification for nonmetallic spur gears

NOTE:
Standards available from: ANSI, 1430 Broadway, New York, NY 10018; or BSI, Linford Wood, Milton Keynes MK146LE, United Kingdom

1.3.2 Symbols

Gear parameters are defined by a set of standardized symbols that are defined in JIS B 0121 (1983). These are reproduced in **Table 1-3**.

The JIS symbols are consistent with the equations given in this text and are consistent with JIS standards. Most differ from typical American symbols, which can be confusing to the first time metric user. To assist, **Table 1-4** is offered as a cross list.

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Table 1-3A The Linear Dimensions and Circular Dimensions

Terms	Symbols
Center Distance	a
Circular Pitch (General)	p
Standard Circular Pitch	p
Radial Circular Pitch	p_t
Circular Pitch Perpendicular to Tooth	p_n
Axial Pitch	p_x
Normal Pitch	p_b
Radial Normal Pitch	p_{bt}
Normal Pitch Perpendicular to Tooth	p_{bn}
Whole Depth	h
Addendum	h_a
Dedendum	h_f
Caliper Tooth Height	$\bar{h}$
Working Depth	h' h_w
Tooth Thickness (General)	s
Circular Tooth Thickness	s
Base Circle Circular Tooth Thickness	s_b
Chordal Tooth Thickness	$\bar{s}$
Span Measurement	W
Root Width	e
Top Clearance	c
Circular Backlash	j_t
Normal Backlash	j_n
Blank Width	b
Working Face Width	b' b_w

Terms	Symbols
Lead	p_z
Contact Length	g_a
Contact Length of Approach	g_f
Contact Length of Recess	g_a
Contact Length of Overlap	g_b
Diameter (General)	d
Standard Pitch Diameter	d
Working Pitch Diameter	d' d_w
Outside Diameter	d_a
Base Diameter	d_b
Root Diameter	d_f
Radius (General)	r
Standard Pitch Radius	r
Working Pitch Radius	r' r_w
Outside Radius	r_a
Base Radius	r_b
Root Radius	r_f
Radius of Curvature	ρ
Cone Distance (General)	R
Cone Distance	R_e
Mean Cone Distance	R_m
Inner Cone Distance	R_i
Back Cone Distance	R_v
Mounting Distance	*A
Offset Distance	*E

* These terms and symbols are specific to JIS Standard

Table 1-3B Angular Dimensions

Terms	Symbols
Pressure Angle (General)	α
Standard Pressure Angle	α
Working Pressure Angle	α' or α_w
Cutter Pressure Angle	α_0
Radial Pressure Angle	α_t
Pressure Angle Normal to Tooth	α_n
Axial Pressure Angle	α_x
Helix Angle (General)	β
Standard Pitch Cylinder Helix Angle	β
Outside Cylinder Helix Angle	β_a
Base Cylinder Helix Angle	β_b
Lead Angle (General)	γ
Standard Pitch Cylinder Lead Angle	γ
Outside Cylinder Lead Angle	γ_a
Base Cylinder Lead Angle	γ_b

Terms	Symbols
Shaft Angle	Σ
Cone Angle (General)	δ
Pitch Cone Angle	δ
Outside Cone Angle	δ_a
Root Cone Angle	δ_f
Addendum Angle	θ_a
Dedendum Angle	θ_f
Radial Contact Angle	ϕ_a
Overlap Contact Angle	ϕ_β
Overall Contact Angle	ϕ_r
Angular Pitch of Crown Gear	τ
Involute Function	inv α

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Table 1-3C Size Number, Ratios & Speed Terms

Terms	Symbols	Terms	Symbols
Number of Teeth	z	Contact Ratio	ε
Equivalent Spur Gear Number of Teeth	z_v	Radial Contact Ratio	ε_α
Number of Threads in Worm	z_w	Overlap Contact Ratio	ε_β
Number of Teeth in Pinion	z_l	Total Contact Ratio	ε_γ
Number of Teeth Ratio	u	Specific Slide	*σ
Speed Ratio	i	Angular Speed	ω
Module	m	Linear or Tangential Speed	v
Radial Module	m_t	Revolutions per Minute	n
Normal Module	m_n	Coefficient of Profile Shift	x
Axial Module	m_x	Coefficient of Center Distance Increase	y

NOTE: The term "Radial" is used to denote parameters in the plane of rotation perpendicular to the axis.

Table 1-3D Accuracy / Error Terms

Terms	Symbols	Terms	Symbols
Single Pitch Error	f_{pt}	Normal Pitch Error	f_{pb}
Pitch Variation	*f_u or f_{pu}	Involute Profile Error	f_f
Partial Accumulating Error (Over Integral k teeth)	F_{pk}	Runout Error	F_r
Total Accumulated Pitch Error	F_p	Lead Error	F_b

*These terms and symbols are specific to JIS Standards

Table 1-4 Equivalence Of American And Japanese Symbols

American Symbol	Japanese Symbol	Nomenclature
B	j	backlash, linear measure along pitch circle
B_{LA}	j_t	backlash, linear measure along line-of-action
B_a	j_n	backlash in arc minutes
C	a	center distance
ΔC	Δa	change in center distance
C_o	a_w	operating center distance
C_{std}		standard center distance
D	d	pitch diameter
D_b	d_b	base circle diameter
D_o	d_a	outside diameter
D_R	d_f	root diameter
F	b	face width
K	K	factor, general
L	L	length, general; also lead of worm
M		measurement over-pins
N	z	number of teeth, usually gear
N_c	z_c	critical number of teeth for no undercutting

American Symbol	Japanese Symbol	Nomenclature
N_v	z_v	virtual number of teeth for helical gear
P_d	p	diametral pitch
P_{dn}	p_n	normal diametral pitch
P_t		horsepower, transmitted
R	r	pitch radius, gear or general use
R_b	r_b	base circle radius, gear
R_o	r_a	outside radius, gear
R_T		testing radius
T	s	tooth thickness, gear
W_b		beam tooth strength
Y		Lewis factor, diametral pitch
Z	i	mesh velocity ratio
a	h_a	addendum
b	h_f	dedendum
c	c	clearance
d	d	pitch diameter, pinion
d_w	d_p	pin diameter, for over-pins measurement
e		eccentricity
h_k	h_w	working depth

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Table 1-4 (Cont.) Equivalence of American and Japanese Symbols

American Symbol	Japanese Symbol	Nomenclature
h_t	h	whole depth
m_p	e	contact ratio
n	z_1	number of teeth, pinion
n_w	z_w	number of threads in worm
p_a	p_x	axial pitch
p_b	p_b	base pitch
p_c	p	circular pitch
p_{cn}	p_n	normal circular pitch
r	r	pitch radius, pinion
r_b	r_b	base circle radius, pinion
r_f	r_f	fillet radius
r_o	r_a	outside radius, pinion
t	s	tooth thickness, and for general use, for tolerance

American Symbol	Japanese Symbol	Nomenclature
y_c		Lewis factor, circular pitch
γ	δ	pitch angle, bevel gear
θ		rotation angle, general
λ	γ	lead angle, worm gearing
μ		mean value
ν		gear stage velocity ratio
ϕ	α	pressure angle
ϕ_o	α_w	operating pressure angle
Ψ	β	helix angle (b_b=base helix angle; b_w= operating helix angle)
ω		angular velocity
inv ϕ	inv α	involute function

1.3.3 Terminology

Terms used in metric gearing are identical or are parallel to those used for inch gearing. The one major exception is that metric gears are based upon the module, which for reference may be considered as the inversion of a metric unit diametral pitch.

Terminology will be appropriately introduced and defined throughout the text.

There are some terminology difficulties with a few of the descriptive words used by the Japanese JIS standards when translated into English. One particular example is the Japanese use of the term "radial" to describe measures such as what Americans term circular pitch. This also crops up with contact ratio. What Americans refer to as contact ratio in the plane of rotation, the Japanese equivalent is called "radial contact ratio". This can be both confusing and annoying. Therefore, since this technical section is being used outside Japan, and the American term is more realistically descriptive, in this text we will use the American term "circular" where it is meaningful. However, the applicable Japanese symbol will be used. Other examples of giving preference to the American terminology will be identified where it occurs.

1.3.4 Conversion

For those wishing to ease themselves into working with metric gears by looking at them in terms of familiar inch gearing relationships and mathematics, **Table 1-5** is offered as a means to make a quick comparison.

Table 1-5 Spur Gear Design Formulas

To Obtain	From Known	Use This Formula*
Pitch Diameter	Module	$D = mN$
Circular Pitch	Module	$p_c = m\pi = \frac{D}{N}\pi$
Module	Diametral Pitch	$m = \frac{25.4}{P_d}$
Number of Teeth	Module and Pitch Diameter	$N = \frac{D}{m}$
Addendum	Module	$a = m$

* All linear dimensions in millimeters
Symbols per **Table 1-4**

Continued on the next page

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Table 1-5 (Cont.) Spur Gear Design Formulas

To Obtain	From Known	Use This Formula*
Dedendum	Module	$b = 1.25m$
Outside Diameter	Module and Pitch Diameter or Number of Teeth	$D_o = D + 2m = m(N + 2)$
Root Diameter	Pitch Diameter and Module	$D_R = D - 2.5m$
Base Circle Diameter	Pitch Diameter and Pressure Angle	$D_b = D \cos\phi$
Base Pitch	Module and Pressure Angle	$p_b = m\pi\cos\phi$
Tooth Thickness at Standard Pitch Diameter	Module	$T_{std} = \frac{\pi}{2}m$
Center Distance	Module and Number of Teeth	$C = \frac{m(N_1 + N_2)}{2}$
Contact Ratio	Outside Radii, Base Circle Radii, Center Distance, Pressure Angle	$m_p = \frac{\sqrt{_1R_o - {_1R_b}} + \sqrt{_2R_o - {_2R_b}} - C\sin\phi}{m\pi\cos\phi}$
Backlash (linear)	Change in Center Distance	$B = 2(\Delta C)\tan\phi$
Backlash (linear)	Change in Tooth Thickness	$B = \Delta T$
Backlash (linear) Along Line-of-action	Linear Backlash Along Pitch Circle	$B_{LA} = B\cos\phi$
Backlash, Angular	Linear Backlash	$B_a = 6880\frac{B}{D}$ (arc minutes)
Min. No. of Teeth for No Undercutting	Pressure Angle	$N_c = \frac{2}{\sin^2\phi}$

*All linear dimensions in millimeters
Symbols per **Table 1-4**

SECTION 2 INTRODUCTION TO GEAR TECHNOLOGY

This section presents a technical coverage of gear fundamentals. It is intended as a broad coverage written in a manner that is easy to follow and to understand by anyone interested in knowing how gear systems function. Since gearing involves specialty components, it is expected that not all designers and engineers possess or have been exposed to every aspect of this subject. However, for proper use of gear components and design of gear systems it is essential to have a minimum understanding of gear basics and a reference source for details.

For those to whom this is their first encounter with gear components, it is suggested this technical treatise be read in the order presented so as to obtain a logical development of the subject. Subsequently, and for those already familiar with gears, this material can be used selectively in random access as a design reference.

2.1 Basic Geometry Of Spur Gears

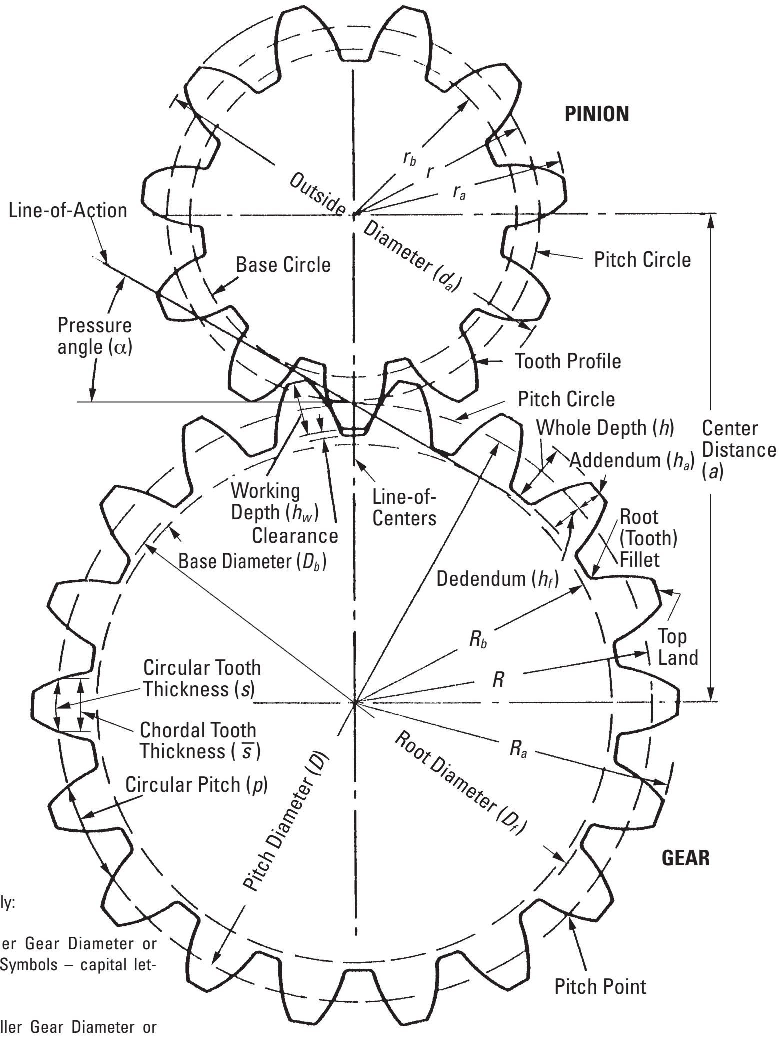
The fundamentals of gearing are illustrated through the spur gear tooth, both because it is the simplest, and hence most comprehensible, and because it is the form most widely used, particularly for instruments and control systems.

The basic geometry and nomenclature of a spur gear mesh is shown in **Figure 2-1**. The essential features of a gear mesh are:

1. Center distance.

2. The pitch circle diameters (or pitch diameters).
3. Size of teeth (or module).
4. Number of teeth.
5. Pressure angle of the contacting involutes.

Details of these items along with their interdependence and definitions are covered in subsequent paragraphs.



Generally:

Larger Gear Diameter or Radius Symbols – capital letters

Smaller Gear Diameter or Radius Symbols – lower case letters

Fig. 2-1 Basic Gear Geometry

2.2 The Law Of Gearing

A primary requirement of gears is the constancy of angular velocities or proportionality of position transmission. Precision instruments require positioning fidelity. High-speed and/or high-power gear trains also require transmission at constant angular velocities in order to avoid severe dynamic problems.

Constant velocity (i.e., constant ratio) motion transmission is defined as "conjugate action" of the gear tooth profiles. A geometric relationship can be derived (2, 12)* for the form of the tooth profiles to provide conjugate action, which is summarized as the Law of Gearing as follows:

"A common normal to the tooth profiles at their point of contact must, in all positions of the contacting teeth, pass through a fixed point on the line-of-centers called the pitch point."

Any two curves or profiles engaging each other and satisfying the law of gearing are conjugate curves.

2.3 The Involute Curve

There is almost an infinite number of curves that can be developed to satisfy the law of gearing, and many different curve forms have been tried in the past. Modern gearing (except for clock gears) is based on involute teeth. This is due to three major advantages of the involute curve:

1. Conjugate action is independent of changes in center distance.
2. The form of the basic rack tooth is straight-sided, and therefore is relatively simple and can be accurately made; as a generating tool it imparts high accuracy to the cut gear tooth.
3. One cutter can generate all gear teeth numbers of the same pitch.

The involute curve is most easily understood as the trace of a point at the end of a taut string that unwinds from a cylinder. It is imagined that a point on a string, which is pulled taut in a fixed direction, projects its trace onto a plane that rotates with the base circle. See **Figure 2-2**. The base cylinder, or base circle as referred to in gear literature, fully defines the form of the involute and in a gear it is an inherent parameter, though invisible.

The development and action of mating teeth can be visualized by imagining the taut string as being unwound from one base circle and wound on to the other, as shown in **Figure 2-3a**. Thus, a single point on the string simultaneously traces an involute on each base circle's rotating plane. This pair of involutes is conjugate, since at all points of contact the common normal is the common tangent which passes through a fixed point on the line-of-centers. If a second winding/unwinding taut string is wound around the base circles in the opposite direction, **Figure 2-3b**, oppositely curved involutes are generated which can accommodate motion reversal. When the involute pairs are properly spaced, the result is the involute gear tooth, **Figure 2-3c**.

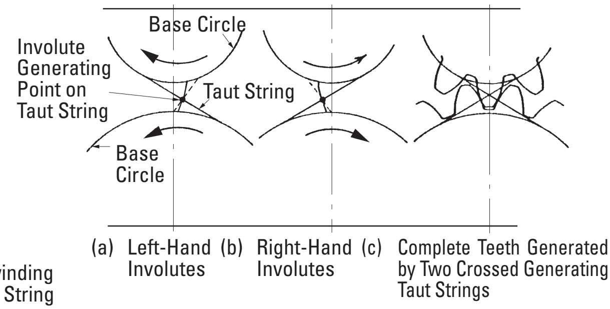


Fig. 2-3 Generation and Action of Gear Teeth

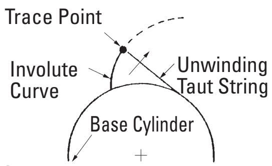


Fig. 2-2 Generation of an Involute by a Taut String

*Numbers in parentheses refer to references at end of text.

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2.4 Pitch Circles

Referring to **Figure 2-4**, the tangent to the two base circles is the line of contact, or line-of-action in gear vernacular. Where this line crosses the line-of-centers establishes the pitch point, *P*. This in turn sets the size of the pitch circles, or as commonly called, the pitch diameters. The ratio of the pitch diameters gives the velocity ratio:

Velocity ratio of gear 2 to gear 1 is:

$$i = \frac{d_1}{d_2} \qquad \textbf{(2-1)}$$

2.5 Pitch And Module

Essential to prescribing gear geometry is the size, or spacing of the teeth along the pitch circle. This is termed pitch, and there are two basic forms.

Circular pitch — A naturally conceived linear measure along the pitch circle of the tooth spacing. Referring to **Figure 2-5**, it is the linear distance (measured along the pitch circle arc) between corresponding points of adjacent teeth. It is equal to the pitch-circle circumference divided by the number of teeth:

$$p = \text{circular pitch} = \frac{\text{pitch circle circumference}}{\text{number of teeth}} = \frac{\pi d}{z} \qquad \textbf{(2-2)}$$

Module — Metric gearing uses the quantity module m in place of the American inch unit, diametral pitch. The module is the length of pitch diameter per tooth. Thus:

$$m = \frac{d}{z} \qquad \textbf{(2-3)}$$

Relation of pitches: From the geometry that defines the two pitches, it can be shown that module and circular pitch are related by the expression:

$$\frac{p}{m} = \pi \qquad \textbf{(2-4)}$$

This relationship is simple to remember and permits an easy transformation from one to the other.

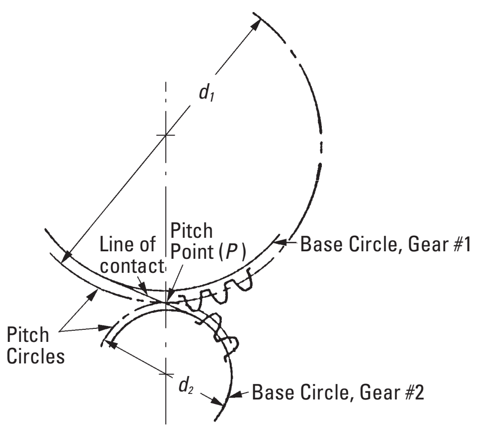


Fig. 2-4 Definition of Pitch Circle and Pitch Point

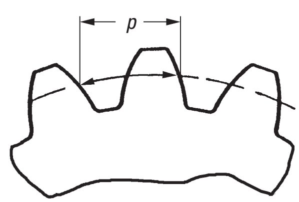


Fig. 2-5 Definition of Circular Pitch

Diametral pitch (P_d)is widely used in England and America to represent the tooth size. The relation between diametral pitch and module is as follows:

$$m = \frac{25.4}{P_d} \tag{2-5}$$

2.6 Module Sizes And Standards

Module m represents the size of involute gear tooth. The unit of module is mm. Module is converted to circular pitch *p*, by the factor π.

$$p = \pi m \tag{2-6}$$

Table 2-1 is extracted from JIS B 1701-[1973] which defines the tooth profile and dimensions of involute gears. It divides the standard module into three series. **Figure 2-6** shows the comparative size of various rack teeth.

Table 2-1 Standard Values of Module **unit: mm**

Series 1	Series 2	Series 3	Series 1	Series 2	Series 3
0.1				3.5	
	0.15				3.75
0.2			4		
	0.25			4.5	
0.3			5		
	0.35			5.5	
0.4			6		
	0.45				6.5
0.5				7	
	0.55		8		
0.6				9	
		0.65	10		
	0.7			11	
	0.75		12		
0.8				14	
	0.9		16		
1				18	
1.25			20		
1.5				22	
	1.75		25		
2				28	
	2.25		32		
2.5				36	
	2.75		40		
3				45	
		3.25	50		

Note: The preferred choices are in the series order beginning with 1.

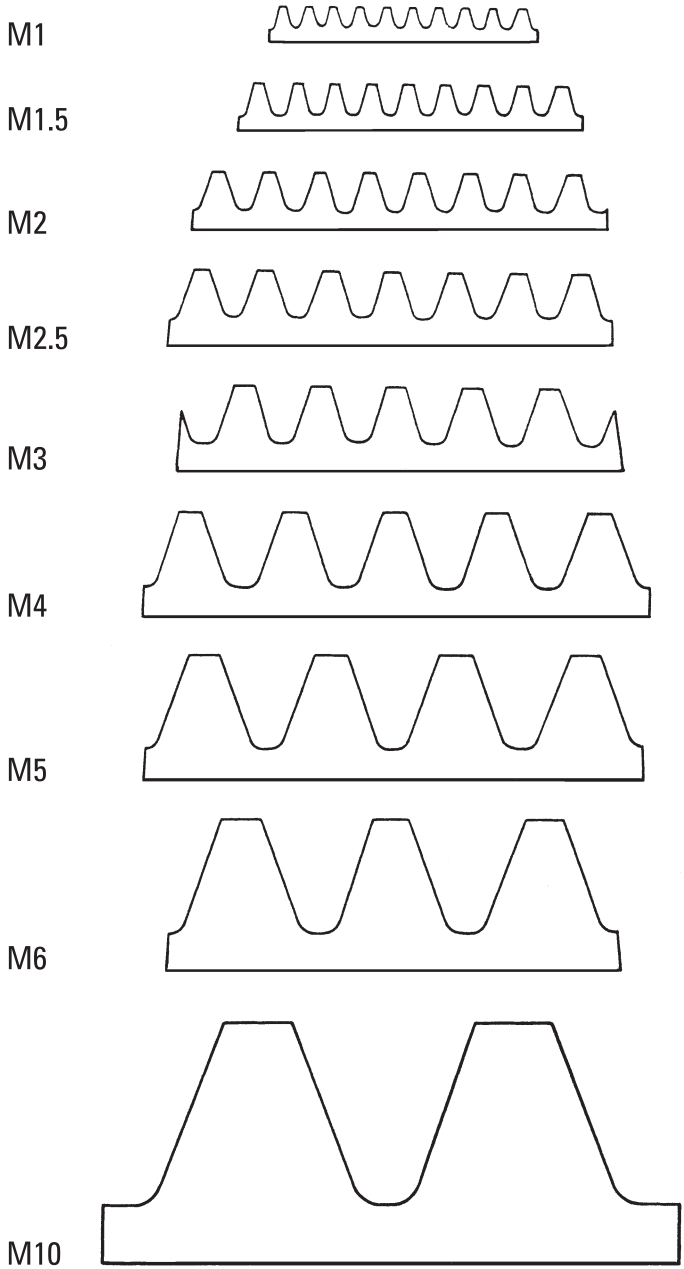


Fig. 2-6 Comparative Size of Various Rack Teeth

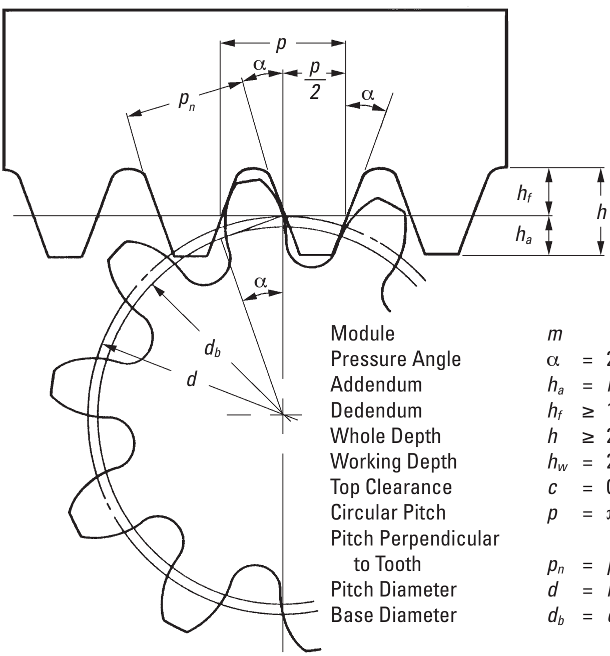
Circular pitch, p, is also used to represent tooth size when a special desired spacing is wanted, such as to get an integral feed in a mechanism. In this case, a circular pitch is chosen that is an integer or a special fractional value. This is often the choice in designing position control systems. Another particular usage is the drive of printing plates to provide a given feed.

Most involute gear teeth have the standard whole depth and a standard pressure angle α = 20°. **Figure 2-7** shows the tooth profile of a whole depth standard rack tooth and mating gear. It has an addendum of $h_a = 1m$ and dedendum $h_f \geq 1.25m$. If tooth depth is shorter than whole depth it is called a "stub" tooth; and if deeper than whole depth it is a "high" depth tooth.

The most widely used stub tooth has an addendum $h_a = 0.8m$ and dedendum $h_f = 1m$. Stub teeth have more strength than a whole depth gear, but contact ratio is reduced. On the other hand, a high depth tooth can increase contact ratio, but weakens the tooth.

In the standard involute gear, pitch p times the number of teeth becomes the length of pitch circle:

$$\left.\begin{array}{l} d\pi = \pi mz \\ \text{Pitch diameter } (d) \text{ is then:} \\ d = mz \end{array}\right\} \qquad \textbf{(2-7)}$$



Module	m		
Pressure Angle	α	=	20°
Addendum	h_a	=	m
Dedendum	h_f	≥	$1.25m$
Whole Depth	h	≥	$2.25m$
Working Depth	h_w	=	$2.00m$
Top Clearance	c	=	$0.25m$
Circular Pitch	p	=	πm
Pitch Perpendicular to Tooth	p_n	=	$p \cos\alpha$
Pitch Diameter	d	=	mz
Base Diameter	d_b	=	$d \cos\alpha$

Fig. 2-7 The Tooth Profile and Dimension of Standard Rack

Metric Module and Inch Gear Preferences: Because there is no direct equivalence between the pitches in metric and inch systems, it is not possible to make direct substitutions. Further, there are preferred modules in the metric system. As an aid in using metric gears, **Table 2-2** presents nearest equivalents for both systems, with the preferred sizes in bold type.

Table 2-2 Metric/American Gear Equivalents

Diametral Pitch, *P*	Module, *m*	Circular Pitch		Circular Tooth Thickness		Addendum	
		in	mm	in	mm	in	mm
203.2000	0.125	0.0155	0.393	0.0077	0.196	0.0049	0.125
200	0.12700	0.0157	0.399	0.0079	0.199	0.0050	0.127
180	0.14111	0.0175	0.443	0.0087	0.222	0.0056	0.141
169.333	0.15	0.0186	0.471	0.0093	0.236	0.0059	0.150
150	0.16933	0.0209	0.532	0.0105	0.266	0.0067	0.169
127.000	**0.2**	0.0247	0.628	0.0124	0.314	0.0079	0.200
125	0.20320	0.0251	0.638	0.0126	0.319	0.0080	0.203
120	0.21167	0.0262	0.665	0.0131	0.332	0.0083	0.212
101.600	**0.25**	0.0309	0.785	0.0155	0.393	0.0098	0.250
96	0.26458	0.0327	0.831	0.0164	0.416	0.0104	0.265
92.3636	0.275	0.0340	0.864	0.0170	0.432	0.0108	0.275
84.6667	**0.3**	0.0371	0.942	0.0186	0.471	0.0118	0.300
80	0.31750	0.0393	0.997	0.0196	0.499	0.0125	0.318
78.1538	0.325	0.0402	1.021	0.0201	0.511	0.0128	0.325
72.5714	0.35	0.0433	1.100	0.0216	0.550	0.0138	0.350
72	0.35278	0.0436	1.108	0.0218	0.554	0.0139	0.353
67.733	0.375	0.0464	1.178	0.0232	0.589	0.0148	0.375
64	0.39688	0.0491	1.247	0.0245	0.623	0.0156	0.397
63.500	**0.4**	0.0495	1.257	0.0247	0.628	0.0157	0.400
50.800	**0.5**	0.0618	1.571	0.0309	0.785	0.0197	0.500
50	0.50800	0.0628	1.596	0.0314	0.798	0.0200	0.508
48	0.52917	0.0655	1.662	0.0327	0.831	0.0208	0.529
44	0.57727	0.0714	1.814	0.0357	0.907	0.0227	0.577
42.333	**0.6**	0.0742	1.885	0.0371	0.942	0.0236	0.600
40	0.63500	0.0785	1.995	0.0393	0.997	0.0250	0.635
36.2857	**0.7**	0.0866	2.199	0.0433	1.100	0.0276	0.700
36	0.70556	0.0873	2.217	0.0436	1.108	0.0278	0.706
33.8667	0.75	0.0928	2.356	0.0464	1.178	0.0295	0.750
32	0.79375	0.0982	2.494	0.0491	1.247	0.0313	0.794
31.7500	**0.8**	0.0989	2.513	0.0495	1.257	0.0315	0.800
30	0.84667	0.1047	2.660	0.0524	1.330	0.0333	0.847
28.2222	0.9	0.1113	2.827	0.0557	1.414	0.0354	0.900
28	0.90714	0.1122	2.850	0.0561	1.425	0.0357	0.907
25.4000	**1**	0.1237	3.142	0.0618	1.571	0.0394	1.000
24	1.0583	0.1309	3.325	0.0654	1.662	0.0417	1.058
22	1.1545	0.1428	3.627	0.0714	1.813	0.0455	1.155
20.3200	**1.25**	0.1546	3.927	0.0773	1.963	0.0492	1.250
20	1.2700	0.1571	3.990	0.0785	1.995	0.0500	1.270
18	1.4111	0.1745	4.433	0.0873	2.217	0.0556	1.411
16.9333	**1.5**	0.1855	4.712	0.0928	2.356	0.0591	1.500
16	1.5875	0.1963	4.987	0.0982	2.494	0.0625	1.588
15	1.6933	0.2094	5.320	0.1047	2.660	0.0667	1.693
14.5143	1.75	0.2164	5.498	0.1082	2.749	0.0689	1.750
14	1.8143	0.2244	5.700	0.1122	2.850	0.0714	1.814
13	1.9538	0.2417	6.138	0.1208	3.069	0.0769	1.954
12.7000	**2**	0.2474	6.283	0.1237	3.142	0.0787	2.000
12	2.1167	0.2618	6.650	0.1309	3.325	0.0833	2.117
11.2889	2.25	0.2783	7.069	0.1391	3.534	0.0886	2.250
11	2.3091	0.2856	7.254	0.1428	3.627	0.0909	2.309
10.1600	**2.50**	0.3092	7.854	0.1546	3.927	0.0984	2.500
10	2.5400	0.3142	7.980	0.1571	3.990	0.1000	2.540

NOTE: Bold face diametral pitches and modules designate preferred values.

Continued on the next page

Table 2-2 (Cont.) Metric/American Gear Equivalents

Diametral Pitch, *P*	Module, *m*	Circular Pitch		Circular Tooth Thickness		Addendum	
		in	mm	in	mm	in	mm
9.2364	2.75	0.3401	8.639	0.1701	4.320	0.1083	2.750
9	2.8222	0.3491	8.866	0.1745	4.433	0.1111	2.822
8.4667	**3**	0.3711	9.425	0.1855	4.712	0.1181	3.000
8	3.1750	0.3927	9.975	0.1963	4.987	0.1250	3.175
7.8154	3.25	0.4020	10.210	0.2010	5.105	0.1280	3.250
7.2571	3.5	0.4329	10.996	0.2164	5.498	0.1378	3.500
7	3.6286	0.4488	11.400	0.2244	5.700	0.1429	3.629
6.7733	3.75	0.4638	11.781	0.2319	5.890	0.1476	3.750
6.3500	**4**	0.4947	12.566	0.2474	6.283	0.1575	4.000
6	4.2333	0.5236	13.299	0.2618	6.650	0.1667	4.233
5.6444	4.5	0.5566	14.137	0.2783	7.069	0.1772	4.500
5.3474	4.75	0.5875	14.923	0.2938	7.461	0.1870	4.750
5.0800	**5**	0.6184	15.708	0.3092	7.854	0.1969	5.000
5	5.0800	0.6283	15.959	0.3142	7.980	0.2000	5.080
4.6182	5.5000	0.6803	17.279	0.3401	8.639	0.2165	5.500
4.2333	**6**	0.7421	18.850	0.3711	9.425	0.2362	6.000
4	6.3500	0.7854	19.949	0.3927	9.975	0.2500	6.350
3.9077	6.5000	0.8040	20.420	0.4020	10.210	0.2559	6.500
3.6286	7	0.8658	21.991	0.4329	10.996	0.2756	7.000
3.5000	7.2571	0.8976	22.799	0.4488	11.399	0.2857	7.257
3.1750	**8**	0.9895	25.133	0.4947	12.566	0.3150	8.000
3.1416	8.0851	1.0000	25.400	0.5000	12.700	0.3183	8.085
3	8.4667	1.0472	26.599	0.5236	13.299	0.3333	8.467
2.8222	9	1.1132	28.274	0.5566	14.137	0.3543	9.000
2.5400	**10**	1.2368	31.416	0.6184	15.708	0.3937	10.000
2.5000	10.160	1.2566	31.919	0.6283	15.959	0.4000	10.160
2.3091	11	1.3605	34.558	0.6803	17.279	0.4331	11.000
2.1167	**12**	1.4842	37.699	0.7421	18.850	0.4724	12.000
2	12.700	1.5708	39.898	0.7854	19.949	0.5000	12.700
1.8143	14	1.7316	43.982	0.8658	21.991	0.5512	14.000
1.5875	**16**	1.9790	50.265	0.9895	25.133	0.6299	16.000
1.5000	16.933	2.0944	53.198	1.0472	26.599	0.6667	16.933
1.4111	18	2.2263	56.549	1.1132	28.274	0.7087	18.000
1.2700	**20**	2.4737	62.832	1.2368	31.416	0.7874	20.000
1.1545	22	2.7211	69.115	1.3605	34.558	0.8661	22.000
1.0583	24	2.9684	75.398	1.4842	37.699	0.9449	24.000
1.0160	**25**	3.0921	78.540	1.5461	39.270	0.9843	25.000
1	25.400	3.1416	79.796	1.5708	39.898	1.0000	25.400
0.9407	27	3.3395	84.823	1.6697	42.412	1.0630	27.000
0.9071	28	3.4632	87.965	1.7316	43.982	1.1024	28.000
0.8467	30	3.7105	94.248	1.8553	47.124	1.1811	30.000
0.7938	**32**	3.9579	100.531	1.9790	50.265	1.2598	32.000
0.7697	33	4.0816	103.673	2.0408	51.836	1.2992	33.000
0.7500	33.867	4.1888	106.395	2.0944	53.198	1.3333	33.867
0.7056	36	4.4527	113.097	2.2263	56.549	1.4173	36.000
0.6513	39	4.8237	122.522	2.4119	61.261	1.5354	39.000
0.6350	**40**	4.9474	125.664	2.4737	62.832	1.5748	40.000
0.6048	42	5.1948	131.947	2.5974	65.973	1.6535	42.000
0.5644	45	5.5658	141.372	2.7829	70.686	1.7717	45.000
0.5080	**50**	6.1842	157.080	3.0921	78.540	1.9685	50.000
0.5000	50.800	6.2832	159.593	3.1416	79.796	2.0000	50.800

NOTE: Bold face diametral pitches and modules designate preferred values.

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2.7 Gear Types And Axial Arrangements

In accordance with the orientation of axes, there are three categories of gears:

1. **Parallel Axes Gears**
2. **Intersecting Axes Gears**
3. **Nonparallel and Nonintersecting Axes Gears**

Spur and helical gears are the parallel axes gears. Bevel gears are the intersecting axes gears. Screw or crossed helical, worm and hypoid gears handle the third category. **Table 2-3** lists the gear types per axes orientation.

Also, included in **Table 2-3** is the theoretical efficiency range of the various gear types. These figures do not include bearing and lubricant losses. Also, they assume ideal mounting in regard to axis orientation and center distance. Inclusion of these realistic considerations will downgrade the efficiency numbers.

Table 2-3 Types of Gears and Their Categories

Categories of Gears	Types of Gears	Efficiency (%)
Parallel Axes Gears	Spur Gear Spur Rack Internal Gear Helical Gear Helical Rack Double Helical Gear	98 ... 99.5
Intersecting Axes Gears	Straight Bevel Gear Spiral Bevel Gear Zerol Gear	98 ... 99
Nonparallel and Nonintersecting Axes Gears	Worm Gear Screw Gear Hypoid Gear	30 ... 90 70 ... 95 96 ... 98

2.7.1 Parallel Axes Gears

1. Spur Gear

This is a cylindrical shaped gear in which the teeth are parallel to the axis. It has the largest applications and, also, it is the easiest to manufacture.

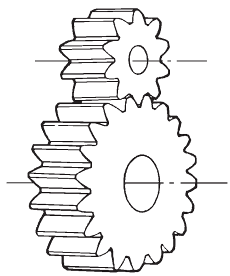

Fig. 2-8 Spur Gear

2. Spur Rack

This is a linear shaped gear which can mesh with a spur gear with any number of teeth. The spur rack is a portion of a spur gear with an infinite radius.

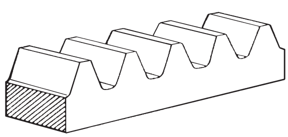

Fig. 2-9 Spur Rack

3. Internal Gear

This is a cylindrical shaped gear but with the teeth inside the circular ring. It can mesh with a spur gear. Internal gears are often used in planetary gear systems.

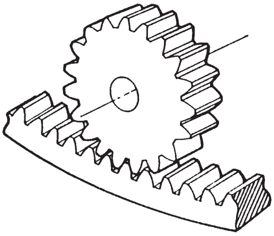

Fig. 2-10 Internal Gear and Spur Gear

4. Helical Gear

This is a cylindrical shaped gear with helicoid teeth. Helical gears can bear more load than spur gears, and work more quietly. They are widely used in industry. A disadvantage is the axial thrust force the helix form causes.

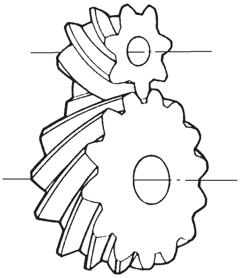

Fig. 2-11 Helical Gear

5. Helical Rack

This is a linear shaped gear which meshes with a helical gear. Again, it can be regarded as a portion of a helical gear with infinite radius.

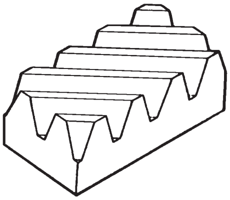

Fig. 2-12 Helical Rack

6. Double Helical Gear

This is a gear with both left-hand and right-hand helical teeth. The double helical form balances the inherent thrust forces.

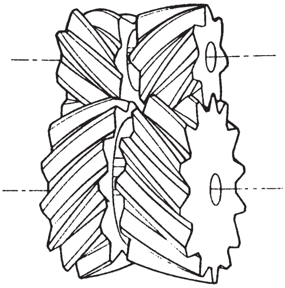

Fig. 2-13 Double Helical Gear

2.7.2 Intersecting Axes Gears

1. Straight Bevel Gear

This is a gear in which the teeth have tapered conical elements that have the same direction as the pitch cone base line (generatrix). The straight bevel gear is both the simplest to produce and the most widely applied in the bevel gear family.

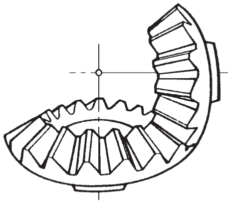

Fig. 2-14 Straight Bevel Gear

2. Spiral Bevel Gear

This is a bevel gear with a helical angle of spiral teeth. It is much more complex to manufacture, but offers a higher strength and lower noise.

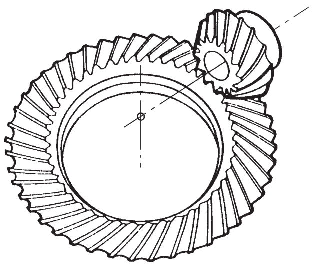

Fig. 2-15 Spiral Bevel Gear

3. Zerol Gear

Zerol gear is a special case of spiral bevel gear. It is a spiral bevel with zero degree of spiral angle tooth advance. It has the characteristics of both the straight and spiral bevel gears. The forces acting upon the tooth are the same as for a straight bevel gear.

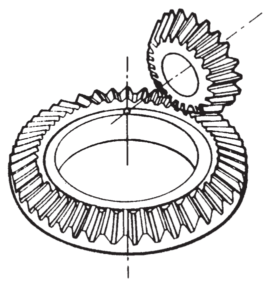

Fig. 2-16 Zerol Gear

2.7.3 Nonparallel And Nonintersecting Axes Gears

1. Worm And Worm Gear

Worm set is the name for a meshed worm and worm gear. The worm resembles a screw thread; and the mating worm gear a helical gear, except that it is made to envelope the worm as seen along the worm's axis. The outstanding feature is that the worm offers a very large gear ratio in a single mesh. However, transmission efficiency is very poor due to a great amount of sliding as the worm tooth engages with its mating worm gear tooth and forces rotation by pushing and sliding. With proper choices of materials and lubrication, wear can be contained and noise is reduced.

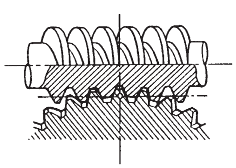

Fig. 2-17 Worm Gear

2. **Screw Gear (Crossed Helical Gear)**

Two helical gears of opposite helix angle will mesh if their axes are crossed. As separate gear components, they are merely conventional helical gears. Installation on crossed axes converts them to screw gears. They offer a simple means of gearing skew axes at any angle. Because they have point contact, their load carrying capacity is very limited.

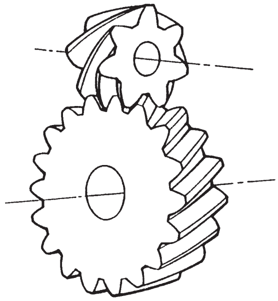

Fig. 2-18 Screw Gear

2.7.4 Other Special Gears

1. **Face Gear**

This is a pseudobevel gear that is limited to 90^0 intersecting axes. The face gear is a circular disc with a ring of teeth cut in its side face; hence the name face gear. Tooth elements are tapered towards its center. The mate is an ordinary spur gear. It offers no advantages over the standard bevel gear, except that it can be fabricated on an ordinary shaper gear generating machine.

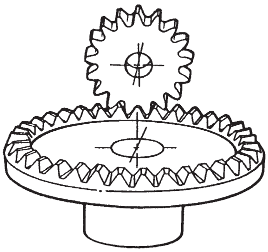

Fig. 2-19 Face Gear

2. **Double Enveloping Worm Gear**

This worm set uses a special worm shape in that it partially envelops the worm gear as viewed in the direction of the worm gear axis. Its big advantage over the standard worm is much higher load capacity. However, the worm gear is very complicated to design and produce, and sources for manufacture are few.

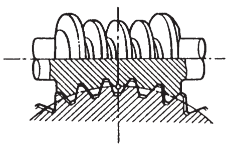

Fig. 2-20 Double Enveloping Worm Gear

3. **Hypoid Gear**

This is a deviation from a bevel gear that originated as a special development for the automobile industry. This permitted the drive to the rear axle to be nonintersecting, and thus allowed the auto body to be lowered. It looks very much like the spiral bevel gear. However, it is complicated to design and is the most difficult to produce on a bevel gear generator.

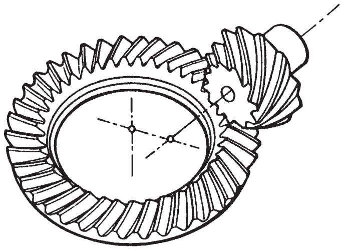

Fig. 2-21 Hypoid Gear

SECTION 3 DETAILS OF INVOLUTE GEARING

3.1 Pressure Angle

The pressure angle is defined as the angle between the line-of-action (common tangent to the base circles in **Figures 2-3** and **2-4**) and a perpendicular to the line-of-centers. See **Figure 3-1**. From the geometry of these figures, it is obvious that the pressure angle varies (slightly) as the center distance of a gear pair is altered. The base circle is related to the pressure angle and pitch diameter by the equation:

$$d_b = d \cos \alpha \qquad \textbf{(3-1)}$$

where d and α are the standard values, or alternately:

$$d_b = d' \cos \alpha' \qquad \textbf{(3-2)}$$

where d' and α' are the exact operating values.

The basic formula shows that the larger the pressure angle the smaller the base circle. Thus, for standard gears, 14.5° pressure angle gears have base circles much nearer to the roots of teeth than 20° gears. It is for this reason that 14.5° gears encounter greater undercutting problems than 20° gears. This is further elaborated on in **SECTION 4.3**.

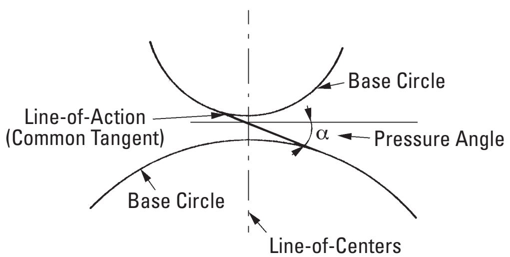

Fig. 3-1 Definition of Pressure Angle

3.2 Proper Meshing And Contact Ratio

Figure 3-2 shows a pair of standard gears meshing together. The contact point of the two involutes, as **Figure 3-2** shows, slides along the common tangent of the two base circles as rotation occurs. The common tangent is called the line-of-contact, or line-of-action.

A pair of gears can only mesh correctly if the pitches and the pressure angles are the same. Pitch comparison can be module (m), circular (p), or base(p_b).

That the pressure angles must be identical becomes obvious from the following equation for base pitch:

$$p_b = \pi \, m \cos \alpha \qquad \textbf{(3-3)}$$

Thus, if the pressure angles are different, the base pitches cannot be identical.

The length of the line-of-action is shown as *ab* in **Figure 3-2**.

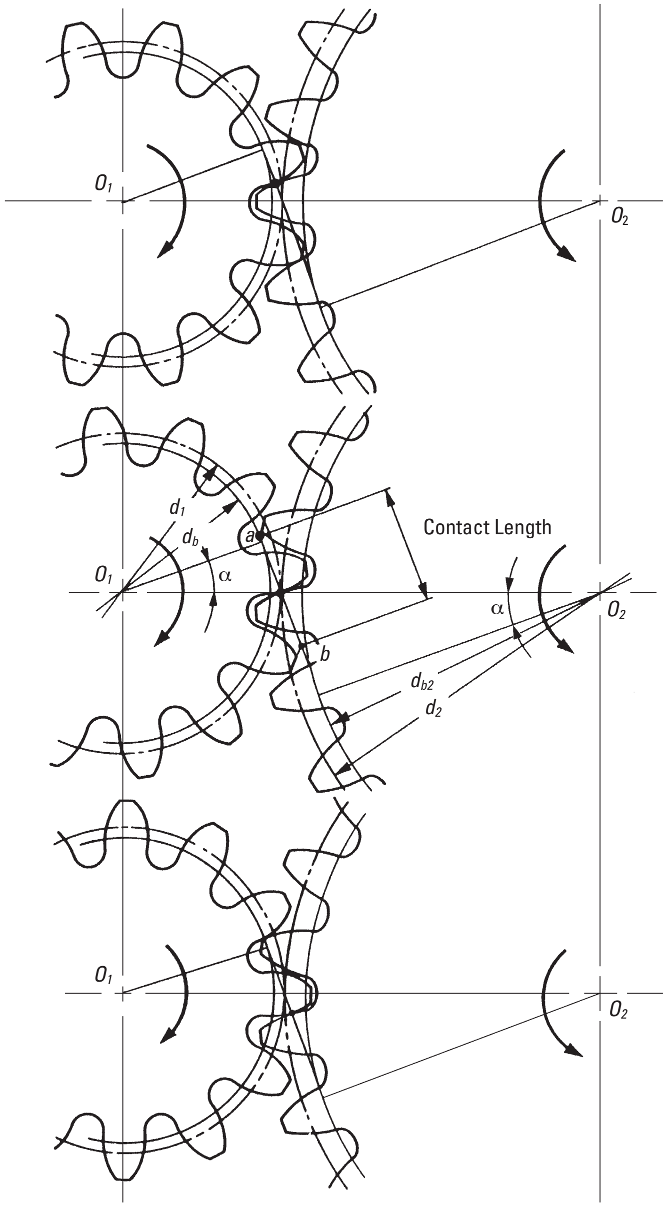

Fig. 3-2 The Meshing of Involute Gear

3.2.1 Contact Ratio

To assure smooth continuous tooth action, as one pair of teeth ceases contact a succeeding pair of teeth must already have come into engagement. It is desirable to have as much overlap as possible. The measure of this overlapping is the contact ratio. This is a ratio of the length of the line-of-action to the base pitch. **Figure 3-3** shows the geometry. The length-of-action is determined from the intersection of the line-of-action and the outside radii. For the simple case of a pair of spur gears, the ratio of the length-of-action to the base pitch is determined from:

$$\varepsilon_\gamma = \frac{\sqrt{(R_a^2 - R_b^2)} + \sqrt{(r_a^2 - r_b^2)} - a \sin\alpha}{p \cos\alpha} \qquad \textbf{(3-4)}$$

It is good practice to maintain a contact ratio of 1.2 or greater. Under no circumstances should the ratio drop below 1.1, calculated for all tolerances at their worst-case values.

A contact ratio between 1 and 2 means that part of the time two pairs of teeth are in contact and during the remaining time one pair is in contact. A ratio between 2 and 3 means 2 or 3 pairs of teeth are always in contact. Such a high contact ratio generally is not obtained with external spur gears, but can be developed in the meshing of an internal and external spur gear pair or specially designed nonstandard external spur gears.

More detail is presented about contact ratio, including calculation equations for specific gear types, in **SECTION 11**.

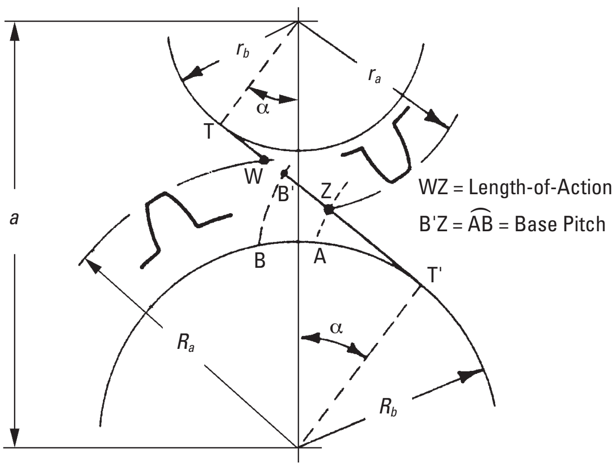

Fig. 3-3 Geometry of Contact Ratio

3.3 The Involute Function

Figure 3-4 shows an element of involute curve. The definition of involute curve is the curve traced by a point on a straight line which rolls without slipping on the circle.

The circle is called the base circle of the involutes. Two opposite hand involute curves meeting at a cusp form a gear tooth curve. We can see, from **Figure 3-4**, the length of base circle arc *ac* equals the length of straight line *bc*.

$$\tan\alpha = \frac{bc}{Oc} = \frac{r_b\theta}{r_b} = \theta\ (\text{radian}) \tag{3-5}$$

The θ in **Figure 3-4** can be expressed as inv α + α, then **Formula (3-5)** will become:

$$\text{inv}\,\alpha = \tan\alpha - \alpha \tag{3-6}$$

Function of α, or inv α, is known as involute function. Involute function is very important in gear design. Involute function values can be obtained from appropriate tables. With the center of the base circle *O* at the origin of a coordinate system, the involute curve can be expressed by values of *x* and *y* as follows:

$$\left.\begin{aligned} x &= r\cos(\text{inv}\,\alpha) = \frac{r_b}{\cos\alpha}\cos(\text{inv}\,\alpha) \\ y &= r\sin(\text{inv}\,\alpha) = \frac{r_b}{\cos\alpha}\sin(\text{inv}\,\alpha) \end{aligned}\right\} \tag{3-7}$$

where, $r = \dfrac{r_b}{\cos\alpha}$

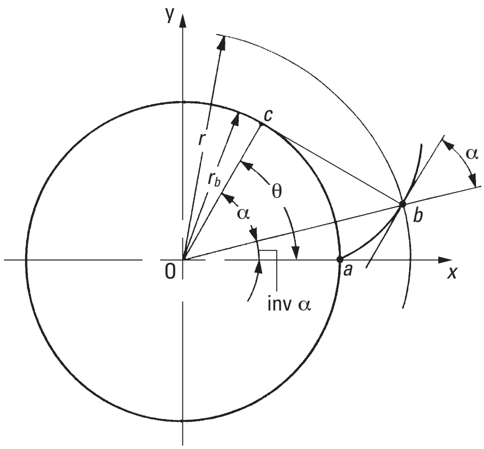

Fig. 3-4 The Involute Curve

SECTION 4 SPUR GEAR CALCULATIONS

4.1 Standard Spur Gear

Figure 4-1 shows the meshing of standard spur gears. The meshing of standard spur gears means pitch circles of two gears contact and roll with each other. The calculation formulas are in **Table 4-1**.

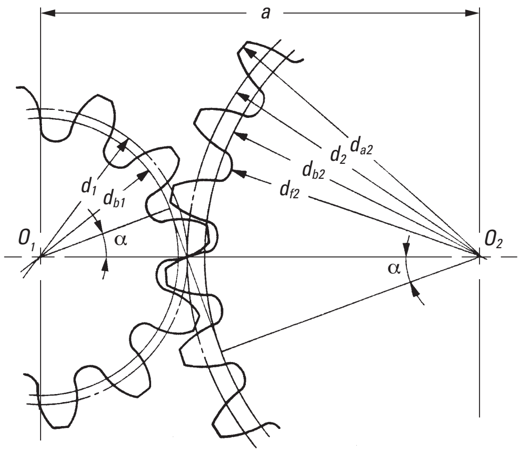

Fig. 4-1 The Meshing of Standard Spur Gears
($\alpha = 20°$, $z_1 = 12$, $z_2 = 24$, $x_1 = x_2 = 0$)

Table 4-1 The Calculation of Standard Spur Gears

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Module	m		3	
2	Pressure Angle	α		20°	
3	Number of Teeth	z_1, z_2 *		12	24
4	Center Distance	a	$\dfrac{(z_1 + z_2)m}{2}$ *	54.000	
5	Pitch Diameter	d	zm	36.000	72.000
6	Base Diameter	d_b	$d \cos \alpha$	33.829	67.658
7	Addendum	h_a	$1.00m$	3.000	
8	Dedendum	h_f	$1.25m$	3.750	
9	Outside Diameter	d_a	$d + 2m$	42.000	78.000
10	Root Diameter	d_f	$d - 2.5m$	28.500	64.500

*The subscripts 1 and 2 of z_1 and z_2 denote pinion and gear.

All calculated values in **Table 4-1** are based upon given module (m) and number of teeth (z_1 and z_2). If instead module (m), center distance (a) and speed ratio (i) are given, then the number of teeth, z_1 and z_2, would be calculated with the formulas as shown in **Table 4-2**.

I R T 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 A

Table 4-2 The Calculation of Teeth Number

No.	Item	Symbol	Formula		Example	
1	Module	m			3	
2	Center Distance	a			54.000	
3	Speed Ratio	i			0.8	
4	Sum of No. of Teeth	$z_1 + z_2$	$\frac{2a}{m}$		36	
5	Number of Teeth	$z_1\,,\ z_2$	$\frac{i\,(z_1 + z_2)}{i + 1}$	$\frac{(z_1 + z_2)}{i + 1}$	16	20

Note that the numbers of teeth probably will not be integer values by calculation with the formulas in **Table 4-2**. Then it is incumbent upon the designer to choose a set of integer numbers of teeth that are as close as possible to the theoretical values. This will likely result in both slightly changed gear ratio and center distance. Should the center distance be inviolable, it will then be necessary to resort to profile shifting. This will be discussed later in this section.

4.2 The Generating Of A Spur Gear

Involute gears can be readily generated by rack type cutters. The hob is in effect a rack cutter. Gear generation is also accomplished with gear type cutters using a shaper or planer machine.

Figure 4-2 illustrates how an involute gear tooth profile is generated. It shows how the pitch line of a rack cutter rolling on a pitch circle generates a spur gear.

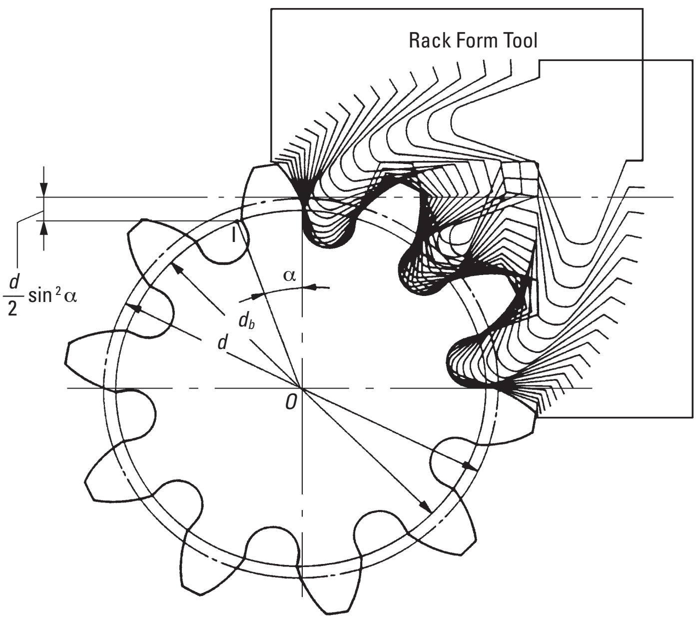

Fig. 4-2 The Generating of a Standard Spur Gear
($\alpha = 20°, z = 10, x = 0$)

4.3 Undercutting

From **Figure 4-3**, it can be seen that the maximum length of the line-of-contact is limited to the length of the common tangent. Any tooth addendum that extends beyond the tangent points (T and T') is not only useless, but interferes with the root fillet area of the mating tooth. This results in the typical undercut tooth, shown in **Figure 4-4**. The undercut not only weakens the tooth with a wasp-like waist, but also removes some of the useful involute adjacent to the base circle.

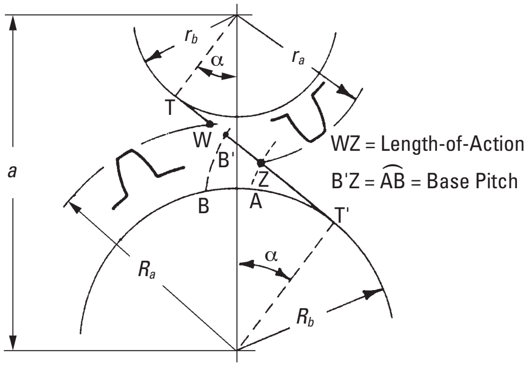

Fig. 4-3 Geometry of Contact Ratio

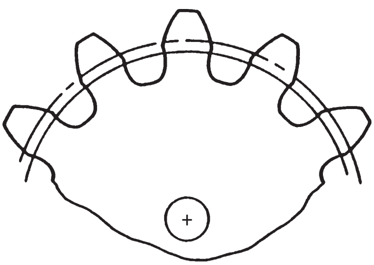

Fig. 4-4 Example of Undercut Standard Design Gear
(12 Teeth, 20° Pressure Angle)

From the geometry of the limiting length-of-contact (T-T', **Figure 4-3**), it is evident that interference is first encountered by the addenda of the gear teeth digging into the mating-pinion tooth flanks. Since addenda are standardized by a fixed value ($h_a = m$), the interference condition becomes more severe as the number of teeth on the mating gear increases. The limit is reached when the gear becomes a rack. This is a realistic case since the hob is a rack-type cutter. The result is that standard gears with teeth numbers below a critical value are automatically undercut in the generating process. The condition for no undercutting in a standard spur gear is given by the expression:

$$\left.\begin{aligned} &\text{Max addendum} = h_a \le \frac{mz}{2}\sin^2\alpha \\ &\text{and the minimum number of teeth is:} \\ &z_c \ge \frac{2}{\sin^2\alpha} \end{aligned}\right\} \qquad \textbf{(4-1)}$$

This indicates that the minimum number of teeth free of undercutting decreases with increasing pressure angle. For 14.5° the value of z_c is 32, and for 20° it is 18. Thus, 20° pressure angle gears with low numbers of teeth have the advantage of much less undercutting and, therefore, are both stronger and smoother acting.

4.4 Enlarged Pinions

Undercutting of pinion teeth is undesirable because of losses of strength, contact ratio and smoothness of action. The severity of these faults depends upon how far below z_c the teeth number is. Undercutting for the first few numbers is small and in many applications its adverse effects can be neglected.

For very small numbers of teeth, such as ten and smaller, and for high-precision applications, undercutting should be avoided. This is achieved by pinion enlargement (or correction as often termed), wherein the pinion teeth, still generated with a standard cutter, are shifted radially outward to form a full involute tooth free of undercut. The tooth is enlarged both radially and circumferentially. Comparison of a tooth form before and after enlargement is shown in **Figure 4-5**.

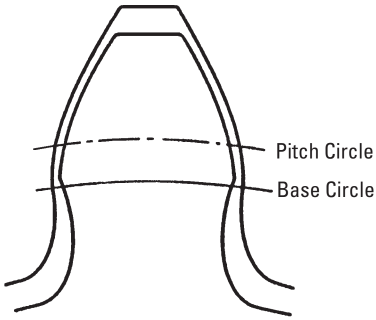

Fig. 4-5 Comparison of Enlarged and Undercut Standard Pinion
(13 Teeth, 20° Pressure Angle, Fine Pitch Standard)

4.5 Profile Shifting

As **Figure 4-2** shows, a gear with 20 degrees of pressure angle and 10 teeth will have a huge undercut volume. To prevent undercut, a positive correction must be introduced. A positive correction, as in **Figure 4-6**, can prevent undercut.

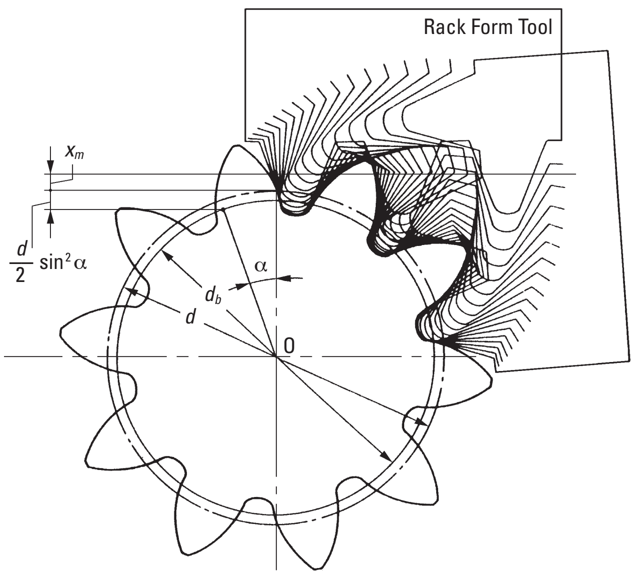

Fig. 4-6 Generating of Positive Shifted Spur Gear
(α = 20°, z = 10, x = +0.5)

I
R
T
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
A

Undercutting will get worse if a negative correction is applied. See **Figure 4-7**.

The extra feed of gear cutter (xm) in **Figures 4-6** and **4-7** is the amount of shift or correction. And x is the shift coefficient.

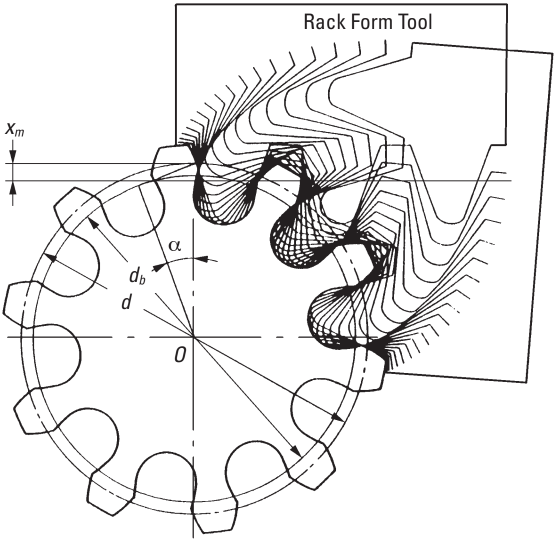

Fig. 4-7 The Generating of Negative Shifted Spur Gear
(α = 20°, z = 10, x = -0.5)

The condition to prevent undercut in a spur gear is:

$$m - xm \le \frac{zm}{2} \sin^2 \alpha \qquad \textbf{(4-2)}$$

The number of teeth without undercut will be:

$$z_c = \frac{2\,(1 - x)}{\sin^2 \alpha} \qquad \textbf{(4-3)}$$

The coefficient without undercut is:

$$x = 1 - \frac{z_c}{2} \sin^2 \alpha \qquad \textbf{(4-4)}$$

Profile shift is not merely used to prevent undercut. It can be used to adjust center distance between two gears.

If a positive correction is applied, such as to prevent undercut in a pinion, the tooth thickness at top is thinner.

Table 4-3 presents the calculation of top land thickness.

Table 4-3 The Calculations of Top Land Thickness

No.	Item	Symbol	Formula	Example
1	Pressure angle at outside circle of gear	α_a	$\cos^{-1}\left(\frac{d_b}{d_a}\right)$	$m = 2$, $\alpha = 20°$, $z = 16$, $x = +0.3$, $d = 32$, $d_b = 30.07016$, $d_a = 37.2$, $\alpha_a = 36.06616°$, $\text{inv}\,\alpha_a = 0.098835$, $\text{inv}\,\alpha = 0.014904$, $\theta = 1.59815°$ (0.027893 radian), $s_a = 1.03762$
2	Half of top land angle of outside circle	θ	$\frac{\pi}{2z} + \frac{2x\tan\alpha}{z} + (\text{inv}\,\alpha - \text{inv}\,\alpha_a)$ (radian)	
3	Top land thickness	s_a	θd_a	

4.6 Profile Shifted Spur Gear

Figure 4-8 shows the meshing of a pair of profile shifted gears. The key items in profile shifted gears are the operating (working) pitch diameters (d_w)and the working (operating) pressure angle (α_w). These values are obtainable from the operating (or i.e., actual) center distance and the following formulas:

$$\left.\begin{aligned} d_{w1} &= 2a_x \frac{z_1}{z_1 + z_2} \\ d_{w2} &= 2a_x \frac{z_2}{z_1 + z_2} \\ \alpha_w &= \cos^{-1}\left(\frac{d_{b1} + d_{b2}}{2a_x}\right) \end{aligned}\right\} \quad (4\text{-}5)$$

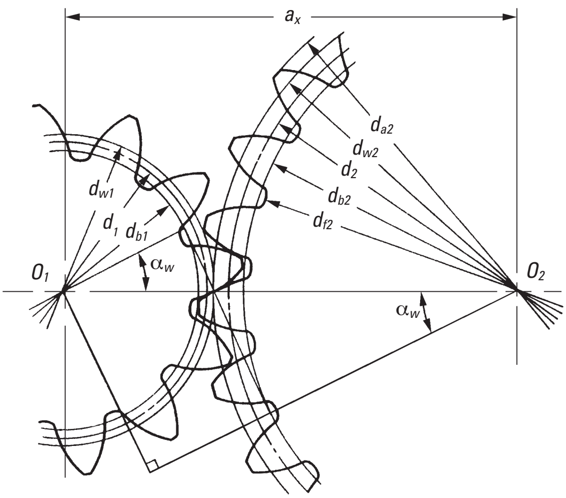

Fig. 4-8 The Meshing of Profile Shifted Gears
($\alpha = 20°$, $z_1 = 12$, $z_2 = 24$, $x_1 = +0.6$, $x_2 = +0.36$)

In the meshing of profile shifted gears, it is the operating pitch circles that are in contact and roll on each other that portrays gear action. The standard pitch circles no longer are of significance; and the operating pressure angle is what matters.

A standard spur gear is, according to **Table 4-4**, a profile shifted gear with 0 coefficient of shift; that is, $x_1 = x_2 = 0$.

Table 4-4 The Calculation of Positive Shifted Gear (1)

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Module	m		3	
2	Pressure Angle	α		20°	
3	Number of Teeth	z_1, z_2		12	24
4	Coefficient of Profile Shift	x_1, x_2		0.6	0.36
5	Involute Function α_w	inv α_w	$2\tan\alpha\left(\frac{x_1+x_2}{z_1+z_2}\right)+\text{inv}\,\alpha$	0.034316	
6	Working Pressure Angle	α_w	Find from Involute Function Table	26.0886°	
7	Center Distance Increment Factor	y	$\frac{z_1+z_2}{2}\left(\frac{\cos\alpha}{\cos\alpha_w}-1\right)$	0.83329	
8	Center Distance	a_x	$\left(\frac{z_1+z_2}{2}+y\right)m$	56.4999	
9	Pitch Diameter	d	zm	36.000	72.000
10	Base Diameter	d_b	$d\cos\alpha$	33.8289	67.6579
11	Working Pitch Diameter	d_w	$\frac{d_b}{\cos\alpha_w}$	37.667	75.333
12	Addendum	h_{a1} h_{a2}	$(1+y-x_2)m$ $(1+y-x_1)m$	4.420	3.700
13	Whole Depth	h	$[2.25+y-(x_1+x_2)]m$	6.370	
14	Outside Diameter	d_a	$d+2h_a$	44.840	79.400
15	Root Diameter	d_f	d_a-2h	32.100	66.660

Table 4-5 is the inverse formula of items from 4 to 8 of **Table 4-4**.

Table 4-5 The Calculation of Positive Shifted Gear (2)

No.	Item	Symbol	Formula	Example	
1	Center Distance	a_x		56.4999	
2	Center Distance Increment Factor	y	$\frac{a_x}{m}-\frac{z_1+z_2}{2}$	0.8333	
3	Working Pressure Angle	α_w	$\cos^{-1}\left[\frac{(z_1+z_2)\cos\alpha}{2y+z_1+z_2}\right]$	26.0886°	
4	Sum of Coefficient of Profile Shift	x_1+x_2	$\frac{(z_1+z_2)(\text{inv}\,\alpha_w-\text{inv}\,\alpha)}{2\tan\alpha}$	0.9600	
5	Coefficient of Profile Shift	$x_1,\ x_2$		0.6000	0.3600

There are several theories concerning how to distribute the sum of coefficient of profile shift, $(x_1 + x_2)$ into pinion, (x_1) and gear, (x_2) separately. BSS (British) and DIN (German) standards are the most often used. In the example above, the 12 tooth pinion was given sufficient correction to prevent undercut, and the residual profile shift was given to the mating gear.

4.7 Rack And Spur Gear

Table 4-6 presents the method for calculating the mesh of a rack and spur gear. **Figure 4-9a** shows the pitch circle of a standard gear and the pitch line of the rack.

One rotation of the spur gear will displace the rack (l) one circumferential length of the gear's pitch circle, per the formula:

$$l = \pi m z \qquad \textbf{(4-6)}$$

Figure 4-9b shows a profile shifted spur gear, with positive correction xm, meshed with a rack. The spur gear has a larger pitch radius than standard, by the amount xm. Also, the pitch line of the rack has shifted outward by the amount xm.

Table 4-6 presents the calculation of a meshed profile shifted spur gear and rack. If the correction factor x_1 is 0, then it is the case of a standard gear meshed with the rack.

The rack displacement, l, is not changed in any way by the profile shifting. **Equation (4-6)** remains applicable for any amount of profile shift.

Table 4-6 The Calculation of Dimensions of a Profile Shifted Spur Gear and a Rack

No.	Item	Symbol	Formula	Example: Spur Gear	Example: Rack
1	Module	m		3	
2	Pressure Angle	α		20°	
3	Number of Teeth	z		12	—
4	Coefficient of Profile Shift	x		0.6	—
5	Height of Pitch Line	H		—	32.000
6	Working Pressure Angle	α_w		20°	
7	Center Distance	a_x	$\frac{zm}{2} + H + xm$	51.800	
8	Pitch Diameter	d	zm	36.000	—
9	Base Diameter	d_b	$d \cos \alpha$	33.829	—
10	Working Pitch Diameter	d_w	$\frac{d_b}{\cos \alpha_w}$	36.000	—
11	Addendum	h_a	$m(1 + x)$	4.800	3.000
12	Whole Depth	h	$2.25m$	6.750	
13	Outside Diameter	d_a	$d + 2h_a$	45.600	—
14	Root Diameter	d_f	$d_a - 2h$	32.100	—

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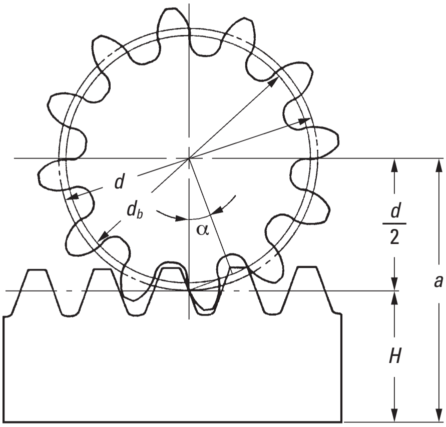

Fig. 4-9a The Meshing of Standard Spur Gear and Rack
($\alpha = 20°$, $z_1 = 12$, $x_1 = 0$)

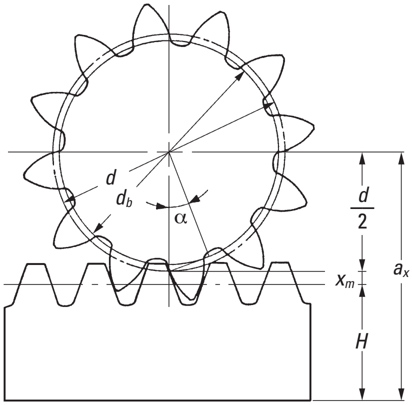

Fig. 4-9b The Meshing of Profile Shifted Spur Gear and Rack
($\alpha = 20°$, $z_1 = 12$, $x_1 = +0.6$)

SECTION 5 INTERNAL GEARS

5.1 Internal Gear Calculations

Calculation of a Profile Shifted Internal Gear

Figure 5-1 presents the mesh of an internal gear and external gear. Of vital importance is the operating (working) pitch diameters, d_w, and operating (working) pressure angle, α_w. They can be derived from center distance, a_x, and **Equations (5-1).**

$$\left.\begin{aligned} d_{w1} &= 2a_x\left(\frac{z_1}{z_2 - z_1}\right) \\ d_{w2} &= 2a_x\left(\frac{z_2}{z_2 - z_1}\right) \\ \alpha_w &= \cos^{-1}\left(\frac{d_{b2} - d_{b1}}{2a_x}\right) \end{aligned}\right\} \qquad \textbf{(5-1)}$$

Table 5-1 shows the calculation steps.
It will become a standard gear calculation if $x_1 = x_2 = 0$.

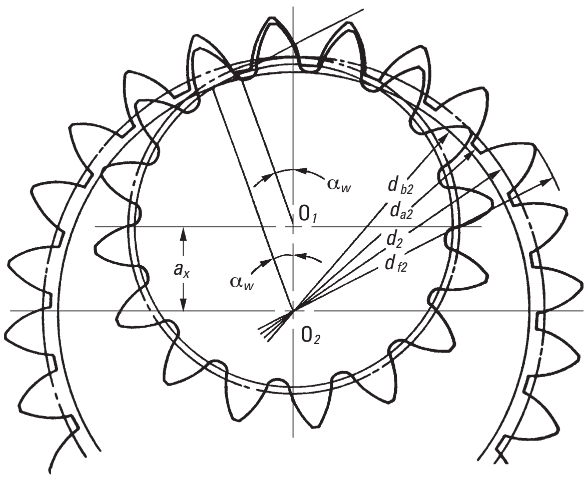

Fig. 5-1 The Meshing of Internal Gear and External Gear
(α = 20°, z_1 = 16, z_2 = 24, $x_1 = x_2$ = 0.5)

Table 5-1 The Calculation of a Profile Shifted Internal Gear and External Gear (1)

No.	Item	Symbol	Formula	Example	
				External Gear (1)	Internal Gear (2)
1	Module	m		3	
2	Pressure Angle	α		20°	
3	Number of Teeth	z_1, z_2		16	24
4	Coefficient of Profile Shift	x_1, x_2		0	0.5
5	Involute Function α_w	inv α_w	$2\tan\alpha\left(\frac{x_2-x_1}{z_2-z_1}\right)+\text{inv}\,\alpha$	0.060401	
6	Working Pressure Angle	α_w	Find from Involute Function Table	31.0937°	
7	Center Distance Increment Factor	y	$\frac{z_2-z_1}{2}\left(\frac{\cos\alpha}{\cos\alpha_w}-1\right)$	0.389426	
8	Center Distance	a_x	$\left(\frac{z_2-z_1}{2}+y\right)m$	13.1683	
9	Pitch Diameter	d	zm	48.000	72.000
10	Base Circle Diameter	d_b	$d\cos\alpha$	45.105	67.658
11	Working Pitch Diameter	d_w	$\frac{d_b}{\cos\alpha_w}$	52.673	79.010
12	Addendum	h_{a1} h_{a2}	$(1+x_1)m$ $(1-x_2)m$	3.000	1.500
13	Whole Depth	h	$2.25m$	6.75	
14	Outside Diameter	d_{a1} d_{a2}	d_1+2h_{a1} d_2-2h_{a2}	54.000	69.000
15	Root Diameter	d_{f1} d_{f2}	$d_{a1}-2h$ $d_{a2}+2h$	40.500	82.500

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If the center distance, a_x, is given, x_1 and x_2 would be obtained from the inverse calculation from item 4 to item 8 of **Table 5-1**. These inverse formulas are in **Table 5-2**.

Pinion cutters are often used in cutting internal gears and external gears. The actual value of tooth depth and root diameter, after cutting, will be slightly different from the calculation. That is because the cutter has a coefficient of shifted profile. In order to get a correct tooth profile, the coefficient of cutter should be taken into consideration.

Table 5-2 The Calculation of Shifted Internal Gear and External Gear (2)

No.	Item	Symbol	Formula	Example	
1	Center Distance	a_x		13.1683	
2	Center Distance Increment Factor	y	$\frac{a_x}{m} - \frac{z_2 - z_1}{2}$	0.38943	
3	Working Pressure Angle	α_w	$\cos^{-1}\left[\frac{(z_2 - z_1)\cos\alpha}{2y + z_2 - z_1}\right]$	31.0937°	
4	Difference of Coefficients of Profile Shift	$x_2 - x_1$	$\frac{(z_2 - z_1)(\text{inv}\,\alpha_w - \text{inv}\,\alpha)}{2\tan\alpha}$	0.5	
5	Coefficient of Profile Shift	x_1, x_2		0	0.5

5.2 Interference In Internal Gears

Three different types of interference can occur with internal gears:

(a) Involute Interference
(b) Trochoid Interference
(c) Trimming Interference

(a) Involute Interference

This occurs between the dedendum of the external gear and the addendum of the internal gear. It is prevalent when the number of teeth of the external gear is small. Involute interference can be avoided by the conditions cited below:

$$\frac{z_1}{z_2} \geq 1 - \frac{\tan\alpha_{a2}}{\tan\alpha_w} \qquad \textbf{(5-2)}$$

where α_{a2} is the pressure angle seen at a tip of the internal gear tooth.

$$\alpha_{a2} = \cos^{-1}\left(\frac{d_{b2}}{d_{a2}}\right) \qquad \textbf{(5-3)}$$

and α_w is working pressure angle:

$$\alpha_w = \cos^{-1}\left[\frac{(z_2 - z_1)m\cos\alpha}{2a_x}\right] \qquad \textbf{(5-4)}$$

Equation (5-3) is true only if the outside diameter of the internal gear is bigger than the base circle:

$$d_{a2} \geq d_{b2} \qquad \textbf{(5-5)}$$

For a standard internal gear, where α = 20°, **Equation (5-5)** is valid only if the number of teeth is $z_2 > 34$.

(b) Trochoid Interference

This refers to an interference occurring at the addendum of the external gear and the dedendum of the internal gear during recess tooth action. It tends to happen when the difference between the numbers of teeth of the two gears is small. **Equation (5-6)** presents the condition for avoiding trochoidal interference.

$$\theta_1 \frac{z_1}{z_2} + \text{inv}\,\alpha_w - \text{inv}\,\alpha_{a2} \geq \theta_2 \qquad \textbf{(5-6)}$$

Here

$$\left.\begin{aligned} \theta_1 &= \cos^{-1}\left(\frac{r_{a2}^2 - r_{a1}^2 - a^2}{2ar_{a1}}\right) + \text{inv}\,\alpha_{a1} - \text{inv}\,\alpha_w \\ \theta_2 &= \cos^{-1}\left(\frac{a^2 + r_{a2}^2 - r_{a1}^2}{2ar_{a2}}\right) \end{aligned}\right\} \qquad \textbf{(5-7)}$$

where α_{a1} is the pressure angle of the spur gear tooth tip:

$$\alpha_{a1} = \cos^{-1}\left(\frac{d_{b1}}{d_{a1}}\right) \qquad \textbf{(5-8)}$$

In the meshing of an external gear and a standard internal gear $\alpha = 20°$, trochoid interference is avoided if the difference of the number of teeth, $z_1 - z_2$, is larger than 9.

(c) Trimming Interference

This occurs in the radial direction in that it prevents pulling the gears apart. Thus, the mesh must be assembled by sliding the gears together with an axial motion. It tends to happen when the numbers of teeth of the two gears are very close. **Equation (5-9)** indicates how to prevent this type of interference.

$$\theta_1 + \text{inv}\,\alpha_{a1} - \text{inv}\,\alpha_w \geq \frac{z_2}{z_1}(\theta_2 + \text{inv}\,\alpha_{a2} - \text{inv}\,\alpha_w) \qquad \textbf{(5-9)}$$

Here

$$\left.\begin{aligned} \theta_1 &= \sin^{-1}\sqrt{\frac{1 - (\cos\alpha_{a1}/\cos\alpha_{a2})^2}{1 - (z_1/z_2)^2}} \\ \theta_2 &= \sin^{-1}\sqrt{\frac{(\cos\alpha_{a2}/\cos\alpha_{a1})^2 - 1}{(z_2/z_1)^2 - 1}} \end{aligned}\right\} \qquad \textbf{(5-10)}$$

This type of interference can occur in the process of cutting an internal gear with a pinion cutter. Should that happen, there is danger of breaking the tooling. **Table 5-3a** shows the limit for the pinion cutter to prevent trimming interference when cutting a standard internal gear, with pressure angle 20°, and no profile shift, i.e., $x_c = 0$.

Table 5-3a The Limit to Prevent an Internal Gear from Trimming Interference
($\alpha = 20°$, $x_c = x_2 = 0$)

z_c	15	16	17	18	19	20	21	22	24	25	27
z_2	34	34	35	36	37	38	39	40	42	43	45
z_c	28	30	31	32	33	34	35	38	40	42	
z_2	46	48	49	50	51	52	53	56	58	60	
z_c	44	48	50	56	60	64	66	80	96	100	
z_2	62	66	68	74	78	82	84	98	114	118	

There will be an involute interference between the internal gear and the pinion cutter if the number of teeth of the pinion cutter ranges from 15 to 22 (z_c = 15 to 22). **Table 5-3b** shows the limit for a profile shifted pinion cutter to prevent trimming interference while cutting a standard internal gear. The correction, x_c, is the magnitude of shift which was assumed to be: $x_c = 0.0075\ z_c + 0.05$.

Table 5-3b The Limit to Prevent an Internal Gear from Trimming Interference
(α = 20°, x_2 = 0)

z_c	15	16	17	18	19	20	21	22	24	25	27
x_c	0.1625	0.17	0.1775	0.185	0.1925	0.2	0.2075	0.215	0.23	0.2375	0.2525
z_2	36	38	39	40	41	42	43	45	47	48	50
z_c	28	30	31	32	33	34	35	38	40	42	
x_c	0.26	0.275	0.2825	0.29	0.2975	0.305	0.3125	0.335	0.35	0.365	
z_2	52	54	55	56	58	59	60	64	66	68	
z_c	44	48	50	56	60	64	66	80	96	100	
x_c	0.38	0.41	0.425	0.47	0.5	0.53	0.545	0.65	0.77	0.8	
z_2	71	76	78	86	90	95	98	115	136	141	

There will be an involute interference between the internal gear and the pinion cutter if the number of teeth of the pinion cutter ranges from 15 to 19 (z_c = 15 to 19).

5.3 Internal Gear With Small Differences In Numbers Of Teeth

In the meshing of an internal gear and an external gear, if the difference in numbers of teeth of two gears is quite small, a profile shifted gear could prevent the interference. **Table 5-4** is an example of how to prevent interference under the conditions of z_2 = 50 and the difference of numbers of teeth of two gears ranges from 1 to 8.

Table 5-4 The Meshing of Internal and External Gears of Small Difference of Numbers of Teeth (m = 1, α = 20°)

z_1	49	48	47	46	45	44	43	42
x_1	0							
z_2	50							
x_2	1.00	0.60	0.40	0.30	0.20	0.11	0.06	0.01
α_w	61.0605°	46.0324°	37.4155°	32.4521°	28.2019°	24.5356°	22.3755°	20.3854°
a	0.971	1.354	1.775	2.227	2.666	3.099	3.557	4.010
ε	1.105	1.512	1.726	1.835	1.933	2.014	2.053	2.088

All combinations above will not cause involute interference or trochoid interference, but trimming interference is still there. In order to assemble successfully, the external gear should be assembled by inserting in the axial direction.

A profile shifted internal gear and external gear, in which the difference of numbers of teeth is small, belong to the field of hypocyclic mechanism, which can produce a large reduction ratio in one step, such as 1/100.

$$\text{Speed Ratio} = \frac{z_2 - z_1}{z_1} \qquad \textbf{(5-11)}$$

In **Figure 5-2** the gear train has a difference of numbers of teeth of only 1; $z_1 = 30$ and $z_2 = 31$. This results in a reduction ratio of 1/30.

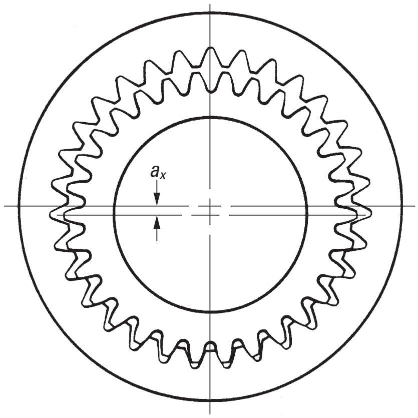

Fig. 5-2 The Meshing of Internal Gear and External Gear in which the Numbers of Teeth Difference is 1
$(z_2 - z_1 = 1)$

SECTION 6 HELICAL GEARS

The helical gear differs from the spur gear in that its teeth are twisted along a helical path in the axial direction. It resembles the spur gear in the plane of rotation, but in the axial direction it is as if there were a series of staggered spur gears. See **Figure 6-1**. This design brings forth a number of different features relative to the spur gear, two of the most important being as follows:

1. Tooth strength is improved because of the elongated helical wraparound tooth base support.
2. Contact ratio is increased due to the axial tooth overlap. Helical gears thus tend to have greater load carrying capacity than spur gears of the same size. Spur gears, on the other hand, have a somewhat higher efficiency.

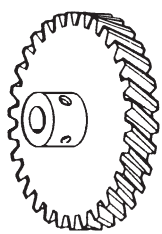

Fig. 6-1 Helical Gear

Helical gears are used in two forms:

1. Parallel shaft applications, which is the largest usage.
2. Crossed-helicals (also called spiral or screw gears) for connecting skew shafts, usually at right angles.

6.1 Generation Of The Helical Tooth

The helical tooth form is involute in the plane of rotation and can be developed in a manner similar to that of the spur gear. However, unlike the spur gear which can be viewed essentially as two dimensional, the helical gear must be portrayed in three dimensions to show changing axial features.

Referring to **Figure 6-2**, there is a base cylinder from which a taut plane is unwrapped, analogous to the unwinding taut string of the spur gear in **Figure 2-2**. On the plane there is a straight line AB, which when wrapped on the base cylinder has a helical trace A_0B_0. As the taut plane is unwrapped, any point on the line AB can be visualized as tracing an involute from the base cylinder. Thus, there is an infinite series of involutes generated by line AB, all alike, but displaced in phase along a helix on the base cylinder.

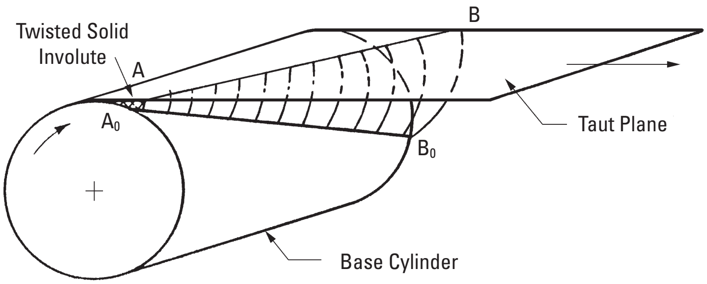

Fig. 6-2 Generation of the Helical Tooth Profile

Again, a concept analogous to the spur gear tooth development is to imagine the taut plane being wound from one base cylinder on to another as the base cylinders rotate in opposite directions. The result is the generation of a pair of conjugate helical involutes. If a reverse direction of rotation is assumed and a second tangent plane is arranged so that it crosses the first, a complete involute helicoid tooth is formed.

6.2 Fundamentals Of Helical Teeth

In the plane of rotation, the helical gear tooth is involute and all of the relationships governing spur gears apply to the helical. However, the axial twist of the teeth introduces a helix angle. Since the helix angle varies from the base of the tooth to the outside radius, the helix angle β is defined as the angle between the tangent to the helicoidal tooth at the intersection of the pitch cylinder and the tooth profile, and an element of the pitch cylinder. See **Figure 6-3**.

The direction of the helical twist is designated as either left or right. The direction is defined by the right-hand rule.

For helical gears, there are two related pitches – one in the plane of rotation and the other in a plane normal to the tooth. In addition, there is an axial pitch.

Referring to **Figure 6-4**, the two circular pitches are defined and related as follows:

$$p_n = p_t \cos\beta = \text{normal circular pitch} \qquad \textbf{(6-1)}$$

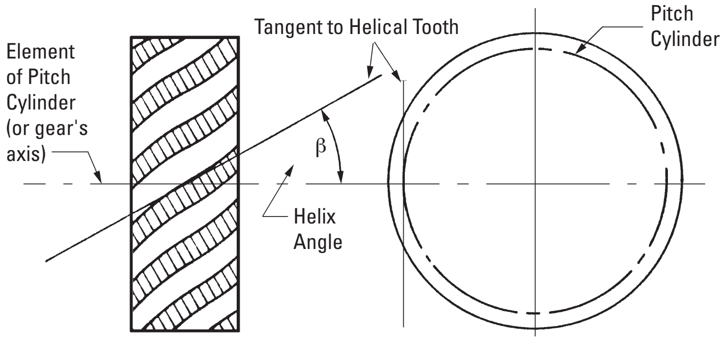

Fig. 6-3 Definition of Helix Angle

The normal circular pitch is less than the transverse radial pitch, p_t, in the plane of rotation; the ratio between the two being equal to the cosine of the helix angle.
Consistent with this, the normal module is less than the transverse (radial) module.

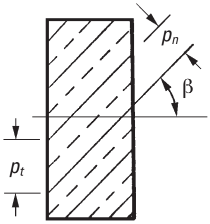

Fig. 6-4 Relationship of Circular Pitches

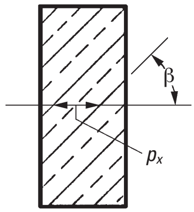

Fig. 6-5 Axial Pitch of a Helical Gear

The axial pitch of a helical gear, p_x, is the distance between corresponding points of adjacent teeth measured parallel to the gear's axis – see **Figure 6-5**. Axial pitch is related to circular pitch by the expressions:

$$p_x = p_t \cot \beta = \frac{p_n}{\sin \beta} = \text{axial pitch} \qquad \textbf{(6-2)}$$

A helical gear such as shown in **Figure 6-6** is a cylindrical gear in which the teeth flank are helicoid. The helix angle in standard pitch circle cylinder is β, and the displacement of one rotation is the lead, L.

The tooth profile of a helical gear is an involute curve from an axial view, or in the plane perpendicular to the axis. The helical gear has two kinds of tooth profiles – one is based on a normal system, the other is based on an axial system.

Circular pitch measured perpendicular to teeth is called normal circular pitch, p_n. And p_n divided by π is then a normal module, m_n.

$$m_n = \frac{p_n}{\pi} \qquad \textbf{(6-3)}$$

The tooth profile of a helical gear with applied normal module, m_n, and normal pressure angle α_n belongs to a normal system.

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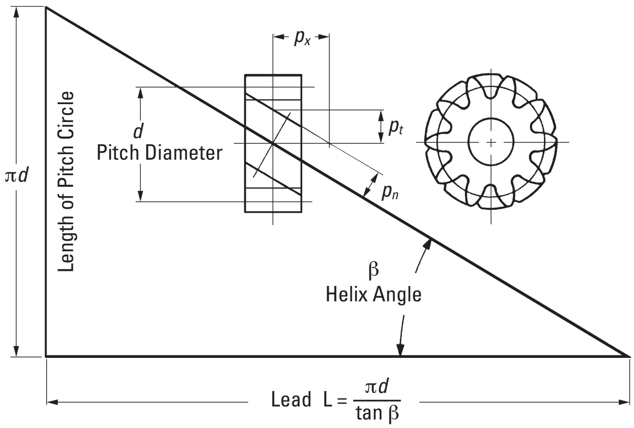

Fig. 6-6 Fundamental Relationship of a Helical Gear (Right-Hand)

In the axial view, the circular pitch on the standard pitch circle is called the radial circular pitch, p_t. And p_t divided by π is the radial module, m_t.

$$m_t = \frac{p_t}{\pi} \tag{6-4}$$

6.3 Equivalent Spur Gear

The true involute pitch and involute geometry of a helical gear is in the plane of rotation. However, in the normal plane, looking at one tooth, there is a resemblance to an involute tooth of a pitch corresponding to the normal pitch. However, the shape of the tooth corresponds to a spur gear of a larger number of teeth, the exact value depending on the magnitude of the helix angle.

The geometric basis of deriving the number of teeth in this equivalent tooth form spur gear is given in **Figure 6-7**. The result of the transposed geometry is an equivalent number of teeth, given as:

$$z_v = \frac{z}{\cos^3 \beta} \tag{6-5}$$

This equivalent number is also called a virtual number because this spur gear is imaginary. The value of this number is used in determining helical tooth strength.

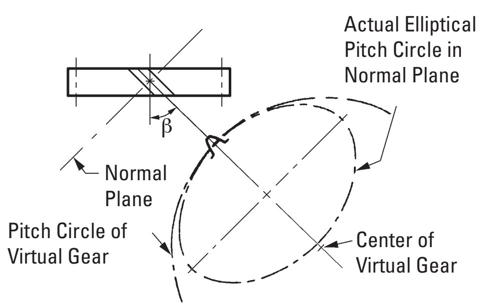

Fig. 6-7 Geometry of Helical Gear's Virtual Number of Teeth

6.4 Helical Gear Pressure Angle

Although, strictly speaking, pressure angle exists only for a gear pair, a nominal pressure angle can be considered for an individual gear. For the helical gear there is a normal pressure, α_n, angle as well as the usual pressure angle in the plane of rotation, α. **Figure 6-8** shows their relationship, which is expressed as:

$$\tan\alpha = \frac{\tan\alpha_n}{\cos\beta} \qquad \textbf{(6-6)}$$

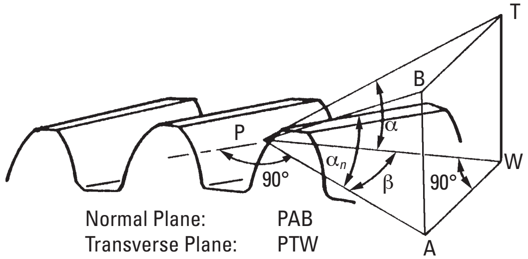

Fig. 6-8 Geometry of Two Pressure Angles

6.5 Importance Of Normal Plane Geometry

Because of the nature of tooth generation with a rack-type hob, a single tool can generate helical gears at all helix angles as well as spur gears. However, this means the normal pitch is the common denominator, and usually is taken as a standard value. Since the true involute features are in the transverse plane, they will differ from the standard normal values. Hence, there is a real need for relating parameters in the two reference planes.

6.6 Helical Tooth Proportions

These follow the same standards as those for spur gears. Addendum, dedendum, whole depth and clearance are the same regardless of whether measured in the plane of rotation or the normal plane. Pressure angle and pitch are usually specified as standard values in the normal plane, but there are times when they are specified as standard in the transverse plane.

6.7 Parallel Shaft Helical Gear Meshes

Fundamental information for the design of gear meshes is as follows:

Helix angle – Both gears of a meshed pair must have the same helix angle. However, the helix direction must be opposite; i.e., a left-hand mates with a right-hand helix.

Pitch diameter – This is given by the same expression as for spur gears, but if the normal module is involved it is a function of the helix angle. The expressions are:

$$d = z\,m_t = \frac{z}{m_n\cos\beta} \qquad \textbf{(6-7)}$$

Center distance – Utilizing **Equation (6-7)**, the center distance of a helical gear mesh is:

$$a = \frac{z_1 + z_2}{2\, m_n \cos\beta} \qquad \textbf{(6-8)}$$

Note that for standard parameters in the normal plane, the center distance will not be a standard value compared to standard spur gears. Further, by manipulating the helix angle, β, the center distance can be adjusted over a wide range of values. Conversely, it is possible:

1. to compensate for significant center distance changes (or errors) without changing the speed ratio between parallel geared shafts; and
2. to alter the speed ratio between parallel geared shafts, without changing the center distance, by manipulating the helix angle along with the numbers of teeth.

6.8 Helical Gear Contact Ratio

The contact ratio of helical gears is enhanced by the axial overlap of the teeth. Thus, the contact ratio is the sum of the transverse contact ratio, calculated in the same manner as for spur gears, and a term involving the axial pitch.

$$\left.\begin{array}{l} (\varepsilon)_{total} = (\varepsilon)_{trans} + (\varepsilon)_{axial} \\ \text{or} \\ \varepsilon_r = \varepsilon_\alpha + \varepsilon_\beta \end{array}\right\} \qquad \textbf{(6-9)}$$

Details of contact ratio of helical gearing are given later in a general coverage of the subject; see **SECTION 11.1**.

6.9 Design Considerations

6.9.1 Involute Interference

Helical gears cut with standard normal pressure angles can have considerably higher pressure angles in the plane of rotation – see **Equation (6-6)** – depending on the helix angle. Therefore, the minimum number of teeth without undercutting can be significantly reduced, and helical gears having very low numbers of teeth without undercutting are feasible.

6.9.2 Normal Vs. Radial Module (Pitch)

In the normal system, helical gears can be cut by the same gear hob if module m_n and pressure angle α_n are constant, no matter what the value of helix angle β.

It is not that simple in the radial system.The gear hob design must be altered in accordance with the changing of helix angle β, even when the module m_t and the pressure angle α_t are the same.

Obviously, the manufacturing of helical gears is easier with the normal system than with the radial system in the plane perpendicular to the axis.

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6.10 Helical Gear Calculations

6.10.1 Normal System Helical Gear

In the normal system, the calculation of a profile shifted helical gear, the working pitch diameter d_w and working pressure angle α_{wt} in the axial system is done per **Equations (6-10)**. That is because meshing of the helical gears in the axial direction is just like spur gears and the calculation is similar.

$$\left.\begin{aligned} d_{w1} &= 2a_x \frac{z_1}{z_1+z_2} \\ d_{w2} &= 2a_x \frac{z_2}{z_1+z_2} \\ \alpha_{wt} &= \cos^{-1}\left(\frac{d_{b1}+d_{b2}}{2a_x}\right) \end{aligned}\right\} \qquad \textbf{(6-10)}$$

Table 6-1 shows the calculation of profile shifted helical gears in the normal system. If normal coefficients of profile shift x_{n1}, x_{n2} are zero, they become standard gears.

Table 6-1 The Calculation of a Profile Shifted Helical Gear in the Normal System (1)

No.	Item	Symbol	Formula	Example Pinion	Example Gear
1	Normal Module	m_n		3	
2	Normal Pressure Angle	α_n		20°	
3	Helix Angle	β		30°	
4	Number of Teeth & Helical Hand	z_1, z_2		12 (L)	60 (R)
5	Radial Pressure Angle	α_t	$\tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right)$	22.79588°	
6	Normal Coefficient of Profile Shift	x_{n1}, x_{n2}		0.09809	0
7	Involute Function α_{wt}	inv α_{wt}	$2\tan\alpha_n\left(\frac{x_{n1}+x_{n2}}{z_1+z_2}\right)+\text{inv}\,\alpha_t$	0.023405	
8	Radial Working Pressure Angle	α_{wt}	Find from Involute Function Table	23.1126°	
9	Center Distance Increment Factor	y	$\frac{z_1+z_2}{2\cos\beta}\left(\frac{\cos\alpha_t}{\cos\alpha_{wt}}-1\right)$	0.09744	
10	Center Distance	a_x	$\left(\frac{z_1+z_2}{2\cos\beta}+y\right)m_n$	125.000	
11	Standard Pitch Diameter	d	$\frac{zm_n}{\cos\beta}$	41.569	207.846
12	Base Diameter	d_b	$d\cos\alpha_t$	38.322	191.611
13	Working Pitch Diameter	h_{a1}	$\frac{d_b}{\cos\alpha_{wt}}$	41.667	208.333
14	Addendum	h_{a2}	$(1+y-x_{n2})\,m_n$ $(1+y-x_{n1})\,m$	3.292	2.998
15	Whole Depth	h	$[2.25+y-(x_{n1}+x_{n2})]m_n$	6.748	
16	Outside Diameter	d_a	$d+2h_a$	48.153	213.842
17	Root Diameter	d_f	d_a-2h	34.657	200.346

If center distance, a_x, is given, the normal coefficient of profile shift x_{n1} and x_{n2} can be calculated from **Table 6-2**. These are the inverse equations from items 4 to 10 of **Table 6-1**.

Table 6-2 The Calculations of a Profile Shifted Helical Gear in the Normal System (2)

No.	Item	Symbol	Formula	Example	
1	Center Distance	a_x		125	
2	Center Distance Increment Factor	y	$\frac{a_x}{m_n} - \frac{z_1 + z_2}{2\cos\beta}$	0.097447	
3	Radial Working Pressure Angle	α_{wt}	$\cos^{-1}\left[\frac{(z_1 + z_2)\cos\alpha_t}{(z_1 + z_2) + 2y\cos\beta}\right]$	23.1126°	
4	Sum of Coefficient of Profile Shift	$x_{n1} + x_{n2}$	$\frac{(z_1 + z_2)(\text{inv}\,\alpha_{wt} - \text{inv}\,\alpha_t)}{2\tan\alpha_n}$	0.09809	
5	Normal Coefficient of Profile Shift	$x_{n1} + x_{n2}$		0.09809	0

The transformation from a normal system to a radial system is accomplished by the following equations:

$$\left.\begin{aligned} x_t &= x_n \cos\beta \\ m_t &= \frac{m_n}{\cos\beta} \\ \alpha_t &= \tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right) \end{aligned}\right\} \qquad (6\text{-}11)$$

6.10.2 Radial System Helical Gear

Table 6-3 shows the calculation of profile shifted helical gears in a radial system. They become standard if $x_{t1} = x_{t2} = 0$.

Table 6-3 The Calculation of a Profile Shifted Helical Gear in the Radial System (1)

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Radial Module	m_t		3	
2	Radial Pressure Angle	α_t		20°	
3	Helix Angle	β		30°	
4	Number of Teeth & Helical Hand	z_1, z_2		12 (L)	60 (R)
5	Radial Coefficient of Profile Shift	x_{t1}, x_{t2}		0.34462	0
6	Involute Function α_{wt}	inv α_{wt}	$2\tan\alpha_t\left(\frac{x_{t1} + x_{t2}}{z_1 + z_2}\right) + \text{inv}\,\alpha_t$	0.0183886	
7	Radial Working Pressure Angle	α_{wt}	Find from Involute Function Table	21.3975°	
8	Center Distance Increment Factor	y	$\frac{z_1 + z_2}{2}\left(\frac{\cos\alpha_t}{\cos\alpha_{wt}} - 1\right)$	0.33333	
9	Center Distance	a_x	$\left(\frac{z_1 + z_2}{2} + y\right)m_t$	109.0000	
10	Standard Pitch Diameter	d	zm_t	36.000	180.000
11	Base Diameter	d_b	$d\cos\alpha_t$	33.8289	169.1447
12	Working Pitch Diameter	d_w	$\frac{d_b}{\cos\alpha_{wt}}$	36.3333	181.6667
13	Addendum	h_{a1} h_{a2}	$(1 + y - x_{t2})\,m_t$ $(1 + y - x_{t1})\,m_t$	4.000	2.966
14	Whole Depth	h	$[2.25 + y - (x_{t1} + x_{t2})]m_t$	6.716	
15	Outside Diameter	d_a	$d + 2\,h_a$	44.000	185.932
16	Root Diameter	d_f	$d_a - 2\,h$	30.568	172.500

Table 6-4 presents the inverse calculation of items 5 to 9 of **Table 6-3**.

Table 6-4 The Calculation of a Shifted Helical Gear in the Radial System (2)

No.	Item	Symbol	Formula	Example	
1	Center Distance	a_x		109	
2	Center Distance Increment Factor	y	$\frac{a_x}{m_t} - \frac{z_1+z_2}{2}$	0.33333	
3	Radial Working Pressure Angle	α_{wt}	$\cos^{-1}\left[\frac{(z_1+z_2)\cos\alpha_t}{(z_1+z_2)+2y}\right]$	21.39752°	
4	Sum of Coefficient of Profile Shift	$x_{t1}+x_{t2}$	$\frac{(z_1+z_2)(\text{inv}\,\alpha_{wt}-\text{inv}\,\alpha_t)}{2\tan\alpha_n}$	0.34462	
5	Normal Coefficient of Profile Shift	x_{t1}, x_{t2}		0.34462	0

The transformation from a radial to a normal system is described by the following equations:

$$\left.\begin{aligned} x_n &= \frac{x_t}{\cos\beta} \\ m_n &= m_t\cos\beta \\ \alpha_n &= \tan^{-1}(\tan\alpha_t\cos\beta) \end{aligned}\right\} \quad \textbf{(6-12)}$$

6.10.3 Sunderland Double Helical Gear

A representative application of radial system is a double helical gear, or herringbone gear, made with the Sunderland machine. The radial pressure angle, α_t, and helix angle, β, are specified as 20° and 22.5°, respectively. The only differences from the radial system equations of **Table 6-3** are those for addendum and whole depth. **Table 6-5** presents equations for a Sunderland gear.

Table 6-5 The Calculation of a Double Helical Gear of SUNDERLAND Tooth Profile

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Radial Module	m_t		3	
2	Radial Pressure Angle	α_t		20°	
3	Helix Angle	β		22.5°	
4	Number of Teeth	z_1, z_2		12	60
5	Radial Coefficient of Profile Shift	x_{t1}, x_{t2}		0.34462	0
6	Involute Function α_{wt}	$\text{inv}\,\alpha_{wt}$	$2\tan\alpha_t\left(\frac{x_{t1}+x_{t2}}{z_1+z_2}\right)+\text{inv}\,\alpha_t$	0.0183886	
7	Radial Working Pressure Angle	α_{wt}	Find from Involute Function Table	21.3975°	
8	Center Distance Increment Factor	y	$\frac{z_1+z_2}{2}\left(\frac{\cos\alpha_t}{\cos\alpha_{wt}}-1\right)$	0.33333	
9	Center Distance	a_x	$\left(\frac{z_1+z_2}{2}+y\right)m_t$	109.0000	
10	Standard Pitch Diameter	d	zm_t	36.000	180.000
11	Base Diameter	d_b	$d\cos\alpha_t$	33.8289	169.1447
12	Working Pitch Diameter	d_w	$\frac{d_b}{\cos\alpha_{wt}}$	36.3333	181.6667
13	Addendum	h_{a1} h_{a2}	$(0.8796+y-x_{t2})\,m_t$ $(0.8796+y-x_{t1})\,m_t$	3.639	2.605
14	Whole Depth	h	$[1.8849+y-(x_{t1}+x_{t2})]m_t$	5.621	
15	Outside Diameter	d_a	$d+2h_a$	43.278	185.210
16	Root Diameter	d_f	d_a-2h	32.036	173.968

6.10.4 Helical Rack

Viewed in the normal direction, the meshing of a helical rack and gear is the same as a spur gear and rack. **Table 6-6** presents the calculation examples for a mated helical rack with normal module and normal pressure angle standard values. Similarly, **Table 6-7** presents examples for a helical rack in the radial system (i.e., perpendicular to gear axis).

Table 6-6 The Calculation of a Helical Rack in the Normal System

No.	Item	Symbol	Formula	Example Gear	Example Rack
1	Normal Module	m_n		2.5	
2	Normal Pressure Angle	α_n		20°	
3	Helix Angle	β		10° 57' 49"	
4	Number of Teeth & Helical Hand	z		20 (R)	– (L)
5	Normal Coefficient of Profile Shift	x_n		0	–
6	Pitch Line Height	H		–	27.5
7	Radial Pressure Angle	α_t	$\tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right)$	20.34160°	
8	Mounting Distance	a_x	$\frac{zm_n}{2\cos\beta} + H + x_n m_n$	52.965	
9	Pitch Diameter	d	$\frac{zm_n}{\cos\beta}$	50.92956	–
10	Base Diameter	d_b	$d\cos\alpha_t$	47.75343	
11	Addendum	h_a	$m_n(1 + x_n)$	2.500	2.500
12	Whole Depth	h	$2.25m_n$	5.625	
13	Outside Diameter	d_a	$d + 2h_a$	55.929	–
14	Root Diameter	d_f	$d_a - 2h$	44.679	

Table 6-7 The Calculation of a Helical Rack in the Radial System

No.	Item	Symbol	Formula	Example Gear	Example Rack
1	Radial Module	m_t		2.5	
2	Radial Pressure Angle	α_t		20°	
3	Helix Angle	β		10° 57' 49"	
4	Number of Teeth & Helical Hand	z		20 (R)	– (L)
5	Radial Coefficient of Profile Shift	x_t		0	–
6	Pitch Line Height	H		–	27.5
7	Mounting Distance	a_x	$\frac{zm_t}{2} + H + x_t m_t$	52.500	
8	Pitch Diameter	d	zm_t	50.000	–
9	Base Diameter	d_b	$d\cos\alpha_t$	46.98463	
10	Addendum	h_a	$m_t(1 + x_t)$	2.500	2.500
11	Whole Depth	h	$2.25\ m_t$	5.625	
12	Outside Diameter	d_a	$d + 2h_a$	55.000	–
13	Root Diameter	d_f	$d_a - 2h$	43.750	

The formulas of a standard helical rack are similar to those of **Table 6-6** with only the normal coefficient of profile shift $x_n = 0$. To mesh a helical gear to a helical rack, they must have the same helix angle but with opposite hands.

The displacement of the helical rack, l, for one rotation of the mating gear is the product of the radial pitch, p_t, and number of teeth.

$$l = \frac{\pi m_n}{\cos\beta} z = p_t z \qquad \textbf{(6-13)}$$

According to the equations of **Table 6-7**, let radial pitch p_t = 8 mm and displacement l = 160 mm. The radial pitch and the displacement could be modified into integers, if the helix angle were chosen properly.

In the axial system, the linear displacement of the helical rack, l, for one turn of the helical gear equals the integral multiple of radial pitch.

$$l = \pi z m_t \qquad \textbf{(6-14)}$$

SECTION 7 SCREW GEAR OR CROSSED HELICAL GEAR MESHES

These helical gears are also known as spiral gears. They are true helical gears and only differ in their application for interconnecting skew shafts, such as in **Figure 7-1**. Screw gears can be designed to connect shafts at any angle, but in most applications the shafts are at right angles.

7.1 Features

7.1.1 Helix Angle And Hands

The helix angles need not be the same. However, their sum must equal the shaft angle:

$$\beta_1 + \beta_2 = \Sigma \qquad \textbf{(7-1)}$$

where β_1 and β_2 are the respective helix angles of the two gears, and Σ is the shaft angle (the acute angle between the two shafts when viewed in a direction paralleling a common perpendicular between the shafts).

Except for very small shaft angles, the helix hands are the same.

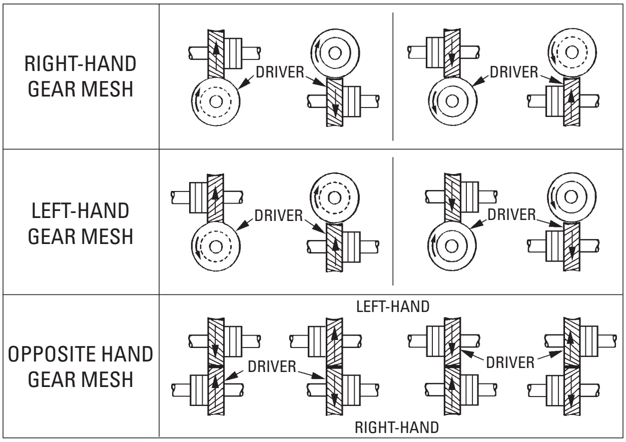

Fig. 7-1 Types of Helical Gear Meshes

NOTES:
1. Helical gears of the same hand operate at right angles.
2. Helical gears of opposite hand operate on parallel shafts.
3. Bearing location indicates the direction of thrust.

7.1.2 Module

Because of the possibility of different helix angles for the gear pair, the radial modules may not be the same. However, the normal modules must always be identical.

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7.1.3 Center Distance

The pitch diameter of a crossed-helical gear is given by **Equation (6-7)**, and the center distance becomes:

$$a = \frac{m_n}{2}\left(\frac{z_1}{\cos\beta_1} + \frac{z_2}{\cos\beta_2}\right) \qquad \textbf{(7-2)}$$

Again, it is possible to adjust the center distance by manipulating the helix angle. However, helix angles of both gears must be altered consistently in accordance with **Equation (7-1)**.

7.1.4 Velocity Ratio

Unlike spur and parallel shaft helical meshes, the velocity ratio (gear ratio) cannot be determined from the ratio of pitch diameters, since these can be altered by juggling of helix angles. The speed ratio can be determined only from the number of teeth, as follows:

$$\text{velocity ratio} = i = \frac{z_1}{z_2} \qquad \textbf{(7-3)}$$

or, if pitch diameters are introduced, the relationship is:

$$i = \frac{z_1 \cos\beta_2}{z_2 \cos\beta_1} \qquad \textbf{(7-4)}$$

7.2 Screw Gear Calculations

Two screw gears can only mesh together under the conditions that normal modules, m_{n1}, and, m_{n2}, and normal pressure angles, α_{n1}, α_{n2}, are the same. Let a pair of screw gears have the shaft angle Σ and helical angles β_1 and β_2:

If they have the same hands, then:
$\Sigma = \beta_1 + \beta_2$

If they have the opposite hands, then:
$\Sigma = \beta_1 - \beta_2$, or $\Sigma = \beta_2 - \beta_1$ **(7-5)**

If the screw gears were profile shifted, the meshing would become a little more complex. Let β_{w1}, β_{w2} represent the working pitch cylinder;

If they have the same hands, then:
$\Sigma = \beta_{w1} + \beta_{w2}$

If they have the opposite hands, then:
$\Sigma = \beta_{w1} - \beta_{w2}$, or $\Sigma = \beta_{w2} - \beta_{w1}$ **(7-6)**

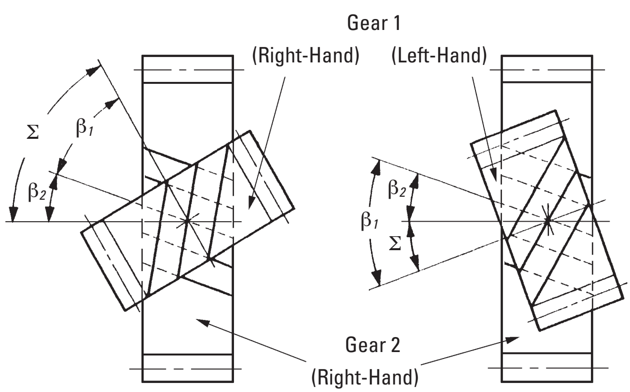


Fig. 7-2 Screw Gears of Nonparallel and Nonintersecting Axes

Table 7-1 presents equations for a profile shifted screw gear pair. When the normal coefficients of profile shift $x_{n1} = x_{n2} = 0$, the equations and calculations are the same as for standard gears.

Table 7-1 The Equations for a Screw Gear Pair on Nonparallel and Nonintersecting Axes in the Normal System

No.	Item	Symbol	Formula	Example Pinion	Example Gear
1	Normal Module	m_n		3	
2	Normal Pressure Angle	α_n		20°	
3	Helix Angle	β		20°	30°
4	Number of Teeth & Helical Hand	z_1, z_2		15 (R)	24 (L)
5	Number of Teeth of an Equivalent Spur Gear	z_v	$\dfrac{z}{\cos^3\beta}$	18.0773	36.9504
6	Radial Pressure Angle	α_t	$\tan^{-1}\left(\dfrac{\tan\alpha_n}{\cos\beta}\right)$	21.1728°	22.7959°
7	Normal Coefficient of Profile Shift	x_n		0.4	0.2
8	Involute Function α_{wn}	inv α_{wn}	$2\tan\alpha_n\left(\dfrac{x_{n1}+x_{n2}}{z_{v1}+z_{v2}}\right)+\text{inv}\,\alpha_n$	0.0228415	
9	Normal Working Pressure Angle	α_{wn}	Find from Involute Function Table	22.9338°	
10	Radial Working Pressure Angle	α_{wt}	$\tan^{-1}\left(\dfrac{\tan\alpha_{wn}}{\cos\beta}\right)$	24.2404°	26.0386°
11	Center Distance Increment Factor	y	$\dfrac{1}{2}(z_{v1}+z_{v2})\left(\dfrac{\cos\alpha_n}{\cos\alpha_{wn}}-1\right)$	0.55977	
12	Center Distance	a_x	$\left(\dfrac{z_1}{2\cos\beta_1}+\dfrac{z_2}{2\cos\beta_2}+y\right)m_n$	67.1925	
13	Pitch Diameter	d	$\dfrac{zm_n}{\cos\beta}$	47.8880	83.1384
14	Base Diameter	d_b	$d\cos\alpha_t$	44.6553	76.6445
15	Working Pitch Diameter	d_{w1} d_{w2}	$2a_x\dfrac{d_1}{d_1+d_2}$ $2a_x\dfrac{d_2}{d_1+d_2}$	49.1155	85.2695
16	Working Helix Angle	β_w	$\tan^{-1}\left(\dfrac{d_w}{d}\tan\beta\right)$	20.4706°	30.6319°
17	Shaft Angle	Σ	$\beta_{w1}+\beta_{w2}$ or $\beta_{w1}-\beta_{w2}$	51.1025°	
18	Addendum	h_{a1} h_{a2}	$(1+y-x_{n2})m_n$ $(1+y-x_{n1})m_n$	4.0793	3.4793
19	Whole Depth	h	$[2.25+y-(x_{n1}+x_{n2})]m_n$	6.6293	
20	Outside Diameter	d_a	$d+2h_a$	56.0466	90.0970
21	Root Diameter	d_f	d_a-2h	42.7880	76.8384

Standard screw gears have relations as follows:

$$\left.\begin{array}{l} d_{w1} = d_1,\ d_{w2} = d_2 \\ \beta_{w1} = \beta_1,\ \beta_{w2} = \beta_2 \end{array}\right\} \qquad \textbf{(7-7)}$$

7.3 Axial Thrust Of Helical Gears

In both parallel-shaft and crossed-shaft applications, helical gears develop an axial thrust load. This is a useless force that loads gear teeth and bearings and must accordingly be considered in the housing and bearing design. In some special instrument designs, this thrust load can be utilized to actuate face clutches, provide a friction drag, or other special purpose. The magnitude of the thrust load depends on the helix angle and is given by the expression:

$$W_T = W^t \tan\beta \qquad \textbf{(7-8)}$$

where

W_T = axial thrust load, and
W^t = transmitted load.

The direction of the thrust load is related to the hand of the gear and the direction of rotation. This is depicted in **Figure 7-1**. When the helix angle is larger than about 20°, the use of double helical gears with opposite hands (**Figure 7-3a**) or herringbone gears (**Figure 7-3b**) is worth considering.

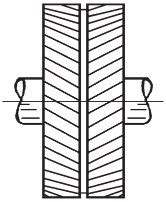


Figure 7-3a

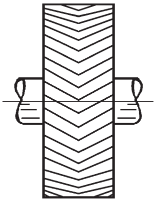


Figure 7-3b

More detail on thrust force of helical gears is presented in **SECTION 16**.

SECTION 8 BEVEL GEARING

For intersecting shafts, bevel gears offer a good means of transmitting motion and power. Most transmissions occur at right angles, **Figure 8-1**, but the shaft angle can be any value. Ratios up to 4:1 are common, although higher ratios are possible as well.

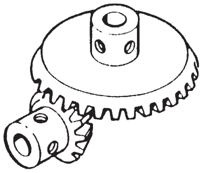
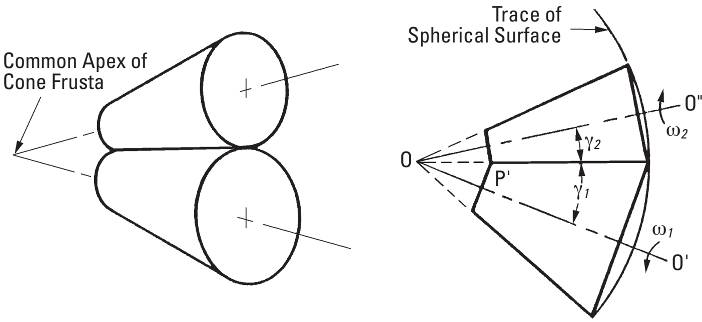


Fig. 8-1 Typical Right Angle Bevel Gear

8.1 Development And Geometry Of Bevel Gears

Bevel gears have tapered elements because they are generated and operate, in theory, on the surface of a sphere. Pitch diameters of mating bevel gears belong to frusta of cones, as shown in **Figure 8-2a**. In the full development on the surface of a sphere, a pair of meshed bevel gears are in conjugate engagement as shown in **Figure 8-2b**.

The crown gear, which is a bevel gear having the largest possible pitch angle (defined in **Figure 8-3**), is analogous to the rack of spur gearing, and is the basic tool for generating bevel gears. However, for practical reasons, the tooth form is not that of a spherical involute, and instead, the crown gear profile assumes a slightly simplified form. Although the deviation from a true spherical involute is minor, it results in a line-of-action having a figure-8 trace in its extreme extension; see **Figure 8-4**. This shape gives rise to the name "octoid" for the tooth form of modern bevel gears.



(a) Pitch Cone Frusta **(b)** Pitch Cones and the Development Sphere

Fig. 8-2 Pitch Cones of Bevel Gears

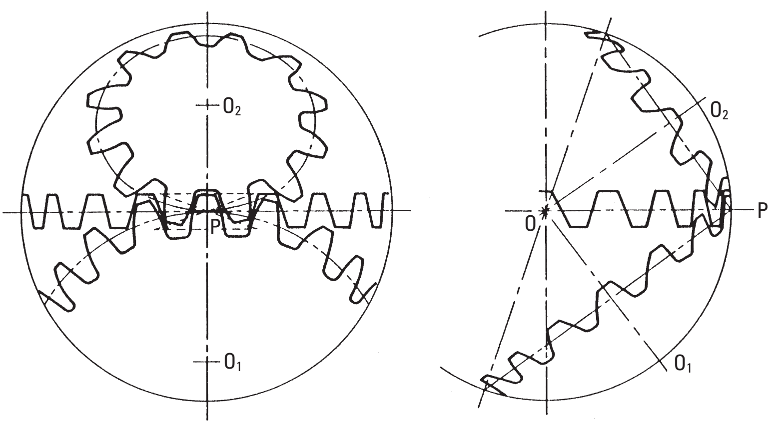


Fig. 8-3 Meshing Bevel Gear Pair with Conjugate Crown Gear

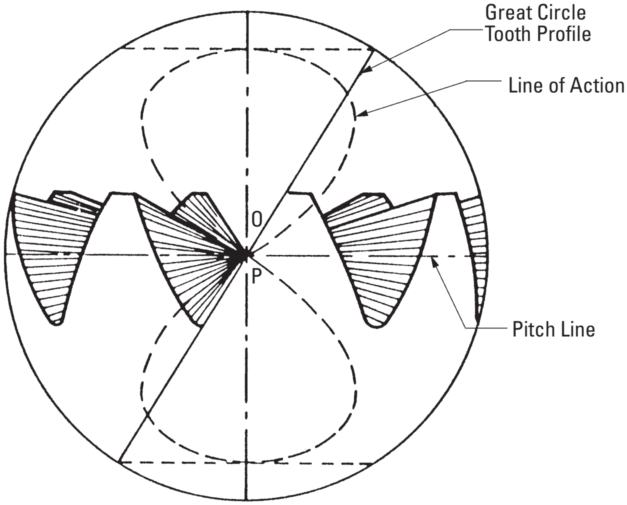


Fig. 8-4 Spherical Basis of Octoid Bevel Crown Gear

8.2 Bevel Gear Tooth Proportions

Bevel gear teeth are proportioned in accordance with the standard system of tooth proportions used for spur gears. However, the pressure angle of all standard design bevel gears is limited to 20°. Pinions with a small number of teeth are enlarged automatically when the design follows the Gleason system.

Since bevel-tooth elements are tapered, tooth dimensions and pitch diameter are referenced to the outer end (heel). Since the narrow end of the teeth (toe) vanishes at the pitch apex (center of reference generating sphere), there is a practical limit to the length (face) of a bevel gear. The geometry and identification of bevel gear parts is given in **Figure 8-5**.

8.3 Velocity Ratio

The velocity ratio, i , can be derived from the ratio of several parameters:

$$i = \frac{z_1}{z_2} = \frac{d_1}{d_2} = \frac{\sin \delta_1}{\sin \delta_2} \qquad \textbf{(8-1)}$$

where: δ = pitch angle (see **Figure 8-5**)

8.4 Forms Of Bevel Teeth *

In the simplest design, the tooth elements are straight radial, converging at the cone apex. However, it is possible to have the teeth curve along a spiral as they converge on the cone apex, resulting in greater tooth overlap, analogous to the overlapping action of helical

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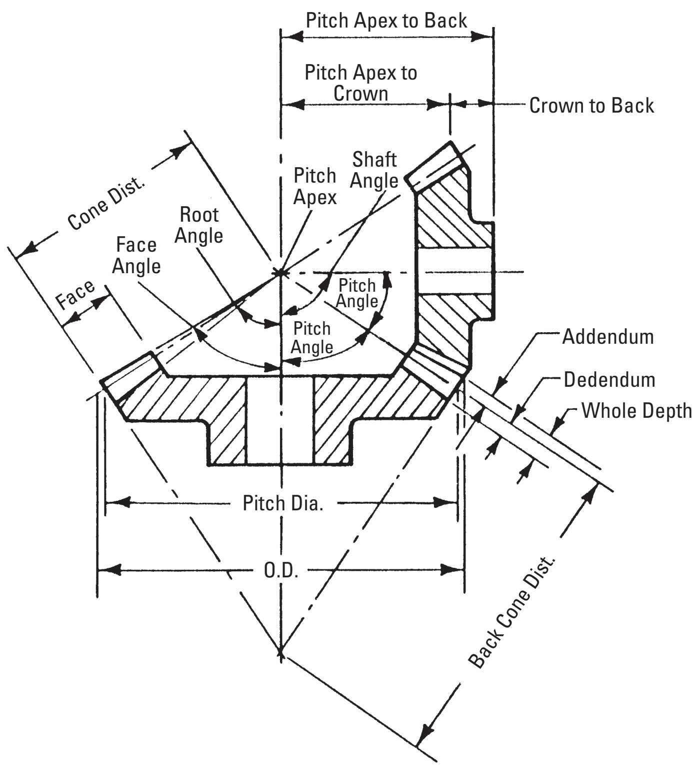

Fig. 8-5 Bevel Gear Pair Design Parameters

teeth. The result is a spiral bevel tooth. In addition, there are other possible variations. One is the zerol bevel, which is a curved tooth having elements that start and end on the same radial line.

Straight bevel gears come in two variations depending upon the fabrication equipment. All current Gleason straight bevel generators are of the Coniflex form which gives an almost imperceptible convexity to the tooth surfaces. Older machines produce true straight elements. See **Figure 8-6a**.

Straight bevel gears are the simplest and most widely used type of bevel gears for the transmission of power and/or motion between intersecting shafts. Straight bevel gears are recommended:

1. When speeds are less than 300 meters/min (1000 feet/min) – at higher speeds, straight bevel gears may be noisy.
2. When loads are light, or for high static loads when surface wear is not a critical factor.
3. When space, gear weight, and mountings are a premium. This includes planetary gear sets, where space does not permit the inclusion of rolling-element bearings.

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Other forms of bevel gearing include the following:

• Coniflex gears (**Figure 8-6b**) are produced by current Gleason straight bevel gear generating machines that crown the sides of the teeth in their lengthwise direction. The teeth, therefore, tolerate small amounts of misalignment in the assembly of the gears and some displacement of the gears under load without concentrating the tooth contact at the ends of the teeth. Thus, for the operating conditions, Coniflex gears are capable of transmitting larger loads than the predecessor Gleason straight bevel gears.

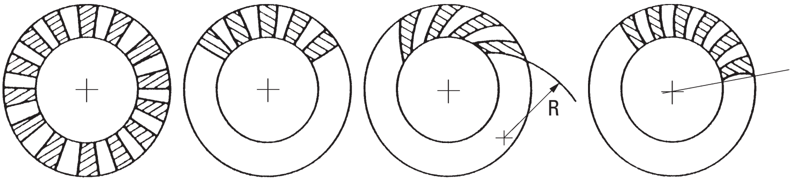
• Spiral bevels (**Figure 8-6c**) have curved oblique teeth which contact each other gradually and smoothly from one end to the other. Imagine cutting a straight bevel into an infinite number of short face width sections, angularly displace one relative to the other, and one has a spiral bevel gear. Well-designed spiral bevels have two or more teeth in contact at all times. The overlapping tooth action transmits motion more smoothly and quietly than with straight bevel gears.

• Zerol bevels (**Figure 8-6d**) have curved teeth similar to those of the spiral bevels, but with zero spiral angle at the middle of the face width; and they have little end thrust.

Both spiral and Zerol gears can be cut on the same machines with the same circular face-mill cutters or ground on the same grinding machines. Both are produced with localized tooth contact which can be controlled for length, width, and shape.

Functionally, however, Zerol bevels are similar to the straight bevels and thus carry the same ratings. In fact, Zerols can be used in the place of straight bevels without mounting changes.

Zerol bevels are widely employed in the aircraft industry, where ground-tooth precision gears are generally required. Most hypoid cutting machines can cut spiral bevel, Zerol or hypoid gears.



(a) Straight Teeth **(b) Coniflex Teeth** (Exaggerated Tooth Curving) **(c) Spiral Teeth** **(d) Zerol Teeth**

Fig. 8-6 Forms of Bevel Gear Teeth

8.5 Bevel Gear Calculations

Let z_1 and z_2 be pinion and gear tooth numbers; shaft angle Σ; and pitch cone angles δ_1 and δ_2; then:

$$\left.\begin{aligned}\tan\delta_1 &= \frac{\sin\Sigma}{\dfrac{z_2}{z_1}+\cos\Sigma}\\ \tan\delta_2 &= \frac{\sin\Sigma}{\dfrac{z_1}{z_2}+\cos\Sigma}\end{aligned}\right\} \quad \text{(8-2)}$$

Generally, shaft angle $\Sigma = 90°$ is most used. Other angles (**Figure 8-7**) are sometimes used. Then, it is called "bevel gear in nonright angle drive". The 90° case is called "bevel gear in right angle drive".

When $\Sigma = 90°$, **Equation (8-2)** becomes:

$$\left.\begin{aligned} \delta_1 &= \tan^{-1}\left(\frac{z_1}{z_2}\right) \\ \delta_2 &= \tan^{-1}\left(\frac{z_2}{z_1}\right) \end{aligned}\right\} \qquad \textbf{(8-3)}$$

Miter gears are bevel gears with $\Sigma = 90°$ and $z_1 = z_2$. Their speed ratio $z_1 / z_2 = 1$. They only change the direction of the shaft, but do not change the speed.

Figure 8-8 depicts the meshing of bevel gears. The meshing must be considered in pairs. It is because the pitch cone angles δ_1 and δ_2 are restricted by the gear ratio z_1 / z_2. In the facial view, which is normal to the contact line of pitch cones, the meshing of bevel gears appears to be similar to the meshing of spur gears.

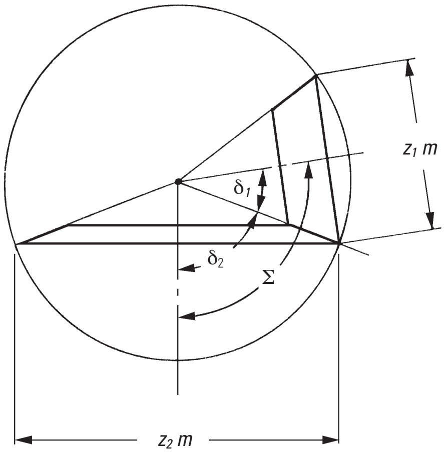


Fig. 8-7 The Pitch Cone Angle of Bevel Gear

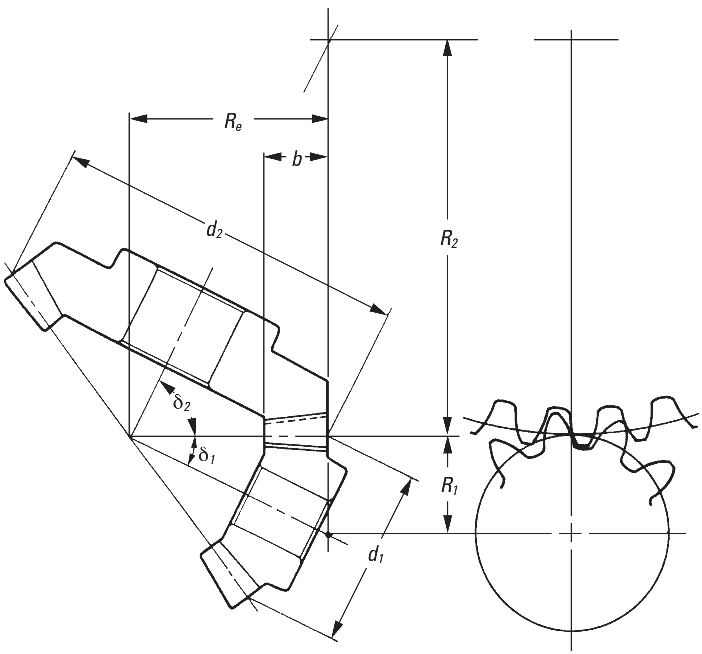


Fig. 8-8 The Meshing of Bevel Gears

8.5.1 Gleason Straight Bevel Gears

The straight bevel gear has straight teeth flanks which are along the surface of the pitch cone from the bottom to the apex. Straight bevel gears can be grouped into the Gleason type and the standard type.

In this section, we discuss the Gleason straight bevel gear. The Gleason Company defined the tooth profile as: whole depth $h = 2.188\ m$; top clearance $c_a = 0.188\ m$; and working depth $h_w = 2.000\ m$.

The characteristics are:

- **Design specified profile shifted gears:**

In the Gleason system, the pinion is positive shifted and the gear is negative shifted. The reason is to distribute the proper strength between the two gears. Miter gears, thus, do not need any shifted tooth profile.

- **The top clearance is designed to be parallel**

The outer cone elements of two paired bevel gears are parallel. That is to ensure that the top clearance along the whole tooth is the same. For the standard bevel gears, top clearance is variable. It is smaller at the toe and bigger at the heel.

Table 8-1 shows the minimum number of teeth to prevent undercut in the Gleason system at the shaft angle $\Sigma = 90°$.

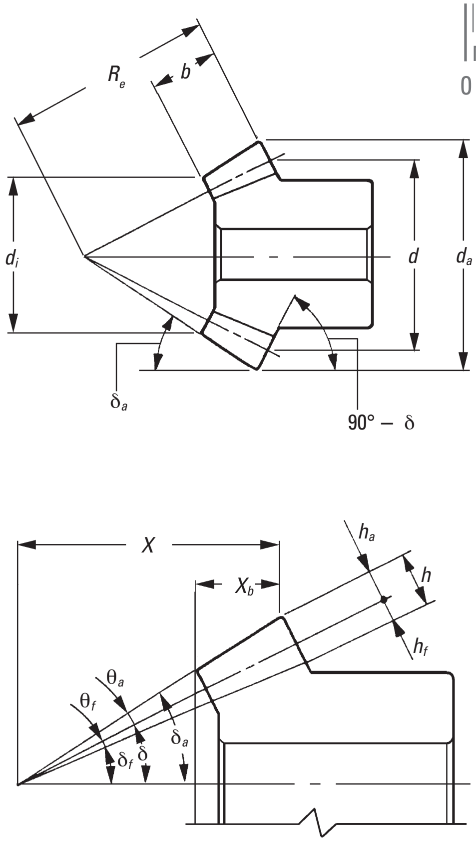


Fig. 8-9 Dimensions and Angles of Bevel Gears

Table 8-1 The Minimum Numbers of Teeth to Prevent Undercut

Pressure Angle	Combination of Numbers of Teeth $\frac{z_1}{z_2}$					
(14.5°)	29 / Over 29	28 / Over 29	27 / Over 31	26 / Over 35	25 / Over 40	24 / Over 57
20°	16 / Over 16	15 / Over 17	14 / Over 20	13 / Over 30	—	—
(25°)	13 / Over 13	—	—	—	—	—

Table 8-2 presents equations for designing straight bevel gears in the Gleason system. The meanings of the dimensions and angles are shown in **Figure 8-9**. All the equations in **Table 8-2** can also be applied to bevel gears with any shaft angle.

Table 8-2 The Calculations of Straight Bevel Gears of the Gleason System

No.	Item	Symbol	Formula	Example Pinion	Example Gear
1	Shaft Angle	Σ		90°	
2	Module	m		3	
3	Pressure Angle	α		20°	
4	Number of Teeth	z_1, z_2		20	40
5	Pitch Diameter	d	zm	60	120
6	Pitch Cone Angle	δ_1 δ_2	$\tan^{-1}\left(\dfrac{\sin\Sigma}{\frac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma-\delta_1$	26.56505°	63.43495°
7	Cone Distance	R_e	$\dfrac{d_2}{2\sin\delta_2}$	67.08204	
8	Face Width	b	It should be less than $R_e/3$ or $10\,m$	22	
9	Addendum	h_{a1} h_{a2}	$2.000\,m-h_{a2}$ $0.540\,m+\dfrac{0.460m}{\left(\frac{z_2\cos\delta_1}{z_1\cos\delta_2}\right)}$	4.035	1.965
10	Dedendum	h_f	$2.188\,m-h_a$	2.529	4.599
11	Dedendum Angle	θ_f	$\tan^{-1}(h_f/R_e)$	2.15903°	3.92194°
12	Addendum Angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	3.92194°	2.15903°
13	Outer Cone Angle	δ_a	$\delta+\theta_a$	30.48699°	65.59398°
14	Root Cone Angle	δ_f	$\delta-\theta_f$	24.40602°	59.51301°
15	Outside Diameter	d_a	$d+2\,h_a\cos\delta$	67.2180	121.7575
16	Pitch Apex to Crown	X	$R_e\cos\delta-h_a\sin\delta$	58.1955	28.2425
17	Axial Face Width	X_b	$\dfrac{b\cos\delta_a}{\cos\theta_a}$	19.0029	9.0969
18	Inner Outside Diameter	d_i	$d_a-\dfrac{2\,b\sin\delta_a}{\cos\theta_a}$	44.8425	81.6609

The straight bevel gear with crowning in the Gleason system is called a Coniflex gear. It is manufactured by a special Gleason "Coniflex" machine. It can successfully eliminate poor tooth wear due to improper mounting and assembly.

The first characteristic of a Gleason straight bevel gear is its profile shifted tooth. From **Figure 8-10**, we can see the positive tooth profile shift in the pinion. The tooth thickness at the root diameter of a Gleason pinion is larger than that of a standard straight bevel gear.

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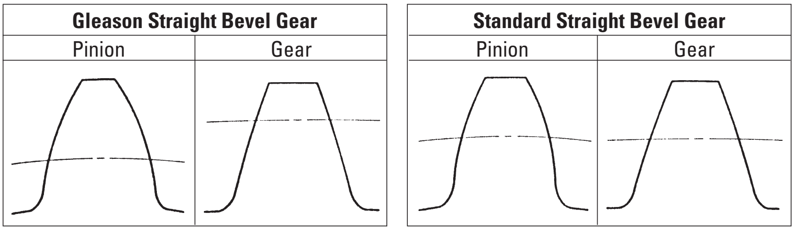

Fig. 8-10 The Tooth Profile of Straight Bevel Gears

8.5.2. Standard Straight Bevel Gears

A bevel gear with no profile shifted tooth is a standard straight bevel gear. The applicable equations are in **Table 8-3**.

Table 8-3 Calculation of a Standard Straight Bevel Gears

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Shaft Angle	Σ		90°	
2	Module	m		3	
3	Pressure Angle	α		20°	
4	Number of Teeth	z_1, z_2		20	40
5	Pitch Diameter	d	zm	60	120
6	Pitch Cone Angle	δ_1 δ_2	$\tan^{-1}\left(\dfrac{\sin\Sigma}{\dfrac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma-\delta_1$	26.56505°	63.43495°
7	Cone Distance	R_e	$\dfrac{d_2}{2\sin\delta_2}$	67.08204	
8	Face Width	b	It should be less than $R_e/3$ or $10\,m$	22	
9	Addendum	h_a	$1.00\,m$	3.00	
10	Dedendum	h_f	$1.25\,m$	3.75	
11	Dedendum Angle	θ_f	$\tan^{-1}(h_f/R_e)$	3.19960°	
12	Addendum Angle	θ_a	$\tan^{-1}(h_a/R_e)$	2.56064°	
13	Outer Cone Angle	δ_a	$\delta+\theta_a$	29.12569°	65.99559°
14	Root Cone Angle	δ_f	$\delta-\theta_f$	23.36545°	60.23535°
15	Outside Diameter	d_a	$d+2\,h_a\cos\delta$	65.3666	122.6833
16	Pitch Apex to Crown	X	$R_e\cos\delta-h_a\sin\delta$	58.6584	27.3167
17	Axial Face Width	X_b	$\dfrac{b\cos\delta_a}{\cos\theta_a}$	19.2374	8.9587
18	Inner Outside Diameter	d_i	$d_a-\dfrac{2\,b\sin\delta_a}{\cos\theta_a}$	43.9292	82.4485

These equations can also be applied to bevel gear sets with other than 90° shaft angle.

8.5.3 Gleason Spiral Bevel Gears

A spiral bevel gear is one with a spiral tooth flank as in **Figure 8-11**. The spiral is generally consistent with the curve of a cutter with the diameter d_c. The spiral angle β is the angle between a generatrix element of the pitch cone and the tooth flank. The spiral angle just at the tooth flank center is called central spiral angle β_m. In practice, spiral angle means central spiral angle.

All equations in **Table 8-6** are dedicated for the manufacturing method of Spread Blade or of Single Side from Gleason. If a gear is not cut per the Gleason system, the equations will be different from these.

The tooth profile of a Gleason spiral bevel gear shown here has the whole depth $h = 1.888\ m$; top clearance $c_a = 0.188\ m$; and working depth $h_w = 1.700\ m$. These Gleason spiral bevel gears belong to a stub gear system. This is applicable to gears with modules $m > 2.1$.

Table 8-4 shows the minimum number of teeth to avoid undercut in the Gleason system with shaft angle $\Sigma = 90°$ and pressure angle $\alpha_n = 20°$.

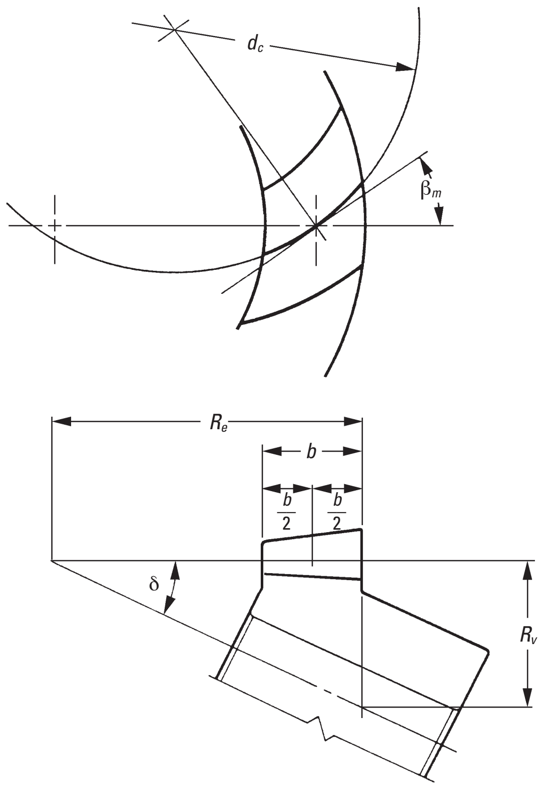

Fig. 8-11 Spiral Bevel Gear (Left-Hand)

Table 8-4 The Minimum Numbers of Teeth to Prevent Undercut $\beta_m = 35°$

Pressure Angle	**Combination of Numbers of Teeth** $\frac{z_1}{z_2}$					
20°	17 / Over 17	16 / Over 18	15 / Over 19	14 / Over 20	13 / Over 22	12 / Over 26

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If the number of teeth is less than 12, **Table 8-5** is used to determine the gear sizes.

Table 8-5 Dimensions for Pinions with Numbers of Teeth Less than 12

Number of Teeth in Pinion		z_1	6	7	8	9	10	11
Number of Teeth in Gear		z_2	Over 34	Over 33	Over 32	Over 31	Over 30	Over 29
Working Depth		h_w	1.500	1.560	1.610	1.650	1.680	1.695
Whole Depth		h	1.666	1.733	1.788	1.832	1.865	1.882
Gear Addendum		h_{a2}	0.215	0.270	0.325	0.380	0.435	0.490
Pinion Addendum		h_{a1}	1.285	1.290	1.285	1.270	1.245	1.205
Circular Tooth Thickness of Gear	s_2	30	0.911	0.957	0.975	0.997	1.023	1.053
		40	0.803	0.818	0.837	0.860	0.888	0.948
		50	—	0.757	0.777	0.828	0.884	0.946
		60	—	—	0.777	0.828	0.883	0.945
Pressure Angle		α_n	20°					
Spiral Angle		β_m	35°... 40°					
Shaft Angle		Σ	90°					

NOTE: All values in the table are based on $m = 1$.

All equations in **Table 8-6** are also applicable to Gleason bevel gears with any shaft angle. A spiral bevel gear set requires matching of hands; left-hand and right-hand as a pair.

8.5.4 Gleason Zerol Spiral Bevel Gears

When the spiral angle $\beta_m = 0$, the bevel gear is called a Zerol bevel gear. The calculation equations of **Table 8-2** for Gleason straight bevel gears are applicable. They also should take care again of the rule of hands; left and right of a pair must be matched. **Figure 8-12** is a left-hand Zerol bevel gear.

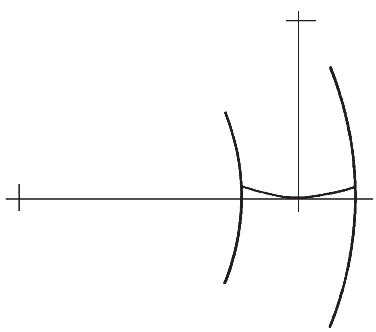

Fig. 8-12 Left-Hand Zerol Bevel Gear

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Table 8-6 The Calculations of Spiral Bevel Gears of the Gleason System

No.	Item	Symbol	Formula	Example: Pinion	Example: Gear
1	Shaft Angle	Σ		90°	
2	Outside Radial Module	m		3	
3	Normal Pressure Angle	α_n		20°	
4	Spiral Angle	β_m		35°	
5	No. of Teeth and Spiral Hand	z_1, z_2		20 (L)	40 (R)
6	Radial Pressure Angle	α_t	$\tan^{-1}\left(\dfrac{\tan\alpha_n}{\cos\beta_m}\right)$	23.95680	
7	Pitch Diameter	d	zm	60	120
8	Pitch Cone Angle	δ_1 δ_2	$\tan^{-1}\left(\dfrac{\sin\Sigma}{\dfrac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma - \delta_1$	26.56505°	63.43495°
9	Cone Distance	R_e	$\dfrac{d_2}{2\sin\delta_2}$	67.08204	
10	Face Width	b	It should be less than R_e / 3 or $10m$	20	
11	Addendum	h_{a1} h_{a2}	$1.700m - h_{a2}$ $0.460m + \dfrac{0.390m}{\left(\dfrac{z_2\cos\delta_1}{z_1\cos\delta_2}\right)}$	3.4275	1.6725
12	Dedendum	h_f	$1.888m - h_a$	2.2365	3.9915
13	Dedendum Angle	θ_f	$\tan^{-1}(h_f / R_e)$	1.90952°	3.40519°
14	Addendum Angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	3.40519°	1.90952°
15	Outer Cone Angle	δ_a	$\delta + \theta_a$	29.97024°	65.34447°
16	Root Cone Angle	δ_f	$\delta - \theta_f$	24.65553°	60.02976°
17	Outside Diameter	d_a	$d + 2h_a\cos\delta$	66.1313	121.4959
18	Pitch Apex to Crown	X	$R_e\cos\delta - h_a\sin\delta$	58.4672	28.5041
19	Axial Face Width	X_b	$\dfrac{b\cos\delta_a}{\cos\theta_a}$	17.3563	8.3479
20	Inner Outside Diameter	d_i	$d_a - \dfrac{2b\sin\delta_a}{\cos\theta_a}$	46.1140	85.1224

SECTION 9 WORM MESH

The worm mesh is another gear type used for connecting skew shafts, usually 90°. See **Figure 9-1**. Worm meshes are characterized by high velocity ratios. Also, they offer the advantage of higher load capacity associated with their line contact in contrast to the point contact of the crossed-helical mesh.

9.1 Worm Mesh Geometry

Although the worm tooth form can be of a variety, the most popular is equivalent to a V-type screw thread, as in **Figure 9-1**. The mating worm gear teeth have a helical lead. (**Note:** The name "worm wheel" is often used interchangeably with "worm gear".) A central section of the mesh, taken through the worm's axis and perpendicular to the worm gear's axis, as shown in **Figure 9-2**, reveals a rack-type tooth of the worm, and a curved involute tooth form for the worm gear. However, the involute features are only true for the central section. Sections on either side of the worm axis reveal nonsymmetric and noninvolute tooth profiles. Thus, a worm gear mesh is not a true involute mesh. Also, for conjugate action, the center distance of the mesh must be an exact duplicate of that used in generating the worm gear.

To increase the length-of-action, the worm gear is made of a throated shape to wrap around the worm.

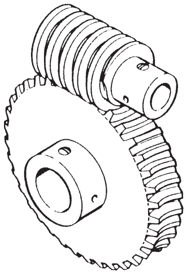


Fig. 9-1 Typical Worm Mesh

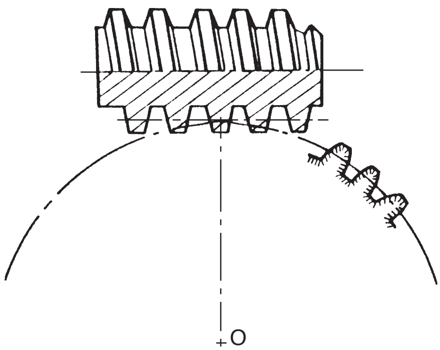


Fig. 9-2 Central Section of a Worm and Worm Gear

9.1.1 Worm Tooth Proportions

Worm tooth dimensions, such as addendum, dedendum, pressure angle, etc., follow the same standards as those for spur and helical gears. The standard values apply to the central section of the mesh. See **Figure 9-3a**. A high pressure angle is favored and in some applications values as high as 25° and 30° are used.

9.1.2 Number Of Threads

The worm can be considered resembling a helical gear with a high helix angle.

For extremely high helix angles, there is one continuous tooth or thread. For slightly smaller angles, there can be two, three or even more threads. Thus, a worm is characterized by the number of threads, z_w.

9.1.3 Pitch Diameters, Lead and Lead Angle

Referring to **Figure 9-3**:

$$\text{Pitch diameter of worm} = d_w = \frac{z_w\, p_n}{\pi \sin \gamma} \qquad \textbf{(9-1)}$$

$$\text{Pitch diameter of worm gear} = d_g = \frac{z_g\, p_n}{\pi \cos \gamma} \qquad \textbf{(9-2)}$$

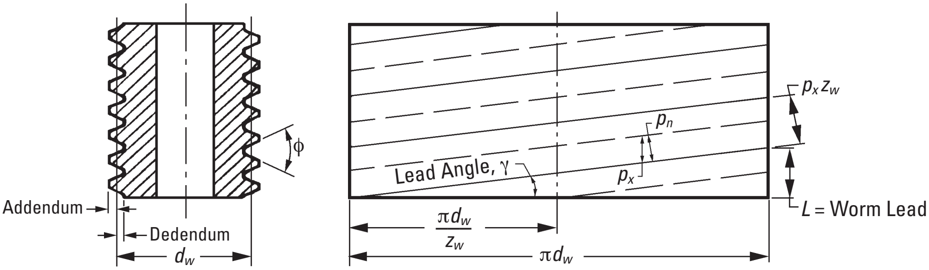
where:

z_w = number of threads of worm; z_g = number of teeth in worm gear

$L = \text{lead of worm} = z_w\, p_x = \dfrac{z_w\, p_n}{\cos \gamma}$

$\gamma = \text{lead angle} = \tan^{-1}\left(\dfrac{z_w\, m}{d_w}\right) = \sin^{-1}\left(\dfrac{z_w\, p_n}{\pi\, d_w}\right)$

$p_n = p_x \cos \gamma$



(a) Tooth Proportion of Central Section

(b) Development of Worm's Pitch Cylinder; Two Thread Example: $z_w = 2$

Fig. 9-3 Worm Tooth Proportions and Geometric Relationships

9.1.4 Center Distance

$$C = \frac{d_w + D_g}{2} = \frac{p_n}{2\pi}\left(\frac{z_g}{\cos \gamma} + \frac{z_w}{\sin \gamma}\right) \qquad \textbf{(9-3)}$$

9.2 Cylindrical Worm Gear Calculations

Cylindrical worms may be considered cylindrical type gears with screw threads. Generally, the mesh has a 90° shaft angle. The number of threads in the worm is equivalent to the number of teeth in a gear of a screw type gear mesh.

Thus, a one-thread worm is equivalent to a one-tooth gear; and two-threads equivalent to two-teeth, etc. Referring to **Figure 9-4**, for a lead angle γ, measured on the pitch cylinder, each rotation of the worm makes the thread advance one lead.

There are four worm tooth profiles in JIS B 1723, as defined below.

Type I Worm: This worm tooth profile is trapezoid in the radial or axial plane.

Type II Worm: This tooth profile is trapezoid viewed in the normal surface.

Type III Worm: This worm is formed by a cutter in which the tooth profile is trapezoid form viewed from the radial surface or axial plane set at the lead angle. Examples are milling and grinding profile cutters.

Type IV Worm: This tooth profile is involute as viewed from the radial surface or at the lead angle. It is an involute helicoid, and is known by that name.

Type III worm is the most popular. In this type, the normal pressure angle α_n has the tendency to become smaller than that of the cutter, α_c.

Per JIS, Type III worm uses a radial module m_t and cutter pressure angle α_c = 20° as the module and pressure angle. A special worm hob is required to cut a Type III worm gear.

Standard values of radial module, m_t, are presented in **Table 9-1**.

Table 9-1 Radial Module of Cylindrical Worm Gears

1	1.25	1.60	2.00	2.50	3.15	4.00	5.00
6.30	8.00	10.00	12.50	16.00	20.00	25.00	—

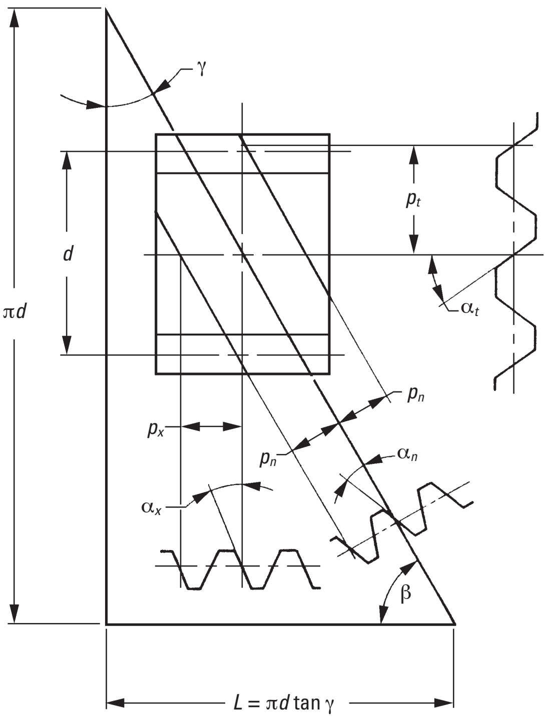


Fig. 9-4 Cylindrical Worm (Right-Hand)

Because the worm mesh couples nonparallel and nonintersecting axes, the radial surface of the worm, or radial cross section, is the same as the normal surface of the worm gear. Similarly, the normal surface of the worm is the radial surface of the worm gear. The common surface of the worm and worm gear is the normal surface. Using the normal module, m_n, is most popular. Then, an ordinary hob can be used to cut the worm gear.

Table 9-2 presents the relationships among worm and worm gear radial surfaces, normal surfaces, axial surfaces, module, pressure angle, pitch and lead.

Table 9-2 The Relations of Cross Sections of Worm Gears

Worm		
Axial Surface	**Normal Surface**	**Radial Surface**
$m_x = \dfrac{m_n}{\cos\gamma}$	m_n	$m_t = \dfrac{m_n}{\sin\gamma}$
$\alpha_x = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\cos\gamma}\right)$	α_n	$\alpha_t = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\sin\gamma}\right)$
$p_x = \pi m_x$	$p_n = \pi m_n$	$p_t = \pi m_t$
$L = \pi m_x z_w$	$L = \dfrac{\pi m_n z_w}{\cos\gamma}$	$L = \pi m_t z_w \tan\gamma$
Radial Surface	**Normal Surface**	**Axial Surface**
Worm Gear		

NOTE: The Radial Surface is the plane perpendicular to the axis.

Reference to **Figure 9-4** can help the understanding of the relationships in **Table 9-2**. They are similar to the relations in **Formulas (6-11)** and **(6-12)** that the helix angle β be substituted by $(90° - \gamma)$. We can consider that a worm with lead angle γ is almost the same as a screw gear with helix angle $(90° - \gamma)$.

9.2.1 Axial Module Worm Gears

Table 9-3 presents the equations, for dimensions shown in **Figure 9-5**, for worm gears with axial module, m_x, and normal pressure angle $\alpha_n = 20°$.

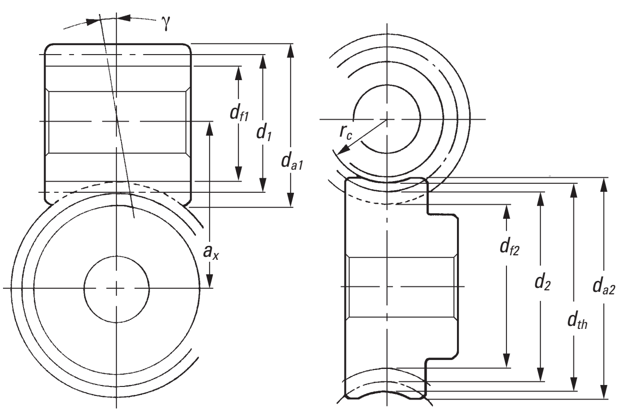


Fig. 9-5 Dimensions of Cylindrical Worm Gears

Table 9-3 The Calculations of Axial Module System Worm Gears (See **Figure 9-5**)

No.	Item	Symbol	Formula	Example	
				Worm	Wheel
1	Axial Module	m_x		3	
2	Normal Pressure Angle	α_n		20°	
3	No. of Threads, No. of Teeth	z_w, z_2		∇	30 (R)
4	Standard Pitch Diameter	d_1 d_2	$Q\,m_x$ **Note 1** $z_2 m_x$	44.000	90.000
5	Lead Angle	γ	$\tan^{-1}\left(\frac{m_x z_w}{d_1}\right)$	7.76517°	
6	Coefficient of Profile Shift	x_{a2}		–	0
7	Center Distance	a_x	$\frac{d_1 + d_2}{2} + x_{a2} m_x$	67.000	
8	Addendum	h_{a1} h_{a2}	$1.00m_x$ $(1.00 + x_{a2})m_x$	3.000	3.000
9	Whole Depth	h	$2.25m_x$	6.750	
10	Outside Diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_2 + 2h_{a2} + m_x$ **Note 2**	50.000	99.000
11	Throat Diameter	d_{th}	$d_2 + 2h_{a2}$	–	96.000
12	Throat Surface Radius	r_i	$\frac{d_1}{2} - h_{a1}$	–	19.000
13	Root Diameter	d_{f1} d_{f2}	$d_{a1} - 2h$ $d_{th} - 2h$	36.500	82.500

∇ Double-Threaded Right-Hand Worm

Note 1: Diameter Factor, Q, means pitch diameter of worm, d_1, over axial module, m_x.

$$Q = \frac{d_1}{m_x}$$

Note 2: There are several calculation methods of worm outside diameter d_{a2} besides those in **Table 9-3**.

Note 3: The length of worm with teeth, b_1, would be sufficient if: $b_1 = \pi\, m_x (4.5 + 0.02 z_2)$

Note 4: Working blank width of worm gear $b_e = 2 m_x \sqrt{(Q + 1)}$. So the actual blank width of $b \geq b_e + 1.5m_x$ would be enough.

9.2.2 Normal Module System Worm Gears

The equations for normal module system worm gears are based on a normal module, m_n, and normal pressure angle, $\alpha_n = 20°$. See **Table 9-4**, on the following page.

Table 9-4 The Calculations of Normal Module System Worm Gears

No.	Item	Symbol	Formula	Example: Worm	Example: Worm Gear
1	Normal Module	m_n		3	
2	Normal Pressure Angle	α_n		20°	
3	No. of Threads, No. of Teeth	z_w, z_2		∇	30 (R)
4	Pitch Diameter of Worm	d_1		44.000	–
5	Lead Angle	γ	$\sin^{-1}\left(\frac{m_n z_w}{d_1}\right)$	7.83748°	
6	Pitch Diameter of Worm Gear	d_2	$\frac{z_2 m_n}{\cos\gamma}$	–	90.8486
7	Coefficient of Profile Shift	x_{n2}		–	-0.1414
8	Center Distance	a_x	$\frac{d_1 + d_2}{2} + x_{n2} m_n$	67.000	
9	Addendum	h_{a1} h_{a2}	$1.00\, m_n$ $(1.00 + x_{n2}) m_n$	3.000	2.5758
10	Whole Depth	h	$2.25 m_n$	6.75	
11	Outside Diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_2 + 2h_{a2} + m_n$	50.000	99.000
12	Throat Diameter	d_{th}	$d_2 + 2h_{a2}$	–	96.000
13	Throat Surface Radius	r_i	$\frac{d_1}{2} - h_{a1}$	–	19.000
14	Root Diameter	d_{f1} d_{f2}	$d_{a1} - 2h$ $d_{th} - 2h$	36.500	82.500

∇ Double-Threaded Right-Hand Worm

Note: All notes are the same as those of **Table 9-3**.

9.3 Crowning Of The Worm Gear Tooth

Crowning is critically important to worm gears (worm wheels). Not only can it eliminate abnormal tooth contact due to incorrect assembly, but it also provides for the forming of an oil film, which enhances the lubrication effect of the mesh. This can favorably impact endurance and transmission efficiency of the worm mesh. There are four methods of crowning worm gears:

1. Cut Worm Gear With A Hob Cutter Of Greater Pitch Diameter Than The Worm.

A crownless worm gear results when it is made by using a hob that has an identical pitch diameter as that of the worm. This crownless worm gear is very difficult to assemble correctly. Proper tooth contact and a complete oil film are usually not possible.

However, it is relatively easy to obtain a crowned worm gear by cutting it with a hob whose pitch diameter is slightly larger than that of the worm. This is shown in **Figure 9-6**. This creates teeth contact in the center region with space for oil film formation.

2. Recut With Hob Center Distance Adjustment.

The first step is to cut the worm gear at standard center distance. This results in no crowning. Then the worm gear is finished with the same hob by recutting with the hob axis shifted parallel to the worm gear axis by $\pm\Delta h$. This results in a crowning effect, shown in **Figure 9-7**.

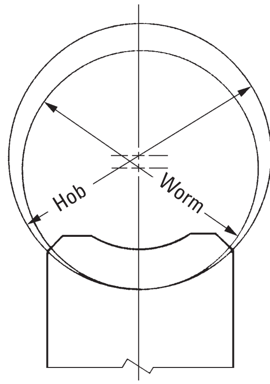


Fig. 9-6 The Method of Using a Greater Diameter Hob

3. Hob Axis Inclining $\Delta\theta$ From Standard Position.

In standard cutting, the hob axis is oriented at the proper angle to the worm gear axis. After that, the hob axis is shifted slightly left and then right, $\Delta\theta$, in a plane parallel to the worm gear axis, to cut a crown effect on the worm gear tooth. This is shown in **Figure 9-8**.

Only method **1** is popular. Methods **2** and **3** are seldom used.

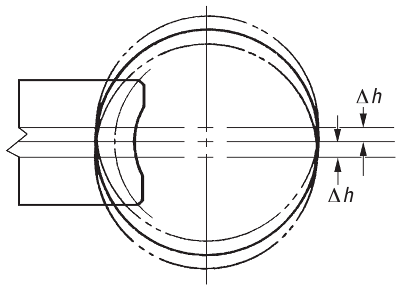


Fig. 9-7 Offsetting Up or Down

4. Use A Worm With A Larger Pressure Angle Than The Worm Gear.

This is a very complex method, both theoretically and practically. Usually, the crowning is done to the worm gear, but in this method the modification is on the worm. That is, to change the pressure angle and pitch of the worm without changing the pitch line parallel to the axis, in accordance with the relationships shown in **Equations 9-4**:

$$p_x \cos\alpha_x = p_x' \cos\alpha_x' \qquad \textbf{(9-4)}$$

In order to raise the pressure angle from before change, α_x', to after change, α_x, it is necessary to increase the axial pitch, p_x', to a new value, p_x, per **Equation (9-4)**. The amount of crowning is represented as the space between the worm and worm gear at the meshing point A in **Figure 9-9**.

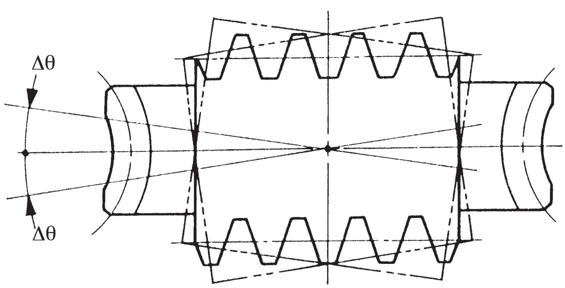


Fig. 9-8 Inclining Right or Left

This amount may be approximated by the following equation:

Amount of Crowning

$$= k \frac{p_x - p_x'}{p_x'} \frac{d_1}{2} \qquad \textbf{(9-5)}$$

where:

d_1 = Pitch diameter of worm
k = Factor from **Table 9-5** and **Figure 9-10**
p_x = Axial pitch after change
p_x' = Axial pitch before change

Table 9-5 The Value of Factor k

α_x	14.5°	17.5°	20°	22.5°
k	0.55	0.46	0.41	0.375

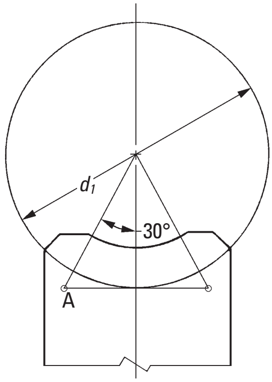


Fig. 9-9 Position A is the Point of Determining Crowning Amount

An example of calculating worm crowning is shown in **Table 9-6**.

Table 9-6 The Calculation of Worm Crowning

No.	Item	Symbol	Formula	Example
	Before Crowning			
1	Axial Module	m_x'		3
2	Normal Pressure Angle	α_n'		20°
3	Number of Threads of Worm	z_w		2
4	Pitch Diameter of Worm	d_1		44.000
5	Lead Angle	γ'	$\tan^{-1}\left(\frac{m_x' z_w}{d_1}\right)$	7.765166°
6	Axial Pressure Angle	α_x'	$\tan^{-1}\left(\frac{\tan \alpha_n'}{\cos \gamma'}\right)$	20.170236°
7	Axial Pitch	p_x'	$\pi m_x'$	9.424778
8	Lead	L'	$\pi m_x' z_v$	18.849556
9	Amount of Crowning	C_R'	*	0.04
10	Factor (k)	k	From **Table 9-5**	0.41
	After Crowning			
11	Axial Pitch	t_x	$t_x' \left(\frac{2C_R}{kd_1} + 1\right)$	9.466573
12	Axial Pressure Angle	α_x	$\cos^{-1}\left(\frac{p_x'}{p_x} \cos \alpha_x'\right)$	20.847973°
13	Axial Module	m_x	$\frac{p_x}{\pi}$	3.013304
14	Lead Angle	γ	$\tan^{-1}\left(\frac{m_x z_w}{d_1}\right)$	7.799179°
15	Normal Pressure Angle	α_n	$\tan^{-1}(\tan \alpha_x \cos \gamma)$	20.671494°
16	Lead	L	$\pi m_x z_w$	18.933146

*It should be determined by considering the size of tooth contact surface.

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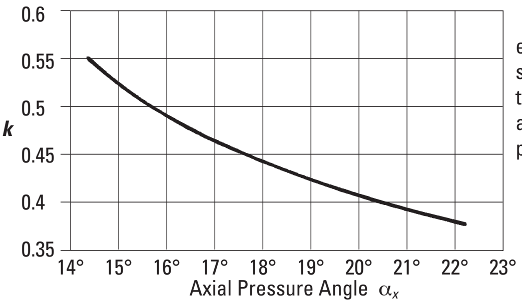

Fig. 9-10 The Value of Factor (k)

Because the theory and equations of these methods are so complicated, they are beyond the scope of this treatment. Usually, all stock worm gears are produced with crowning.

9.4 Self-Locking Of Worm Mesh

Self-locking is a unique characteristic of worm meshes that can be put to advantage. It is the feature that a worm cannot be driven by the worm gear. It is very useful in the design of some equipment, such as lifting, in that the drive can stop at any position without concern that it can slip in reverse. However, in some situations it can be detrimental if the system requires reverse sensitivity, such as a servomechanism.

Self-locking does not occur in all worm meshes, since it requires special conditions as outlined here. In this analysis, only the driving force acting upon the tooth surfaces is considered without any regard to losses due to bearing friction, lubricant agitation, etc. The governing conditions are as follows:

Let F_{u1} = tangential driving force of worm

Then, $F_{u1} = F_n (\cos \alpha_n \sin \gamma - \mu \cos \gamma)$ **(9-6)**

where:
α_n = normal pressure angle
γ = lead angle of worm
μ = coefficient of friction
F_n = normal driving force of worm

If $F_{u1} > 0$ then there is no self-locking effect at all. Therefore, $F_{u1} \le 0$ is the critical limit of self-locking.

Let α_n in **Equation (9-6)** be 20°, then the condition:

$F_{u1} \le 0$ will become:

$(\cos 20° \sin \gamma - \mu \cos \gamma) \le 0$

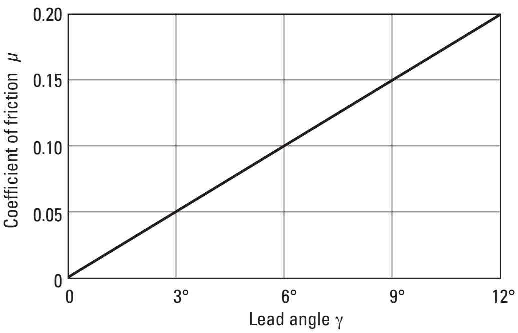

Fig. 9-11 The Critical Limit of Self-locking of Lead Angle γ and Coefficient of Friction μ

Figure 9-11 shows the critical limit of self-locking for lead angle γ and coefficient of friction μ. Practically, it is very hard to assess the exact value of coefficient of friction μ. Further, the bearing loss, lubricant agitation loss, etc. can add many side effects. Therefore, it is not easy to establish precise self-locking conditions. However, it is true that the smaller the lead angle γ, the more likely the self-locking condition will occur.

SECTION 10 TOOTH THICKNESS

There are direct and indirect methods for measuring tooth thickness. In general, there are three methods:

- Chordal Thickness Measurement
- Span Measurement
- Over Pin or Ball Measurement

10.1 Chordal Thickness Measurement

This method employs a tooth caliper that is referenced from the gear's outside diameter. Thickness is measured at the pitch circle. See **Figure 10-1**.

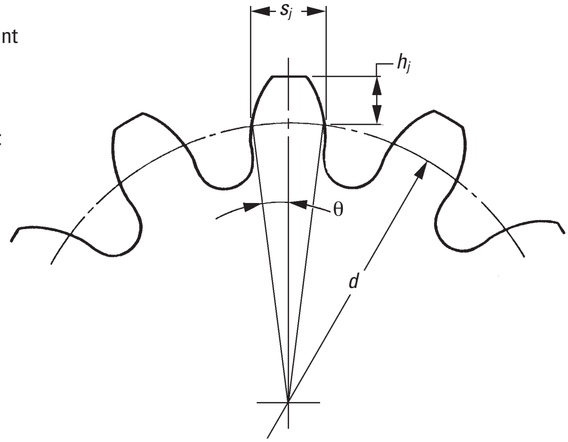

Fig. 10-1 Chordal Thickness Method

10.1.1 Spur Gears

Table 10-1 presents equations for each chordal thickness measurement.

Table 10-1 Equations for Spur Gear Chordal Thickness

No.	Item	Symbol	Formula	Example
1	Circular Tooth Thickness	s	$\left(\frac{\pi}{2} + 2x \tan\alpha\right)m$	m = 10 α = 20° z = 12 x = +0.3 h_a = 13.000 s = 17.8918 θ = 8.54270° s_j = 17.8256 h_j = 13.6657
2	Half of Tooth Angle at Pitch Circle	s_j	$\frac{90}{z} + \frac{360\,x\tan\alpha}{\pi z}$	
3	Chordal Thickness	h_j	$zm\sin\theta$	
4	Chordal Addendum		$\frac{zm}{2}(1-\cos\theta) + h_a$	

10.1.2 Spur Racks And Helical Racks

The governing equations become simple since the rack tooth profile is trapezoid, as shown in **Table 10-2**.

Table 10-2 Chordal Thickness of Racks

No.	Item	Symbol	Formula	Example
1	Chordal Thickness	s_j	$\frac{\pi m}{2}$ or $\frac{\pi m_n}{2}$	m = 3 α = 20° s_j = 4.7124 h_a = 3.0000
2	Chordal Addendum	h_j	h_a	

NOTE: These equations are also applicable to helical racks.

10.1.3 Helical Gears

The chordal thickness of helical gears should be measured on the normal surface basis as shown in **Table 10-3**. **Table 10-4** presents the equations for chordal thickness of helical gears in the radial system.

Table 10-3 Equations for Chordal Thickness of Helical Gears in the Normal System

No.	Item	Symbol	Formula	Example
1	Normal Circular Tooth Thickness	s_n	$\left(\frac{\pi}{2} + 2x_n \tan \alpha_n\right) m_n$	m_n = 5 α_n = 20° β = 25° 00' 00" z = 16 x_n = +0.2 h_a = 6.0000 s_n = 8.5819 z_v = 21.4928 θ_v = 4.57556° s_j = 8.5728 h_j = 6.1712
2	Number of Teeth of an Equivalent Spur Gear	z_v	$\frac{z}{\cos^3 \beta}$	
3	Half of Tooth Angle at Pitch Circle	θ_v	$\frac{90}{z_v} + \frac{360 x_n \tan \alpha_n}{\pi z_v}$	
4	Chordal Thickness	s_j	$z_v m_n \sin \theta_v$	
5	Chordal Addendum	h_j	$\frac{z_v m_n}{2}(1 - \cos \theta_v) + h_a$	

Table 10-4 Equations for Chordal Thickness of Helical Gears in the Radial System

No.	Item	Symbol	Formula	Example
1	Normal Circular Tooth Thickness	s_n	$\left(\frac{\pi}{2} + 2x_t \tan \alpha_t\right) m_t \cos \beta$	m = 4 α_t = 20° β = 22° 30' 00" z = 20 x_t = +0.3 h_a = 4.7184 s_n = 6.6119 z_v = 25.3620 θ_v = 4.04196° s_j = 6.6065 h_j = 4.8350
2	Number of Teeth in an Equivalent Spur Gear	z_v	$\frac{z}{\cos^3 \beta}$	
3	Half of Tooth Angle at Pitch Circle	θ_v	$\frac{90}{z_v} + \frac{360 x_t \tan \alpha_t}{\pi z_v}$	
4	Chordal Thickness	s_j	$z_v m_t \cos \beta \sin \theta_v$	
5	Chordal Addendum	h_j	$\frac{z_v m_t \cos \beta}{2}(1 - \cos \theta_v) + h_a$	

NOTE: Table 10-4 equations are also for the tooth profile of a Sunderland gear.

Table 10-5 Equations for Chordal Thickness of Gleason Straight Bevel Gears

No.	Item	Symbol	Formula	Example
1	Circular Tooth Thickness Factor (Coefficient of Horizontal Profile Shift)	K	Obtain from **Figure 10-2** (on the following page)	m = 4 α = 20° Σ = 90° z_1 = 16 z_2 = 40 $\frac{z_1}{z_2}$ = 0.4 K = 0.0259 h_{a1} = 5.5456 h_{a2} = 2.4544 δ_1 = 21.8014° δ_2 = 68.1986° s_1 = 7.5119 s_2 = 5.0545 s_{j1} = 7.4946 s_{j2} = 5.0536 h_{j1} = 5.7502 h_{j2} = 2.4692
2	Circular Tooth Thickness	s_1 s_2	$\pi m - s_2$ $\frac{\pi m}{2} - (h_{a1} - h_{a2}) \tan \alpha - Km$	
4	Chordal Thickness	s_j	$s - \frac{s^3}{6d^2}$	
5	Chordal Addendum	h_j	$h_a + \frac{s^2 \cos \delta}{4d}$	

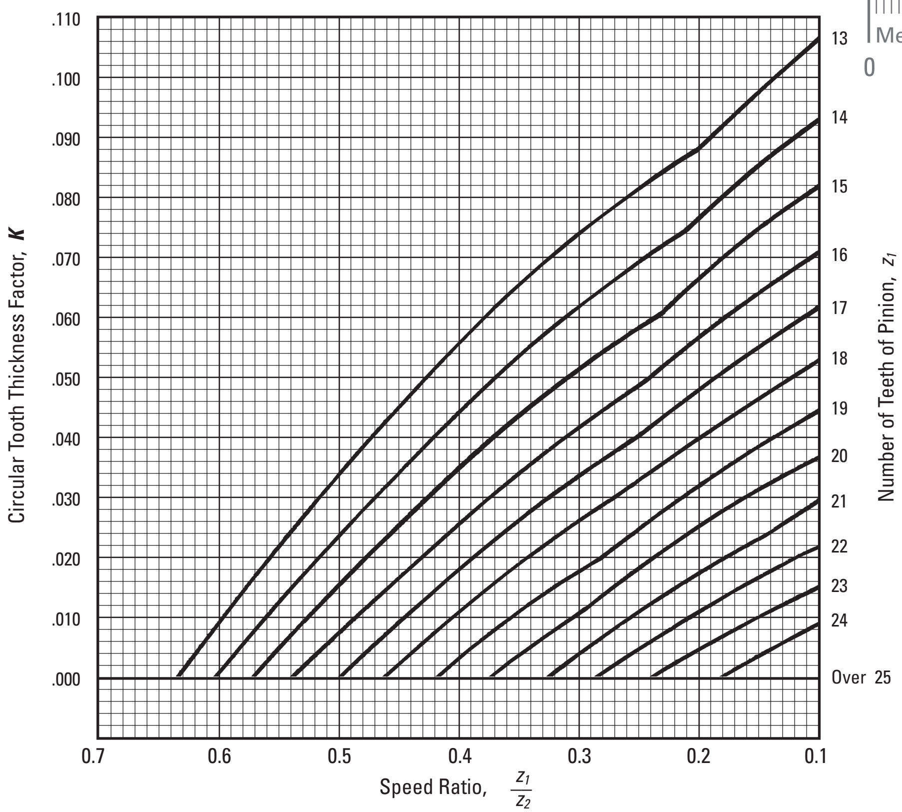

Fig. 10-2 Chart to Determine the Circular Tooth Thickness Factor *K* for Gleason Straight Bevel Gear (See **Table 10-5**)

Table 10-6 presents equations for chordal thickness of a standard straight bevel gear.

Table 10-6 Equations for Chordal Thickness of Standard Straight Bevel Gears

No.	Item	Symbol	Formula	Example
1	Circular Tooth Thickness	s	$\frac{\pi m}{2}$	$m = 4$ $\alpha = 20°$ $\quad \Sigma = 90°$ $z_1 = 16$ $\quad z_2 = 40$ $d_1 = 64$ $\quad d_2 = 160$ $h_a = 4.0000$ $\delta_1 = 21.8014°$ $\quad \delta_2 = 68.1986°$ $s = 6.2832$ $z_{v1} = 17.2325$ $\quad z_{v2} = 107.7033$ $R_{v1} = 34.4650$ $\quad R_{v2} = 215.4066$ $\theta_{v1} = 5.2227°$ $\quad \theta_{v2} = 0.83563°$ $s_{j1} = 6.2745$ $\quad s_{j2} = 6.2830$ $h_{j1} = 4.1431$ $\quad h_{j2} = 4.0229$
2	Number of Teeth of an Equivalent Spur Gear	z_v	$\frac{z}{\cos\delta}$	
3	Back Cone Distance	R_v	$\frac{d}{2\cos\delta}$	
4	Half of Tooth Angle at Pitch Circle	θ_v	$\frac{90}{z_v}$	
5	Chordal Thickness	s_j	$z_v m \sin\theta_v$	
6	Chordal Addendum	h_j	$h_a + R_v(1 - \cos\theta_v)$	

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If a standard straight bevel gear is cut by a Gleason straight bevel cutter, the tooth angle should be adjusted according to:

$$\text{tooth angle } (°) = \frac{180°}{\pi R_e}\left(\frac{s}{2} + h_f \tan\alpha\right) \qquad \textbf{(10-1)}$$

This angle is used as a reference in determining the circular tooth thickness, s, in setting up the gear cutting machine.

Table 10-7 presents equations for chordal thickness of a Gleason spiral bevel gear.

No.	Item	Symbol	Formula	Example
1	Circular Tooth Thickness Factor	K	Obtain from **Figure 10-3**	$\Sigma = 90°$ $m = 3$ $\alpha_n = 20°$ $z_1 = 20$ $z_2 = 40$ $\beta_m = 35°$ $h_{a1} = 3.4275$ $h_{a2} = 1.6725$ $K = 0.060$ $p = 9.4248$ $s_1 = 5.6722$ $s_2 = 3.7526$
2	Circular Tooth Thickness	s_1 s_2	$p - s_2$ $\frac{p}{2} - (h_{a1} - h_{a2})\frac{\tan\alpha_n}{\cos\beta_m} - Km$	

Figure 10-3 is shown on the following page.

The calculations of circular thickness of a Gleason spiral bevel gear are so complicated that we do not intend to go further in this presentation.

10.1.5 Worms And Worm Gears

Table 10-8 presents equations for chordal thickness of axial module worms and worm gears.

Table 10-8 Equations for Chordal Thickness of Axial Module Worms and Worm Gears

No.	Item	Symbol	Formula	Example
1	Axial Circular Tooth Thickness of Worm Radial Circular Tooth Thickness of Worm Gear	s_{x1} s_{x2}	$\frac{\pi m_x}{2}$ $\left(\frac{\pi}{2} + 2x_{x2}\tan\alpha_x\right)m_x$	$m_x = 3$ $\alpha_n = 20°$ $z_w = 2$ $z_2 = 30$ $d_1 = 38$ $d_2 = 90$ $a_x = 65$ $x_{x2} = +0.33333$ $h_{a1} = 3.0000$ $h_{a2} = 4.0000$ $\gamma = 8.97263°$ $\alpha_x = 20.22780°$ $s_{x1} = 4.71239$ $s_{x2} = 5.44934$ $z_{v2} = 31.12885$ $\theta_{v2} = 3.34335°$ $s_{j1} = 4.6547$ $s_{j2} = 5.3796$ $h_{j1} = 3.0035$ $h_{j2} = 4.0785$
2	No. of Teeth in an Equivalent Spur Gear (Worm Gear)	z_{v2}	$\frac{z_2}{\cos^3\gamma}$	
3	Half of Tooth Angle at Pitch Circle (Worm Gear)	θ_{v2}	$\frac{90}{z_{v2}} + \frac{360\, x_{x2}\tan\alpha_x}{\pi z_{v2}}$	
4	Chordal Thickness	s_{j1} s_{j2}	$s_{x1}\cos\gamma$ $z_v m_x \cos\gamma\sin\theta_{v2}$	
5	Chordal Addendum	h_{j1} h_{j2}	$h_{a1} + \frac{(s_{x1}\sin\gamma\cos\gamma)^2}{4d_1}$ $h_{a2} + \frac{z_v m_x\cos\gamma}{2}(1 - \cos\theta_{v2})$	

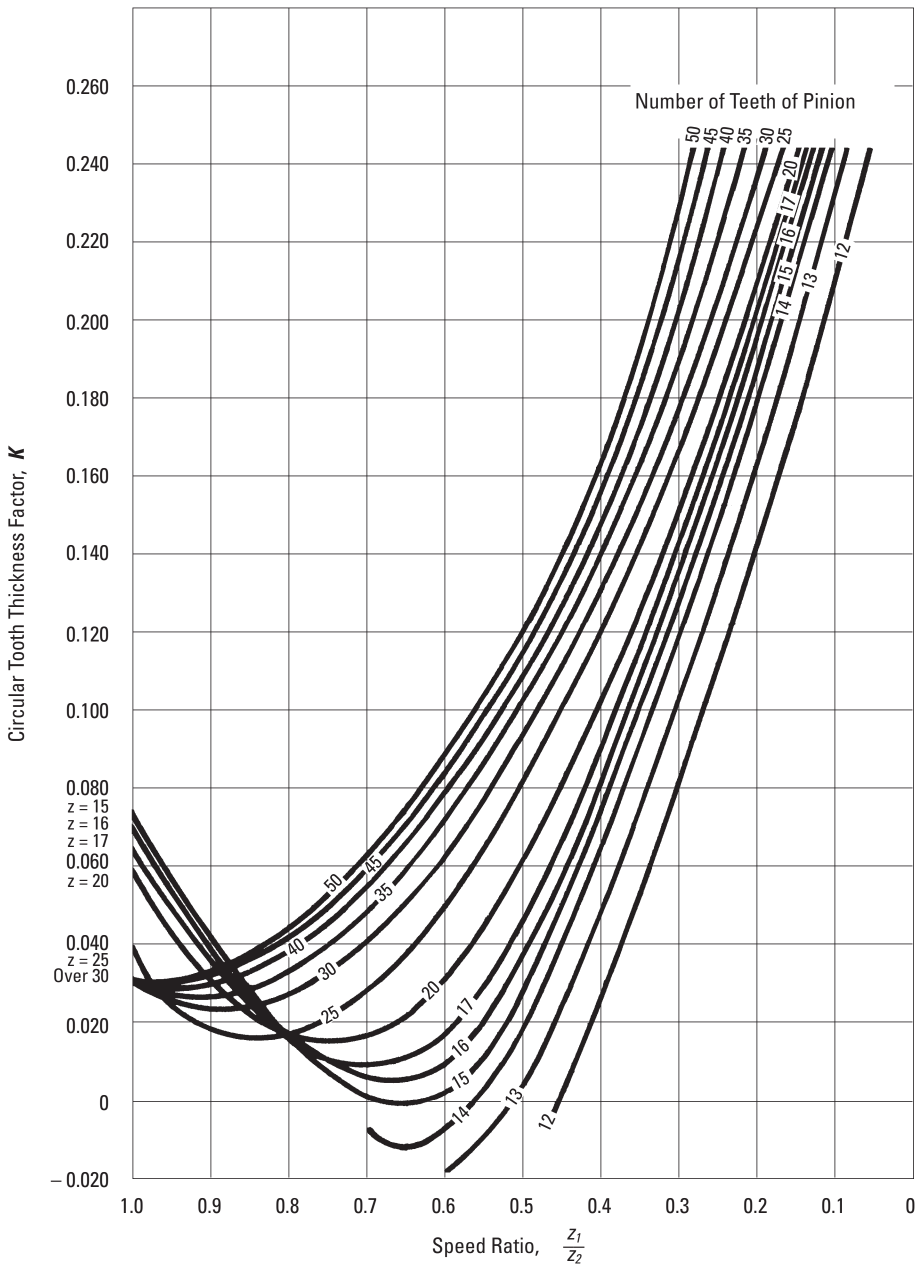

Fig. 10-3 Chart to Determine the Circular Tooth Thickness Factor *K* for Gleason Spiral Bevel Gears

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Table 10-9 contains the equations for chordal thickness of normal module worms and worm gears.

Table 10-9 Equations for Chordal Thickness of Normal Module Worms and Worm Gears

No.	Item	Symbol	Formula	Example
1	Axial Circular Tooth Thickness of Worm Radial Circular Tooth Thickness of Worm Gear	s_{n1} s_{n2}	$\frac{\pi m_n}{2}$ $(\frac{\pi}{2} + 2x_{n2} \tan \alpha_n) m_n$	$m_n = 3$ $\alpha_n = 20°$ $z_w = 2$ $z_2 = 30$ $d_1 = 38$ $d_2 = 91.1433$ $a_x = 65$ $x_{n2} = 0.14278$
2	No. of Teeth in an Equivalent Spur Gear (Worm Gear)	z_{v2}	$\frac{z_2}{\cos^3 \gamma}$	$h_{a1} = 3.0000$ $h_{a2} = 3.42835$ $\gamma = 9.08472°$ $s_{n1} = 4.71239$ $s_{n2} = 5.02419$
3	Half of Tooth Angle at Pitch Circle (Worm Gear)	θ_{v2}	$\frac{90}{z_{v2}} + \frac{360\, x_{n2} \tan \alpha_n}{\pi z_{v2}}$	$z_{v2} = 31.15789$ $\theta_{v2} = 3.07964°$
4	Chordal Thickness	s_{j1} s_{j2}	$s_{n1} \cos \gamma$ $z_v m_n \cos \gamma \sin \theta_{v2}$	$s_{j1} = 4.7124$ $s_{j2} = 5.0218$ $h_{j1} = 3.0036$ $h_{j2} = 3.4958$
5	Chordal Addendum	h_{j1} h_{j2}	$h_{a1} + \frac{(s_{n1} \sin \gamma)^2}{4d_1}$ $h_{a2} + \frac{z_v m_n \cos \gamma}{2}(1 - \cos \theta_{v2})$	

10.2 Span Measurement Of Teeth

Span measurement of teeth, s_m, is a measure over a number of teeth, z_m, made by means of a special tooth thickness micrometer. The value measured is the sum of normal circular tooth thickness on the base circle, s_{bn}, and normal pitch, $p_{en}(z_m - 1)$.

10.2.1 Spur And Internal Gears

The applicable equations are presented in **Table 10-10**.

Table 10-10 Span Measurement of Spur and Internal Gear Teeth

No.	Item	Symbol	Formula	Example
1	Span Number of Teeth	z_m	$z_{mth} = zK(f) + 0.5$ See **NOTE** Select the nearest natural number of z_{mth} as z_m.	$m = 3$ $\alpha = 20°$ $z = 24$ $x = +0.4$
2	Span Measurement	s_m	$m \cos \alpha \; [\pi (z_m - 0.5) + z \operatorname{inv} \alpha] + 2xm \sin \alpha$	$z_{mth} = 3.78787$ $z_m = 4$ $s_m = 32.8266$

NOTE:

$$K(f) = \frac{1}{\pi}\left[\sec \alpha \sqrt{(1 + 2f)^2 - \cos^2 \alpha} - \operatorname{inv} \alpha - 2f \tan \alpha\right] \quad \textbf{(10-2)}$$

where $f = \frac{x}{z}$

Figure 10-4 shows the span measurement of a spur gear. This measurement is on the outside of the teeth.

For internal gears the tooth profile is opposite to that of the external spur gear. Therefore, the measurement is between the inside of the tooth profiles.

10.2.2 Helical Gears

Tables 10-11 and **10-12** present equations for span measurement of the normal and the radial systems, respectively, of helical gears.

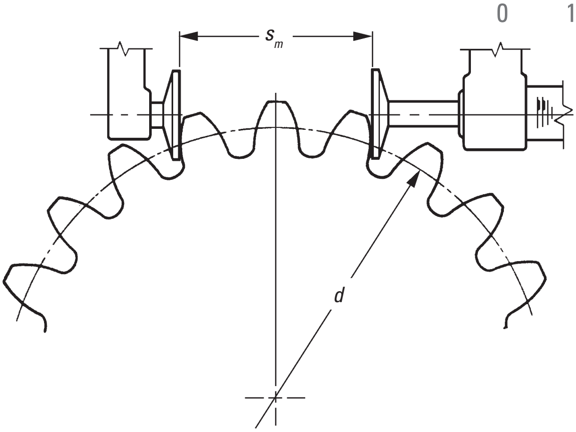

Fig. 10-4 Span Measurement of Teeth (Spur Gear)

Table 10-11 Equations for Span Measurement of the Normal System Helical Gears

No.	Item	Symbol	Formula	Example
1	Span Number of Teeth	z_m	$z_{mth} = zK(f, \beta) + 0.5$ See **NOTE** Select the nearest natural number of z_{mth} as z_m.	$m_n = 3, \ \alpha_n = 20°, \ z = 24$ $\beta = 25° \ 00' \ 00''$ $x_n = +0.4$ $\alpha_s = 21.88023°$ $z_{mth} = 4.63009$ $z_m = 5$ $s_m = 42.0085$
2	Span Measurement	s_m	$m_n \cos\alpha_n [\pi (z_m - 0.5) + z \,\mathrm{inv}\,\alpha_t] + 2x_n m_n \sin\alpha_n$	

NOTE:

$$K(f, \beta) = \frac{1}{\pi}\left[\left(1 + \frac{\sin^2\beta}{\cos^2\beta + \tan^2\alpha_n}\right)\sqrt{(\cos^2\beta + \tan^2\alpha_n)(\sec\beta + 2f)^2 - 1} - \mathrm{inv}\,\alpha_t - 2f\tan\alpha_n\right] \qquad \textbf{(10-3)}$$

where $f = \dfrac{x_n}{z}$

Table 10-12 Equations for Span Measurement of the Radial System Helical Gears

No.	Item	Symbol	Formula	Example
1	Span Number of Teeth	z_m	$z_{mth} = zK(f, \beta) + 0.5$ See **NOTE** Select the nearest natural number of z_{mth} as z_m.	$m_t = 3, \ \alpha_t = 20°, \ z = 24$ $\beta = 22° \ 30' \ 00''$ $x_t = +0.4$ $\alpha_n = 18.58597°$ $z_{mth} = 4.31728$ $z_m = 4$ $s_m = 30.5910$
2	Span Measurement	s_m	$m_t \cos\beta \cos\alpha_n [\pi (z_m - 0.5) + z \,\mathrm{inv}\,\alpha_t] + 2x_t m_t \sin\alpha_n$	

NOTE:

$$K(f, \beta) = \frac{1}{\pi}\left[\left(1 + \frac{\sin^2\beta}{\cos^2\beta + \tan^2\alpha_n}\right)\sqrt{(\cos^2\beta + \tan^2\alpha_n)(\sec\beta + 2f)^2 - 1} - \mathrm{inv}\,\alpha_t - 2f\tan\alpha_n\right] \qquad \textbf{(10-4)}$$

where $f = \dfrac{x_t}{z\cos\beta}$

There is a requirement of a minimum blank width to make a helical gear span measurement. Let b_{min} be the minimum value for blank width. Then

$$b_{min} = s_m \sin \beta_b + \Delta b \qquad \textbf{(10-5)}$$

where β_b is the helix angle at the base cylinder,

$$\begin{aligned} \beta_b &= \tan^{-1} (\tan \beta \cos \alpha_t) \\ &= \sin^{-1} (\sin \beta \cos \alpha_n) \end{aligned} \qquad \textbf{(10-6)}$$

From the above, we can determine that at least 3mm of Δb is required to make a stable measurement of s_m.

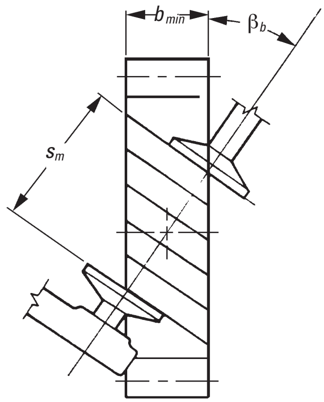

Fig. 10-5 Blank Width of Helical Gear

10.3 Over Pin (Ball) Measurement

As shown in **Figures 10-6** and **10-7**, measurement is made over the outside of two pins that are inserted in diametrically opposite tooth spaces, for even tooth number gears; and as close as possible for odd tooth number gears.

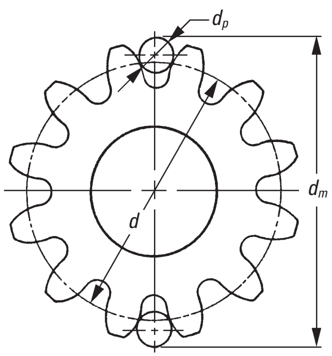

Fig. 10-6 Even Number of Teeth

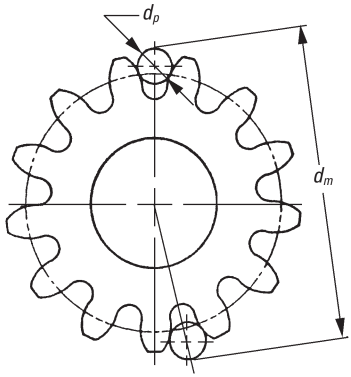

Fig. 10-7 Odd Number of Teeth

The procedure for measuring a rack with a pin or a ball is as shown in **Figure 10-9** by putting pin or ball in the tooth space and using a micrometer between it and a reference surface. Internal gears are similarly measured, except that the measurement is between the pins. See **Figure 10-10**. Helical gears can only be measured with balls. In the case of a worm, three pins are used, as shown in **Figure 10-11**. This is similar to the procedure of measuring a screw thread. All these cases are discussed in detail in the following sections.

Note that gear literature uses "over pins" and "over wires" terminology interchangeably. The "over wires" term is often associated with very fine pitch gears because the diameters are accordingly small.

10.3.1 Spur Gears

In measuring a gear, the size of the pin must be such that the over pins measurement is larger than the gear's outside diameter. An ideal value is one that would place the point of contact (tangent point) of pin and tooth profile at the pitch radius. However, this is not a necessary requirement. Referring to **Figure 10-8**, following are the equations for calculating the over pins measurement for a specific tooth thickness, s, regardless of where the pin contacts the tooth profile:

For even number of teeth:

$$d_m = \frac{d \cos \phi}{\cos \phi_1} + d_p \quad \textbf{(10-7)}$$

For odd number of teeth:

$$d_m = \frac{d \cos \phi}{\cos \phi_1} \cos\left(\frac{90°}{z}\right) + d_p \quad \textbf{(10-8)}$$

where the value of ϕ_1 is obtained from:

$$\text{inv } \phi_1 = \frac{s}{d} + \text{inv } \phi + \frac{d_p}{d \cos \phi} - \frac{\pi}{z} \quad \textbf{(10-9)}$$

When tooth thickness, s, is to be calculated from a known over pins measurement, d_m, the above equations can be manipulated to yield:

$$s = d\left(\frac{\pi}{z} + \text{inv } \phi_c - \text{inv } \phi + \frac{d_p}{d \cos \phi}\right) \quad \textbf{(10-10)}$$

where

$$\cos \phi_c = \frac{d \cos \phi}{2R_c} \quad \textbf{(10-11)}$$

For even number of teeth:

$$R_c = \frac{d_m - d_p}{2} \quad \textbf{(10-12)}$$

For odd number of teeth:

$$R_c = \frac{d_m - d_p}{2 \cos\left(\frac{90°}{z}\right)} \quad \textbf{(10-13)}$$

In measuring a standard gear, the size of the pin must meet the condition that its surface should have the tangent point at the standard pitch circle. While, in measuring a shifted gear, the surface of the pin should have the tangent point at the $d + 2xm$ circle.

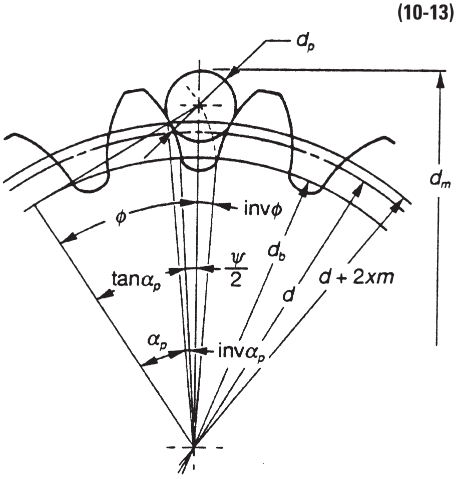

Fig. 10-8 Over Pins Diameter of Spur Gear

I R T 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 A

I R T 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 A

The ideal diameters of pins when calculated from the equations of **Table 10-13** may not be practical. So, in practice, we select a standard pin diameter close to the ideal value. After the actual diameter of pin d_p is determined, the over pin measurement d_m can be calculated from **Table 10-14**.

Table 10-13 Equations for Calculating Ideal Pin Diameters

No.	Item	Symbol	Formula	Example
1	Half Tooth Space Angle at Base Circle	$\frac{\psi}{2}$	$\left(\frac{\pi}{2z} - \text{inv}\,\alpha\right) - \frac{2x\tan\alpha}{z}$	$m = 1$ $\alpha = 20°$ $z = 20$ $x = 0$ $\frac{\psi}{2} = 0.0636354$ $\alpha_p = 20°$ $\phi = 0.4276057$ $d_p = 1.7245$
2	The Pressure Angle at the Point Pin is Tangent to Tooth Surface	α_p	$\cos^{-1}\left[\frac{zm\cos\alpha}{(z+2x)m}\right]$	
3	The Pressure Angle at Pin Center	ϕ	$\tan\alpha_p + \frac{\psi}{2}$	
4	Ideal Pin Diameter	d_p	$zm\cos\alpha\,(\text{inv}\,\phi + \frac{\psi}{2})$	

NOTE: The units of angles $\psi/2$ and ϕ are radians.

Table 10-14 Equations for Over Pins Measurement for Spur Gears

No.	Item	Symbol	Formula	Example
1	Actual Diameter of Pin	d_p	See **NOTE**	Let d_p = 1.7, then: inv ϕ = 0.0268197 ϕ = 24.1350° d_m = 22.2941
2	Involute Function ϕ	inv ϕ	$\frac{d_p}{mz\cos\alpha} - \frac{\pi}{2z} + \text{inv}\,\alpha + \frac{2x\tan\alpha}{z}$	
3	The Pressure Angle at Pin Center	ϕ	Find from Involute Function Table	
4	Over Pins Measurement	d_m	Even Teeth $\frac{zm\cos\alpha}{\cos\phi} + d_p$ Odd Teeth $\frac{zm\cos\alpha}{\cos\phi}\cos\frac{90°}{z} + d_p$	

NOTE: The value of the ideal pin diameter from **Table 10-13**, or its approximate value, is applied as the actual diameter of pin d_p here.

Table 10-15 is a dimensional table under the condition of module $m = 1$ and pressure angle $\alpha = 20°$ with which the pin has the tangent point at $d + 2xm$ circle.

Table 10-15 The Size of Pin which Has the Tangent Point at $d + 2xm$ Circle of Spur Gears

Number of Teeth z	Coefficient of Profile Shift, x $m = 1$, $\alpha = 20°$							
	– 0.4	– 0.2	0	0.2	0.4	0.6	0.8	1.0
10	—	1.6348	1.7886	1.9979	2.2687	2.6079	3.0248	3.5315
20	1.6231	1.6599	1.7245	1.8149	1.9306	2.0718	2.2389	2.4329
30	1.6418	1.6649	1.7057	1.7632	1.8369	1.9267	2.0324	2.1542
40	1.6500	1.6669	1.6967	1.7389	1.7930	1.8589	1.9365	2.0257
50	1.6547	1.6680	1.6915	1.7248	1.7675	1.8196	1.8810	1.9516
60	1.6577	1.6687	1.6881	1.7155	1.7509	1.7940	1.8448	1.9032
70	1.6598	1.6692	1.6857	1.7090	1.7392	1.7759	1.8193	1.8691
80	1.6614	1.6695	1.6839	1.7042	1.7305	1.7625	1.8003	1.8438
90	1.6625	1.6698	1.6825	1.7005	1.7237	1.7521	1.7857	1.8242
100	1.6635	1.6700	1.6814	1.6975	1.7184	1.7439	1.7740	1.8087
110	1.6642	1.6701	1.6805	1.6951	1.7140	1.7372	1.7645	1.7960
120	1.6649	1.6703	1.6797	1.6931	1.7104	1.7316	1.7567	1.7855
130	1.6654	1.6704	1.6791	1.6914	1.7074	1.7269	1.7500	1.7766
140	1.6659	1.6705	1.6785	1.6900	1.7048	1.7229	1.7444	1.7690
150	1.6663	1.6706	1.6781	1.6887	1.7025	1.7195	1.7394	1.7625
160	1.6666	1.6706	1.6777	1.6877	1.7006	1.7164	1.7351	1.7567
170	1.6669	1.6707	1.6773	1.6867	1.6989	1.7138	1.7314	1.7517
180	1.6672	1.6708	1.6770	1.6858	1.6973	1.7114	1.7280	1.7472
190	1.6674	1.6708	1.6767	1.6851	1.6960	1.7093	1.7250	1.7432
200	1.6676	1.6708	1.6764	1.6844	1.6947	1.7074	1.7223	1.7396

10.3.2 Spur Racks And Helical Racks

In measuring a rack, the pin is ideally tangent with the tooth flank at the pitch line. The equations in **Table 10-16** can, thus, be derived. In the case of a helical rack, module m, and pressure angle α, in **Table 10-16**, can be substituted by normal module, m_n, and normal pressure angle, α_n, resulting in **Table 10-16A**.

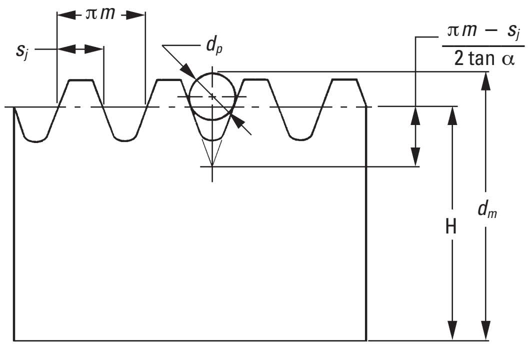

Fig. 10-9 Over Pins Measurement for a Rack Using a Pin or a Ball

Table 10-16 Equations for Over Pins Measurement of Spur Racks

No.	Item	Symbol	Formula	Example
1	Ideal Pin Diameter	d_p'	$\dfrac{\pi m - s_j}{\cos \alpha}$	m = 1 α = 20° s_j = 1.5708 Ideal Pin Diameter d_p' = 1.6716 Actual Pin Diameter d_p = 1.7 H = 14.0000 d_m = 15.1774
2	Over Pins Measurement	d_m	$H - \dfrac{\pi m - s_j}{2 \tan \alpha} + \dfrac{d_p}{2}\left(1 + \dfrac{1}{\sin \alpha}\right)$	

Table 10-16A Equations for Over Pins Measurement of Helical Racks

No.	Item	Symbol	Formula	Example
1	Ideal Pin Diameter	$d_p{}'$	$\dfrac{\pi m_n - s_j}{\cos\alpha_n}$	m_n = 1, α_n = 20°; s_j = 1.5708, β = 15°; Ideal Pin Diameter $d_p{}'$ = 1.6716
2	Over Pins Measurement	d_m	$H - \dfrac{\pi m_n - s_j}{2\tan\alpha_n} + \dfrac{d_p}{2}\left(1 + \dfrac{1}{\sin\alpha_n}\right)$	Actual Pin Diameter d_p = 1.7; H = 14.0000, d_m = 15.1774

10.3.3 Internal Gears

As shown in **Figure 10-10**, measuring an internal gear needs a proper pin which has its tangent point at $d + 2xm$ circle. The equations are in **Table 10-17** for obtaining the ideal pin diameter. The equations for calculating the between pin measurement, d_m, are given in **Table 10-18**.

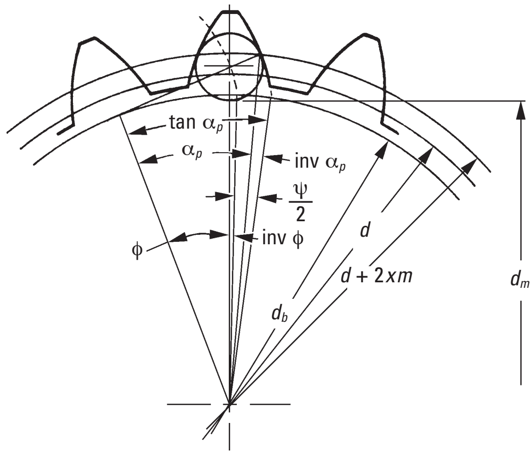

Fig. 10-10 Between Pin Dimension of Internal Gears

Table 10-17 Equations for Calculating Pin Size for Internal Gears

No.	Item	Symbol	Formula	Example
1	Half of Tooth Space Angle at Base Circle	$\dfrac{\psi}{2}$	$\left(\dfrac{\pi}{2z} + \text{inv}\,\alpha\right) + \dfrac{2x\tan\alpha}{z}$	m = 1; α = 20°; z = 40; x = 0
2	The Pressure Angle at the Point Pin is Tangent to Tooth Surface	α_p	$\cos^{-1}\left[\dfrac{zm\cos\alpha}{(z+2x)m}\right]$	$\dfrac{\psi}{2}$ = 0.054174
3	The Pressure Angle at Pin Center	ϕ	$\tan\alpha_p - \dfrac{\psi}{2}$	α_p = 20°; ϕ = 0.309796
4	Ideal Pin Diameter	d_p	$zm\cos\alpha\left(\dfrac{\psi}{2} - \text{inv}\,\phi\right)$	d_p = 1.6489

NOTE: The units of angles $\psi/2$ and ϕ are radians.

I R T 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 A

Table 10-18 Equations for Between Pins Measurement of Internal Gears

No.	Item	Symbol	Formula	Example
1	Actual Diameter of Pin	d_p	See **NOTE**	Let $d_p = 1.7$, then:
2	Involute Function ϕ	inv ϕ	$\left(\frac{\pi}{2z} + \text{inv}\,\alpha\right) - \frac{d_p}{zm\cos\alpha} + \frac{2x\tan\alpha}{z}$	
3	The Pressure Angle at Pin Center	ϕ	Find from Involute Function Table	inv ϕ = 0.0089467 ϕ = 16.9521° d_m = 37.5951
4	Between Pins Measurement	d_m	Even Teeth $\frac{zm\cos\alpha}{\cos\phi} - d_p$	
			Odd Teeth $\frac{zm\cos\alpha}{\cos\phi}\cos\frac{90°}{z} - d_p$	

NOTE: First, calculate the ideal pin diameter. Then, choose the nearest practical actual pin size.

Table 10-19 lists ideal pin diameters for standard and profile shifted gears under the condition of module $m = 1$ and pressure angle $\alpha = 20°$, which makes the pin tangent to the pitch circle $d + 2xm$.

Table 10-19 The Size of Pin that is Tangent at Pitch Circle $d + 2xm$ of Internal Gears

Number of Teeth z	Coefficient of Profile Shift, x $m = 1$, $\alpha = 20°$							
	– 0.4	– 0.2	0	0.2	0.4	0.6	0.8	1.0
10	—	1.4789	1.5936	1.6758	1.7283	1.7519	1.7460	1.7092
20	1.4687	1.5604	1.6284	1.6759	1.7047	1.7154	1.7084	1.6837
30	1.5309	1.5942	1.6418	1.6751	1.6949	1.7016	1.6956	1.6771
40	1.5640	1.6123	1.6489	1.6745	1.6895	1.6944	1.6893	1.6744
50	1.5845	1.6236	1.6533	1.6740	1.6862	1.6900	1.6856	1.6732
60	1.5985	1.6312	1.6562	1.6737	1.6839	1.6870	1.6832	1.6725
70	1.6086	1.6368	1.6583	1.6734	1.6822	1.6849	1.6815	1.6721
80	1.6162	1.6410	1.6600	1.6732	1.6810	1.6833	1.6802	1.6718
90	1.6222	1.6443	1.6612	1.6731	1.6800	1.6820	1.6792	1.6717
100	1.6270	1.6470	1.6622	1.6729	1.6792	1.6810	1.6784	1.6716
110	1.6310	1.6492	1.6631	1.6728	1.6785	1.6801	1.6778	1.6715
120	1.6343	1.6510	1.6638	1.6727	1.6779	1.6794	1.6772	1.6714
130	1.6371	1.6525	1.6644	1.6727	1.6775	1.6788	1.6768	1.6714
140	1.6396	1.6539	1.6649	1.6726	1.6771	1.6783	1.6764	1.6714
150	1.6417	1.6550	1.6653	1.6725	1.6767	1.6779	1.6761	1.6713
160	1.6435	1.6561	1.6657	1.6725	1.6764	1.6775	1.6758	1.6713
170	1.6451	1.6570	1.6661	1.6724	1.6761	1.6772	1.6755	1.6713
180	1.6466	1.6578	1.6664	1.6724	1.6759	1.6768	1.6753	1.6713
190	1.6479	1.6585	1.6666	1.6724	1.6757	1.6766	1.6751	1.6713
200	1.6491	1.6591	1.6669	1.6723	1.6755	1.6763	1.6749	1.6713

I R T 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 A

10.3.4 Helical Gears

The ideal pin that makes contact at the $d + 2x_n m_n$ pitch circle of a helical gear can be obtained from the same above equations, but with the teeth number z substituted by the equivalent (virtual) teeth number z_v.

Table 10-20 presents equations for deriving over pin diameters.

Table 10-20 Equations for Calculating Pin Size for Helical Gears in the Normal System

No.	Item	Symbol	Formula	Example
1	Number of Teeth of an Equivalent Spur Gear	z_v	$\dfrac{z}{\cos^3\beta}$	$m_n = 1$ $\alpha_n = 20°$ $z = 20$ $\beta = 15°\ 00'\ 00''$ $x_n = +0.4$ $z_v = 22.19211$ $\dfrac{\psi_v}{2} = 0.0427566$ $\alpha_v = 24.90647°$ $\phi_v = 0.507078$ $d_p = 1.9020$
2	Half Tooth Space Angle at Base Circle	$\dfrac{\psi_v}{2}$	$\dfrac{\pi}{2z_v} - \mathrm{inv}\ \alpha_n - \dfrac{2x_n \tan\alpha_n}{z_v}$	
3	Pressure Angle at the Point Pin is Tangent to Tooth Surface	α_v	$\cos^{-1}\left(\dfrac{z_v \cos\alpha_n}{z_v + 2x_n}\right)$	
4	Pressure Angle at Pin Center	ϕ_v	$\tan\alpha_v + \dfrac{\psi_v}{2}$	
5	Ideal Pin Diameter	d_p	$z_v m_n \cos\alpha_n \left(\mathrm{inv}\ \phi_v + \dfrac{\psi_v}{2}\right)$	

NOTE: The units of angles $\psi_v/2$ and ϕ_v are radians.

Table 10-21 presents equations for calculating over pin measurements for helical gears in the normal system.

Table 10-21 Equations for Calculating Over Pins Measurement for Helical Gears in the Normal System

No.	Item	Symbol	Formula	Example
1	Actual Pin Diameter	d_p	See **NOTE**	Let $d_p = 2$, then $\alpha_t = 20.646896°$ $\mathrm{inv}\ \phi = 0.058890$ $\phi = 30.8534$ $d_m = 24.5696$
2	Involute Function ϕ	$\mathrm{inv}\ \phi$	$\dfrac{d_p}{m_n z \cos\alpha_n} - \dfrac{\pi}{2z} + \mathrm{inv}\ \alpha_t + \dfrac{2x_n \tan\alpha_n}{z}$	
3	Pressure Angle at Pin Center	ϕ	Find from Involute Function Table	
4	Over Pins Measurement	d_m	Even Teeth: $\dfrac{z m_n \cos\alpha_t}{\cos\beta\cos\phi} + d_p$ Odd Teeth: $\dfrac{z m_n \cos\alpha_t}{\cos\beta\cos\phi}\cos\dfrac{90°}{z} + d_p$	

NOTE: The ideal pin diameter of **Table 10-20**, or its approximate value, is entered as the actual diameter of d_p.

Table 10-22 and **Table 10-23** present equations for calculating pin measurements for helical gears in the radial (perpendicular to axis) system.

Table 10-22 Equations for Calculating Pin Size for Helical Gears in the Radial System

No.	Item	Symbol	Formula	Example
1	Number of Teeth of an Equivalent Spur Gear	z_v	$\dfrac{z}{\cos^3\beta}$	$m_t = 3$ $\alpha_t = 20°$ $z = 36$ $\beta = 33°\ 33'\ 26.3''$ $\alpha_n = 16.87300°$ $x_t = +0.2$ $z_v = 62.20800$ $\dfrac{\psi_v}{2} = 0.014091$ $\alpha_v = 18.26390$ $\phi_v = 0.34411$ $\text{inv}\ \phi_v = 0.014258$ $d_p = 4.2190$
2	Half Tooth Space Angle at Base Circle	$\dfrac{\psi_v}{2}$	$\dfrac{\pi}{2z_v} - \text{inv}\ \alpha_n - \dfrac{2x_t \tan\alpha_t}{z_v}$	
3	Pressure Angle at the Point Pin is tangent to Tooth Surface	α_v	$\cos^{-1}\left(\dfrac{z_v\cos\alpha_n}{z_v + 2\dfrac{x_t}{\cos\beta}}\right)$	
4	Pressure Angle at Pin Center	ϕ_v	$\tan\alpha_v + \dfrac{\psi_v}{2}$	
5	Ideal Pin Diameter	d_p	$z_v m_t \cos\beta\cos\alpha_n\left(\text{inv}\ \phi_v + \dfrac{\psi_v}{2}\right)$	

NOTE: The units of angles $\psi_v/2$ and ϕ_v *are radians.*

Table 10-23 Equations for Calculating Over Pins Measurement for Helical Gears in the Radial System

No.	Item	Symbol	Formula	Example
1	Actual Pin Diameter	d_p	See **NOTE**	$d_p = 4.2190$ $\text{inv}\ \phi = 0.024302$ $\phi = 23.3910$ $d_m = 114.793$
2	Involute Function ϕ	$\text{inv}\ \phi$	$\dfrac{d_p}{m_t z\cos\beta\cos\alpha_n} - \dfrac{\pi}{2z} + \text{inv}\ \alpha_t + \dfrac{2x_t\tan\alpha_t}{z}$	
3	Pressure Angle at Pin Center	ϕ	Find from Involute Function Table	
4	Over Pins Measurement	d_m	Even Teeth: $\dfrac{zm_t\cos\alpha_t}{\cos\phi} + d_p$ Odd Teeth: $\dfrac{zm_t\cos\alpha_t}{\cos\phi}\cos\dfrac{90°}{z} + d_p$	

NOTE: The ideal pin diameter of **Table 10-22**, or its approximate value, is applied as the actual diameter of pin d_p here.

10.3.5 Three Wire Method Of Worm Measurement

The teeth profile of Type III worms which are most popular are cut by standard cutters with a pressure angle $\alpha_c = 20°$. This results in the normal pressure angle of the worm being a bit smaller than 20°.The equation below shows how to calculate a Type III worm in an AGMA system.

$$\alpha_n = \alpha_c - \frac{90°}{z_w} \frac{r}{r_c \cos^2\gamma + r} \sin^3\gamma \qquad \textbf{(10-14)}$$

where:

r = Worm Pitch Radius
r_c = Cutter Radius
z_w = Number of Threads
γ = Lead Angle of Worm

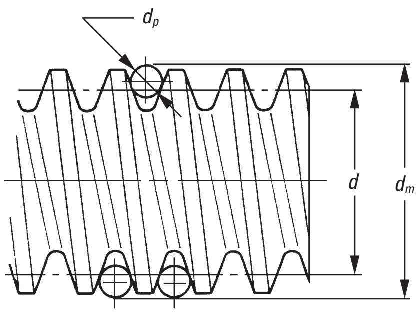

Fig. 10-11 Three Wire Method of a Worm

The exact equation for a three wire method of Type III worm is not only difficult to comprehend, but also hard to calculate precisely. We will introduce two approximate calculation methods here:

(a) Regard the tooth profile of the worm as a linear tooth profile of a rack and apply its equations. Using this system, the three wire method of a worm can be calculated by **Table 10-24**.

Table 10-24 Equations for Three Wire Method of Worm Measurement, (a)-1

No.	Item	Symbol	Formula	Example
1	Ideal Pin Diameter	d_p'	$\dfrac{\pi m_x}{2\cos\alpha_x}$	m_x = 2, α_n = 20° z_w = 1, d_1 = 31 γ = 3.691386°, α_x = 20.03827° d_p' = 3.3440; let d_p = 3.3 d_m = 35.3173
2	Three Wire Measurement	d_m	$d_1 - \dfrac{\pi m_x}{2\tan\alpha_x} + d_p\left(1 + \dfrac{1}{\sin\alpha_x}\right)$	

These equations presume the worm lead angle to be very small and can be neglected. Of course, as the lead angle gets larger, the equations' error gets correspondingly larger. If the lead angle is considered as a factor, the equations are as in **Table 10-25**.

Table 10-25 Equations for Three Wire Method of Worm Measurement, (a)-2

No.	Item	Symbol	Formula	Example
1	Ideal Pin Diameter	d_p'	$\dfrac{\pi m_n}{2\cos\alpha_n}$	m_x = 2, α_n = 20° z_w = 1, d_1 = 31 γ = 3.691386° m_n = 1.99585 d_p' = 3.3363; let d_p = 3.3 d_m = 35.3344
2	Three Wire Measurement	d_m	$d_1 - \dfrac{\pi m_n}{2\tan\alpha_n} + d_p\left(1 + \dfrac{1}{\sin\alpha_n}\right) - \dfrac{(d_p \cos\alpha_n \sin\gamma)^2}{2d_1}$	

(b) Consider a worm to be a helical gear.

This means applying the equations for calculating over pins measurement of helical gears to the case of three wire method of a worm. Because the tooth profile of Type III worm is not an involute curve, the method yields an approximation. However, the accuracy is adequate in practice.

Tables 10-26 and **10-27** contain equations based on the axial system. **Tables 10-28** and **10-29** are based on the normal system.

Table 10-26 Equation for Calculating Pin Size for Worms in the Axial System, (b)-1

No.	Item	Symbol	Formula	Example
1	Number of Teeth of an Equivalent Spur Gear	z_v	$\dfrac{z_w}{\cos^3(90-\gamma)}$	$m_x = 2$ $\alpha_n = 20°$ $z_w = 1$ $d_1 = 31$ $\gamma = 3.691386°$ $z_v = 3747.1491$ $\dfrac{\psi_v}{2} = -0.014485$ $\alpha_v = 20°$ $\phi_v = 0.349485$ inv $\phi_v = 0.014960$ $d_p = 3.3382$
2	Half Tooth Space Angle at Base Circle	$\dfrac{\psi_v}{2}$	$\dfrac{\pi}{2z_v} - \text{inv}\,\alpha_n$	
3	Pressure Angle at the Point Pin is Tangent to Tooth Surface	α_v	$\cos^{-1}\left(\dfrac{z_v\cos\alpha_n}{z_v}\right)$	
4	Pressure Angle at Pin Center	ϕ_v	$\tan\alpha_v + \dfrac{\psi_v}{2}$	
5	Ideal Pin Diameter	d_p	$z_v m_x \cos\gamma\ \cos\alpha_n\left(\text{inv}\,\phi_v + \dfrac{\psi_v}{2}\right)$	

NOTE: The units of angles $\psi_v/2$ and ϕ_v are radians.

Table 10-27 Equation for Three Wire Method for Worms in the Axial System, (b)-2

No.	Item	Symbol	Formula	Example
1	Actual Pin Size	d_p	See **NOTE 1**	Let $d_p = 3.3$ $\alpha_t = 79.96878°$ inv $\alpha_t = 4.257549$ inv $\phi = 4.446297$ $\phi = 80.2959°$ $d_m = 35.3345$
2	Involute Function ϕ	inv ϕ	$\dfrac{d_p}{m_x z_w \cos\gamma\cos\alpha_n} - \dfrac{\pi}{2z_w} + \text{inv}\,\alpha_t$	
3	Pressure Angle at Pin Center	ϕ	Find from Involute Function Table	
4	Three Wire Measurement	d_m	$\dfrac{z_w m_x \cos\alpha_t}{\tan\gamma\cos\phi} + d_p$	

NOTE: 1. The value of ideal pin diameter from **Table 10-26**, or its approximate value, is to be used as the actual pin diameter, d_p.

2. $\alpha_t = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\sin\gamma}\right)$

Table 10-28 shows the calculation of a worm in the normal module system. Basically, the normal module system and the axial module system have the same form of equations. Only the notations of module make them different.

Table 10-28 Equation for Calculating Pin Size for Worms in the Normal System, (b)-3

No.	Item	Symbol	Formula	Example
1	Number of Teeth of an Equivalent Spur Gear	z_v	$\dfrac{z_w}{\cos^3(90-\gamma)}$	$m_n = 2.5$ $\alpha_n = 20°$ $z_w = 1$ $d_1 = 37$ $\gamma = 3.874288°$ $z_v = 3241.792$ $\dfrac{\psi_v}{2} = -0.014420$ $\alpha_v = 20°$ $\phi_v = 0.349550$ $\text{inv}\,\phi_v = 0.0149687$ $d_p = 4.1785$
2	Half of Tooth Space Angle at Base Circle	$\dfrac{\psi_v}{2}$	$\dfrac{\pi}{2z_v} - \text{inv}\,\alpha_n$	
3	Pressure Angle at the Point Pin is Tangent to Tooth Surface	α_v	$\cos^{-1}\left(\dfrac{z_v \cos\alpha_n}{z_v}\right)$	
4	Pressure Angle at Pin Center	ϕ_v	$\tan\alpha_v + \dfrac{\psi_v}{2}$	
5	Ideal Pin Diameter	d_p	$z_v m_n \cos\alpha_n\left(\text{inv}\,\phi_v + \dfrac{\psi_v}{2}\right)$	

NOTE: The units of angles $\psi_v/2$ and ϕ_v are radians.

Table 10-29 Equations for Three Wire Method for Worms in the Normal System, (b)-4

No.	Item	Symbol	Formula	Example
1	Actual Pin Size	d_p	See **NOTE 1**	$d_p = 4.2$ $\alpha_t = 79.48331°$ $\text{inv}\,\alpha_t = 3.999514$ $\text{inv}\,\phi = 4.216536$ $\phi = 79.8947°$ $d_m = 42.6897$
2	Involute Function ϕ	$\text{inv}\,\phi$	$\dfrac{d_p}{m_n z_w \cos\alpha_n} - \dfrac{\pi}{2\,z_w} + \text{inv}\,\alpha_t$	
3	Pressure Angle at Pin Center	ϕ	Find from Involute Function Table	
4	Three Wire Measurement	d_m	$\dfrac{z_w m_n \cos\alpha_t}{\sin\gamma\cos\phi} + d_p$	

NOTE: 1. The value of ideal pin diameter from **Table 10-28**, or its approximate value, is to be used as the actual pin diameter, d_p.

2. $\alpha_t = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\sin\gamma}\right)$

10.4 Over Pins Measurements For Fine Pitch Gears With Specific Numbers Of Teeth

Table 10-30 presents measurements for metric gears. These are for standard ideal tooth thicknesses. Measurements can be adjusted accordingly to backlash allowance and tolerance; i.e., tooth thinning.

TABLE 10-30 METRIC GEAR OVER PINS MEASUREMENT

Pitch Diameter and Measurement Over Wires for External, Module Type Gears, 20-Degree Pressure Angle

No. of Teeth	Module 0.30 Wire Size = 0.5184mm; 0.0204 Inch				Module 0.40 Wire Size = 0.6912mm; 0.0272 Inch				No. of Teeth
	Pitch Diameter		Meas. Over Wire		Pitch Diameter		Meas. Over Wire		
	mm	Inch	mm	Inch	mm	Inch	mm	Inch	
5	1.500	0.0591			2.000	0.0787			5
6	1.800	0.0709			2.400	0.0945			6
7	2.100	0.0827			2.800	0.1102			7
8	2.400	0.0945			3.200	0.1260			8
9	2.700	0.1063			3.600	0.1417			9
10	3.000	0.1181			4.000	0.1575			10
11	3.300	0.1299			4.400	0.1732			11
12	3.600	0.1417			4.800	0.1890			12
13	3.900	0.1535			5.200	0.2047			13
14	4.200	0.1654			5.600	0.2205			14
15	4.500	0.1772			6.000	0.2362			15
16	4.800	0.1890			6.400	0.2520			16
17	5.100	0.2008			6.800	0.2677			17
18	5.400	0.2126	6.115	0.2408	7.200	0.2835	8.154	0.3210	18
19	5.700	0.2244	6.396	0.2518	7.600	0.2992	8.528	0.3357	19
20	6.000	0.2362	6.717	0.2644	8.000	0.3150	8.956	0.3526	20
21	6.300	0.2480	7.000	0.2756	8.400	0.3307	9.333	0.3674	21
22	6.600	0.2598	7.319	0.2881	8.800	0.3465	9.758	0.3842	22
23	6.900	0.2717	7.603	0.2993	9.200	0.3622	10.137	0.3991	23
24	7.200	0.2835	7.920	0.3118	9.600	0.3780	10.560	0.4157	24
25	7.500	0.2953	8.205	0.3230	10.000	0.3937	10.940	0.4307	25
26	7.800	0.3071	8.521	0.3355	10.400	0.4094	11.361	0.4473	26
27	8.100	0.3189	8.808	0.3468	10.800	0.4252	11.743	0.4623	27
28	8.400	0.3307	9.122	0.3591	11.200	0.4409	12.163	0.4789	28
29	8.700	0.3425	9.410	0.3705	11.600	0.4567	12.546	0.4939	29
30	9.000	0.3543	9.723	0.3828	12.000	0.4724	12.964	0.5104	30
31	9.300	0.3661	10.011	0.3941	12.400	0.4882	13.348	0.5255	31
32	9.600	0.3780	10.324	0.4065	12.800	0.5039	13.765	0.5419	32
33	9.900	0.3898	10.613	0.4178	13.200	0.5197	14.150	0.5571	33
34	10.200	0.4016	10.925	0.4301	13.600	0.5354	14.566	0.5735	34
35	10.500	0.4134	11.214	0.4415	14.000	0.5512	14.952	0.5887	35
36	10.800	0.4252	11.525	0.4538	14.400	0.5669	15.367	0.6050	36
37	11.100	0.4370	11.815	0.4652	14.800	0.5827	15.754	0.6202	37
38	11.400	0.4488	12.126	0.4774	15.200	0.5984	16.168	0.6365	38
39	11.700	0.4606	12.417	0.4888	15.600	0.6142	16.555	0.6518	39
40	12.000	0.4724	12.727	0.5010	16.000	0.6299	16.969	0.6681	40
41	12.300	0.4843	13.018	0.5125	16.400	0.6457	17.357	0.6833	41
42	12.600	0.4961	13.327	0.5247	16.800	0.6614	17.769	0.6996	42
43	12.900	0.5079	13.619	0.5362	17.200	0.6772	18.158	0.7149	43
44	13.200	0.5197	13.927	0.5483	17.600	0.6929	18.570	0.7311	44
45	13.500	0.5315	14.219	0.5598	18.000	0.7087	18.959	0.7464	45
46	13.800	0.5433	14.528	0.5720	18.400	0.7244	19.371	0.7626	46
47	14.100	0.5551	14.820	0.5835	18.800	0.7402	19.760	0.7780	47
48	14.400	0.5669	15.128	0.5956	19.200	0.7559	20.171	0.7941	48
49	14.700	0.5787	15.421	0.6071	19.600	0.7717	20.561	0.8095	49
50	15.000	0.5906	15.729	0.6192	20.000	0.7874	20.972	0.8257	50
51	15.300	0.6024	16.022	0.6308	20.400	0.8031	21.362	0.8410	51
52	15.600	0.6142	16.329	0.6429	20.800	0.8189	21.772	0.8572	52
53	15.900	0.6260	16.622	0.6544	21.200	0.8346	22.163	0.8726	53
54	16.200	0.6378	16.929	0.6665	21.600	0.8504	22.573	0.8887	54
55	16.500	0.6496	17.223	0.6781	22.000	0.8661	22.964	0.9041	55
56	16.800	0.6614	17.530	0.6901	22.400	0.8819	23.373	0.9202	56
57	17.100	0.6732	17.823	0.7017	22.800	0.8976	23.764	0.9356	57
58	17.400	0.6850	18.130	0.7138	23.200	0.9134	24.173	0.9517	58
59	17.700	0.6969	18.424	0.7253	23.600	0.9291	24.565	0.9671	59
60	18.000	0.7087	18.730	0.7374	24.000	0.9449	24.974	0.9832	60
61	18.300	0.7205	19.024	0.7490	24.400	0.9606	25.366	0.9987	61
62	18.600	0.7323	19.331	0.7610	24.800	0.9764	25.774	1.0147	62
63	18.900	0.7441	19.625	0.7726	25.200	0.9921	26.166	1.0302	63
64	19.200	0.7559	19.931	0.7847	25.600	1.0079	26.574	1.0462	64
65	19.500	0.7677	20.225	0.7963	26.000	1.0236	26.967	1.0617	65
66	19.800	0.7795	20.531	0.8083	26.400	1.0394	27.375	1.0777	66
67	20.100	0.7913	20.826	0.8199	26.800	1.0551	27.767	1.0932	67
68	20.400	0.8031	21.131	0.8319	27.200	1.0709	28.175	1.1093	68
69	20.700	0.8150	21.426	0.8435	27.600	1.0866	28.568	1.1247	69
70	21.000	0.8268	21.731	0.8556	28.000	1.1024	28.975	1.1408	70
71	21.300	0.8386	22.026	0.8672	28.400	1.1181	29.368	1.1562	71
72	21.600	0.8504	22.332	0.8792	28.800	1.1339	29.776	1.1723	72
73	21.900	0.8622	22.627	0.8908	29.200	1.1496	30.169	1.1877	73
74	22.200	0.8740	22.932	0.9028	29.600	1.1654	30.576	1.2038	74

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TABLE 10-30 (Cont.) **METRIC GEAR OVER PINS MEASUREMENT**
Pitch Diameter and Measurement Over Wires for External, Module Type Gears, 20-Degree Pressure Angle

No. of Teeth	Module 0.30 Wire Size = 0.5184mm; 0.0204 Inch				Module 0.40 Wire Size = 0.6912mm; 0.0272 Inch				No. of Teeth
	Pitch Diameter		Meas. Over Wire		Pitch Diameter		Meas. Over Wire		
	mm	Inch	mm	Inch	mm	Inch	mm	Inch	
75	22.500	0.8858	23.227	0.9144	30.000	1.1811	30.969	1.2193	75
76	22.800	0.8976	23.532	0.9265	30.400	1.1969	31.376	1.2353	76
77	23.100	0.9094	23.827	0.9381	30.800	1.2126	31.770	1.2508	77
78	23.400	0.9213	24.132	0.9501	31.200	1.2283	32.176	1.2668	78
79	23.700	0.9331	24.428	0.9617	31.600	1.2441	32.570	1.2823	79
80	24.000	0.9449	24.732	0.9737	32.000	1.2598	32.977	1.2983	80
81	24.300	0.9567	25.028	0.9853	32.400	1.2756	33.370	1.3138	81
82	24.600	0.9685	25.333	0.9973	32.800	1.2913	33.777	1.3298	82
83	24.900	0.9803	25.628	1.0090	33.200	1.3071	34.171	1.3453	83
84	25.200	0.9921	25.933	1.0210	33.600	1.3228	34.577	1.3613	84
85	25.500	1.0039	26.228	1.0326	34.000	1.3386	34.971	1.3768	85
86	25.800	1.0157	26.533	1.0446	34.400	1.3543	35.377	1.3928	86
87	26.100	1.0276	26.829	1.0562	34.800	1.3701	35.771	1.4083	87
88	26.400	1.0394	27.133	1.0682	35.200	1.3858	36.177	1.4243	88
89	26.700	1.0512	27.429	1.0799	35.600	1.4016	36.572	1.4398	89
90	27.000	1.0630	27.733	1.0919	36.000	1.4173	36.977	1.4558	90
91	27.300	1.0748	28.029	1.1035	36.400	1.4331	37.372	1.4713	91
92	27.600	1.0866	28.333	1.1155	36.800	1.4488	37.778	1.4873	92
93	27.900	1.0984	28.629	1.1271	37.200	1.4646	38.172	1.5029	93
94	28.200	1.1102	28.933	1.1391	37.600	1.4803	38.578	1.5188	94
95	28.500	1.1220	29.230	1.1508	38.000	1.4961	38.973	1.5344	95
96	28.800	1.1339	29.533	1.1627	38.400	1.5118	39.378	1.5503	96
97	29.100	1.1457	29.830	1.1744	38.800	1.5276	39.773	1.5659	97
98	29.400	1.1575	30.134	1.1864	39.200	1.5433	40.178	1.5818	98
99	29.700	1.1693	30.430	1.1980	39.600	1.5591	40.573	1.5974	99
100	30.000	1.1811	30.734	1.2100	40.000	1.5748	40.978	1.6133	100
101	30.300	1.1929	31.030	1.2217	40.400	1.5906	41.373	1.6289	101
102	30.600	1.2047	31.334	1.2336	40.800	1.6063	41.778	1.6448	102
103	30.900	1.2165	31.630	1.2453	41.200	1.6220	42.174	1.6604	103
104	31.200	1.2283	31.934	1.2572	41.600	1.6378	42.579	1.6763	104
105	31.500	1.2402	32.230	1.2689	42.000	1.6535	42.974	1.6919	105
106	31.800	1.2520	32.534	1.2809	42.400	1.6693	43.379	1.7078	106
107	32.100	1.2638	32.831	1.2925	42.800	1.6850	43.774	1.7234	107
108	32.400	1.2756	33.134	1.3045	43.200	1.7008	44.179	1.7393	108
109	32.700	1.2874	33.431	1.3162	43.600	1.7165	44.574	1.7549	109
110	33.000	1.2992	33.734	1.3281	44.000	1.7323	44.979	1.7708	110
111	33.300	1.3110	34.031	1.3398	44.400	1.7480	45.374	1.7864	111
112	33.600	1.3228	34.334	1.3517	44.800	1.7638	45.779	1.8023	112
113	33.900	1.3346	34.631	1.3634	45.200	1.7795	46.175	1.8179	113
114	34.200	1.3465	34.934	1.3754	45.600	1.7953	46.579	1.8338	114
115	34.500	1.3583	35.231	1.3871	46.000	1.8110	46.975	1.8494	115
116	34.800	1.3701	35.534	1.3990	46.400	1.8268	47.379	1.8653	116
117	35.100	1.3819	35.831	1.4107	46.800	1.8425	47.775	1.8809	117
118	35.400	1.3937	36.135	1.4226	47.200	1.8583	48.179	1.8968	118
119	35.700	1.4055	36.431	1.4343	47.600	1.8740	48.575	1.9124	119
120	36.000	1.4173	36.735	1.4462	48.000	1.8898	48.979	1.9283	120
121	36.300	1.4291	37.032	1.4579	48.400	1.9055	49.375	1.9439	121
122	36.600	1.4409	37.335	1.4699	48.800	1.9213	49.780	1.9598	122
123	36.900	1.4528	37.632	1.4816	49.200	1.9370	50.176	1.9754	123
124	37.200	1.4646	37.935	1.4935	49.600	1.9528	50.580	1.9913	124
125	37.500	1.4764	38.232	1.5052	50.000	1.9685	50.976	2.0069	125
126	37.800	1.4882	38.535	1.5171	50.400	1.9843	51.380	2.0228	126
127	38.100	1.5000	38.832	1.5288	50.800	2.0000	51.776	2.0384	127
128	38.400	1.5118	39.135	1.5407	51.200	2.0157	52.180	2.0543	128
129	38.700	1.5236	39.432	1.5524	51.600	2.0315	52.576	2.0699	129
130	39.000	1.5354	39.735	1.5644	52.000	2.0472	52.980	2.0858	130
131	39.300	1.5472	40.032	1.5761	52.400	2.0630	53.376	2.1014	131
132	39.600	1.5591	40.335	1.5880	52.800	2.0787	53.780	2.1173	132
133	39.900	1.5709	40.632	1.5997	53.200	2.0945	54.176	2.1329	133
134	40.200	1.5827	40.935	1.6116	53.600	2.1102	54.580	2.1488	134
135	40.500	1.5945	41.232	1.6233	54.000	2.1260	54.976	2.1644	135
136	40.800	1.6063	41.535	1.6352	54.400	2.1417	55.380	2.1803	136
137	41.100	1.6181	41.832	1.6469	54.800	2.1575	55.777	2.1959	137
138	41.400	1.6299	42.135	1.6589	55.200	2.1732	56.180	2.2118	138
139	41.700	1.6417	42.433	1.6706	55.600	2.1890	56.577	2.2274	139
140	42.000	1.6535	42.735	1.6825	56.000	2.2047	56.980	2.2433	140
141	42.300	1.6654	43.033	1.6942	56.400	2.2205	57.377	2.2589	141
142	42.600	1.6772	43.335	1.7061	56.800	2.2362	57.780	2.2748	142
143	42.900	1.6890	43.633	1.7178	57.200	2.2520	58.177	2.2904	143
144	43.200	1.7008	43.935	1.7297	57.600	2.2677	58.580	2.3063	144

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TABLE 10-30 (Cont.) **METRIC GEAR OVER PINS MEASUREMENT**

Pitch Diameter and Measurement Over Wires for External, Module Type Gears, 20-Degree Pressure Angle

No. of Teeth	Module 0.30 Wire Size = 0.5184mm; 0.0204 Inch				Module 0.40 Wire Size = 0.6912mm; 0.0272 Inch				No. of Teeth
	Pitch Diameter		Meas. Over Wire		Pitch Diameter		Meas. Over Wire		
	mm	Inch	mm	Inch	mm	Inch	mm	Inch	
145	43.500	1.7126	44.233	1.7414	58.000	2.2835	58.977	2.3219	145
146	43.800	1.7244	44.535	1.7534	58.400	2.2992	59.381	2.3378	146
147	44.100	1.7362	44.833	1.7651	58.800	2.3150	59.777	2.3534	147
148	44.400	1.7480	45.135	1.7770	59.200	2.3307	60.181	2.3693	148
149	44.700	1.7598	45.433	1.7887	59.600	2.3465	60.577	2.3849	149
150	45.000	1.7717	45.735	1.8006	60.000	2.3622	60.981	2.4008	150
151	45.300	1.7835	46.033	1.8123	60.400	2.3780	61.377	2.4164	151
152	45.600	1.7953	46.336	1.8242	60.800	2.3937	61.781	2.4323	152
153	45.900	1.8071	46.633	1.8360	61.200	2.4094	62.178	2.4479	153
154	46.200	1.8189	46.936	1.8479	61.600	2.4252	62.581	2.4638	154
155	46.500	1.8307	47.233	1.8596	62.000	2.4409	62.978	2.4794	155
156	46.800	1.8425	47.536	1.8715	62.400	2.4567	63.381	2.4953	156
157	47.100	1.8543	47.833	1.8832	62.800	2.4724	63.778	2.5109	157
158	47.400	1.8661	48.136	1.8951	63.200	2.4882	64.181	2.5268	158
159	47.700	1.8780	48.433	1.9068	63.600	2.5039	64.578	2.5424	159
160	48.000	1.8898	48.736	1.9187	64.000	2.5197	64.981	2.5583	160
161	48.300	1.9016	49.033	1.9305	64.400	2.5354	65.378	2.5739	161
162	48.600	1.9134	49.336	1.9424	64.800	2.5512	65.781	2.5898	162
163	48.900	1.9252	49.633	1.9541	65.200	2.5669	66.178	2.6054	163
164	49.200	1.9370	49.936	1.9660	65.600	2.5827	66.581	2.6213	164
165	49.500	1.9488	50.234	1.9777	66.000	2.5984	66.978	2.6369	165
166	49.800	1.9606	50.536	1.9896	66.400	2.6142	67.381	2.6528	166
167	50.100	1.9724	50.834	2.0013	66.800	2.6299	67.778	2.6684	167
168	50.400	1.9843	51.136	2.0132	67.200	2.6457	68.181	2.6843	168
169	50.700	1.9961	51.434	2.0249	67.600	2.6614	68.578	2.6999	169
170	51.000	2.0079	51.736	2.0368	68.000	2.6772	68.981	2.7158	170
171	51.300	2.0197	52.034	2.0486	68.400	2.6929	69.378	2.7314	171
172	51.600	2.0315	52.336	2.0605	68.800	2.7087	69.781	2.7473	172
173	51.900	2.0433	52.634	2.0722	69.200	2.7244	70.178	2.7629	173
174	52.200	2.0551	52.936	2.0841	69.600	2.7402	70.581	2.7788	174
175	52.500	2.0669	53.234	2.0958	70.000	2.7559	70.979	2.7944	175
176	52.800	2.0787	53.536	2.1077	70.400	2.7717	71.381	2.8103	176
177	53.100	2.0906	53.834	2.1194	70.800	2.7874	71.779	2.8259	177
178	53.400	2.1024	54.136	2.1313	71.200	2.8031	72.181	2.8418	178
179	53.700	2.1142	54.434	2.1431	71.600	2.8189	72.579	2.8574	179
180	54.000	2.1260	54.736	2.1550	72.000	2.8346	72.981	2.8733	180
181	54.300	2.1378	55.034	2.1667	72.400	2.8504	73.379	2.8889	181
182	54.600	2.1496	55.336	2.1786	72.800	2.8661	73.782	2.9048	182
183	54.900	2.1614	55.634	2.1903	73.200	2.8819	74.179	2.9204	183
184	55.200	2.1732	55.936	2.2022	73.600	2.8976	74.582	2.9363	184
185	55.500	2.1850	56.234	2.2139	74.000	2.9134	74.979	2.9519	185
186	55.800	2.1969	56.536	2.2258	74.400	2.9291	75.382	2.9678	186
187	56.100	2.2087	56.834	2.2376	74.800	2.9449	75.779	2.9834	187
188	56.400	2.2205	57.136	2.2495	75.200	2.9606	76.182	2.9993	188
189	56.700	2.2323	57.434	2.2612	75.600	2.9764	76.579	3.0149	189
190	57.000	2.2441	57.736	2.2731	76.000	2.9921	76.982	3.0308	190
191	57.300	2.2559	58.036	2.2849	76.400	3.0079	77.382	3.0465	191
192	57.600	2.2677	58.336	2.2967	76.800	3.0236	77.782	3.0623	192
193	57.900	2.2795	58.636	2.3085	77.200	3.0394	78.182	3.0780	193
194	58.200	2.2913	58.936	2.3203	77.600	3.0551	78.582	3.0938	194
195	58.500	2.3031	59.236	2.3321	78.000	3.0709	78.982	3.1095	195
196	58.800	2.3150	59.536	2.3440	78.400	3.0866	79.382	3.1253	196
197	59.100	2.3268	59.836	2.3558	78.800	3.1024	79.782	3.1410	197
198	59.400	2.3386	60.136	2.3676	79.200	3.1181	80.182	3.1568	198
199	59.700	2.3504	60.436	2.3794	79.600	3.1339	80.582	3.1725	199
200	60.000	2.3622	60.736	2.3912	80.000	3.1496	80.982	3.1883	200
201	60.300	2.3740	61.035	2.4029	80.400	3.1654	81.379	3.2039	201
202	60.600	2.3858	61.335	2.4147	80.800	3.1811	81.780	3.2197	202
203	60.900	2.3976	61.635	2.4266	81.200	3.1969	82.180	3.2354	203
204	61.200	2.4094	61.935	2.4384	81.600	3.2126	82.580	3.2512	204
205	61.500	2.4213	62.235	2.4502	82.000	3.2283	82.980	3.2669	205
240	72.000	2.8346	72.737	2.8637	96.000	3.7795	96.982	3.8182	240
280	84.000	3.3071	84.737	3.3361	112.000	4.4094	112.983	4.4481	280
300	90.000	3.5433	90.737	3.5723	120.000	4.7244	120.983	4.7631	300
340	102.000	4.0157	102.738	4.0448	136.000	5.3543	136.983	5.3930	340
380	114.000	4.4882	114.738	4.5172	152.000	5.9843	152.984	6.0230	380
400	120.000	4.7244	120.738	4.7535	160.000	6.2992	160.984	6.3379	400
440	132.000	5.1969	132.738	5.2259	176.000	6.9291	176.984	6.9679	440
480	144.000	5.6693	144.738	5.6984	192.000	7.5591	192.984	7.5978	480
500	150.000	5.9055	150.738	5.9346	200.000	7.8740	200.984	7.9128	500

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TABLE 10-30 (Cont.) **METRIC GEAR OVER PINS MEASUREMENT**

Pitch Diameter and Measurement Over Wires for External, Module Type Gears, 20-Degree Pressure Angle

No. of Teeth	Module 0.50 Wire Size = 0.8640mm; 0.0340 Inch				Module 0.75 Wire Size = 1.2960mm; 0.0510 Inch				No. of Teeth
	Pitch Diameter		Meas. Over Wire		Pitch Diameter		Meas. Over Wire		
	mm	Inch	mm	Inch	mm	Inch	mm	Inch	
5	2.500	0.0984			3.750	0.1476			5
6	3.000	0.1181			4.500	0.1772			6
7	3.500	0.1378			5.250	0.2067			7
8	4.000	0.1575			6.000	0.2362			8
9	4.500	0.1772			6.750	0.2657			9
10	5.000	0.1969			7.500	0.2953			10
11	5.500	0.2165			8.250	0.3248			11
12	6.000	0.2362			9.000	0.3543			12
13	6.500	0.2559			9.750	0.3839			13
14	7.000	0.2756			10.500	0.4134			14
15	7.500	0.2953			11.250	0.4429			15
16	8.000	0.3150			12.000	0.4724			16
17	8.500	0.3346			12.750	0.5020			17
18	9.000	0.3543	10.192	0.4013	13.500	0.5315	15.288	0.6019	18
19	9.500	0.3740	10.660	0.4197	14.250	0.5610	15.990	0.6295	19
20	10.000	0.3937	11.195	0.4407	15.000	0.5906	16.792	0.6611	20
21	10.500	0.4134	11.666	0.4593	15.750	0.6201	17.499	0.6889	21
22	11.000	0.4331	12.198	0.4802	16.500	0.6496	18.296	0.7203	22
23	11.500	0.4528	12.671	0.4989	17.250	0.6791	19.007	0.7483	23
24	12.000	0.4724	13.200	0.5197	18.000	0.7087	19.800	0.7795	24
25	12.500	0.4921	13.676	0.5384	18.750	0.7382	20.513	0.8076	25
26	13.000	0.5118	14.202	0.5591	19.500	0.7677	21.303	0.8387	26
27	13.500	0.5315	14.679	0.5779	20.250	0.7972	22.019	0.8669	27
28	14.000	0.5512	15.204	0.5986	21.000	0.8268	22.805	0.8978	28
29	14.500	0.5709	15.683	0.6174	21.750	0.8563	23.524	0.9261	29
30	15.000	0.5906	16.205	0.6380	22.500	0.8858	24.308	0.9570	30
31	15.500	0.6102	16.685	0.6569	23.250	0.9154	25.028	0.9854	31
32	16.000	0.6299	17.206	0.6774	24.000	0.9449	25.810	1.0161	32
33	16.500	0.6496	17.688	0.6964	24.750	0.9744	26.532	1.0446	33
34	17.000	0.6693	18.208	0.7168	25.500	1.0039	27.312	1.0753	34
35	17.500	0.6890	18.690	0.7358	26.250	1.0335	28.036	1.1038	35
36	18.000	0.7087	19.209	0.7563	27.000	1.0630	28.813	1.1344	36
37	18.500	0.7283	19.692	0.7753	27.750	1.0925	29.539	1.1629	37
38	19.000	0.7480	20.210	0.7957	28.500	1.1220	30.315	1.1935	38
39	19.500	0.7677	20.694	0.8147	29.250	1.1516	31.041	1.2221	39
40	20.000	0.7874	21.211	0.8351	30.000	1.1811	31.816	1.2526	40
41	20.500	0.8071	21.696	0.8542	30.750	1.2106	32.544	1.2813	41
42	21.000	0.8268	22.212	0.8745	31.500	1.2402	33.318	1.3117	42
43	21.500	0.8465	22.698	0.8936	32.250	1.2697	34.046	1.3404	43
44	22.000	0.8661	23.212	0.9139	33.000	1.2992	34.819	1.3708	44
45	22.500	0.8858	23.699	0.9330	33.750	1.3287	35.548	1.3995	45
46	23.000	0.9055	24.213	0.9533	34.500	1.3583	36.320	1.4299	46
47	23.500	0.9252	24.700	0.9725	35.250	1.3878	37.051	1.4587	47
48	24.000	0.9449	25.214	0.9927	36.000	1.4173	37.821	1.4890	48
49	24.500	0.9646	25.702	1.0119	36.750	1.4469	38.552	1.5178	49
50	25.000	0.9843	26.215	1.0321	37.500	1.4764	39.322	1.5481	50
51	25.500	1.0039	26.703	1.0513	38.250	1.5059	40.054	1.5769	51
52	26.000	1.0236	27.215	1.0715	39.000	1.5354	40.823	1.6072	52
53	26.500	1.0433	27.704	1.0907	39.750	1.5650	41.556	1.6360	53
54	27.000	1.0630	28.216	1.1109	40.500	1.5945	42.324	1.6663	54
55	27.500	1.0827	28.705	1.1301	41.250	1.6240	43.057	1.6952	55
56	28.000	1.1024	29.216	1.1502	42.000	1.6535	43.824	1.7254	56
57	28.500	1.1220	29.706	1.1695	42.750	1.6831	44.558	1.7543	57
58	29.000	1.1417	30.217	1.1896	43.500	1.7126	45.325	1.7845	58
59	29.500	1.1614	30.706	1.2089	44.250	1.7421	46.060	1.8134	59
60	30.000	1.1811	31.217	1.2290	45.000	1.7717	46.826	1.8435	60
61	30.500	1.2008	31.707	1.2483	45.750	1.8012	47.561	1.8725	61
62	31.000	1.2205	32.218	1.2684	46.500	1.8307	48.326	1.9026	62
63	31.500	1.2402	32.708	1.2877	47.250	1.8602	49.062	1.9316	63
64	32.000	1.2598	33.218	1.3078	48.000	1.8898	49.827	1.9617	64
65	32.500	1.2795	33.709	1.3271	48.750	1.9193	50.563	1.9907	65
66	33.000	1.2992	34.218	1.3472	49.500	1.9488	51.328	2.0208	66
67	33.500	1.3189	34.709	1.3665	50.250	1.9783	52.064	2.0498	67
68	34.000	1.3386	35.219	1.3866	51.000	2.0079	52.828	2.0799	68
69	34.500	1.3583	35.710	1.4059	51.750	2.0374	53.565	2.1089	69
70	35.000	1.3780	36.219	1.4260	52.500	2.0669	54.329	2.1389	70
71	35.500	1.3976	36.710	1.4453	53.250	2.0965	55.066	2.1679	71
72	36.000	1.4173	37.219	1.4653	54.000	2.1260	55.829	2.1980	72
73	36.500	1.4370	37.711	1.4847	54.750	2.1555	56.567	2.2270	73
74	37.000	1.4567	38.220	1.5047	55.500	2.1850	57.330	2.2571	74

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TABLE 10-30 (Cont.) **METRIC GEAR OVER PINS MEASUREMENT**

Pitch Diameter and Measurement Over Wires for External, Module Type Gears, 20-Degree Pressure Angle

No. of Teeth	Module 0.50 Wire Size = 0.8640mm; 0.0340 Inch				Module 0.75 Wire Size = 1.2960mm; 0.0510 Inch				No. of Teeth
	Pitch Diameter		Meas. Over Wire		Pitch Diameter		Meas. Over Wire		
	mm	Inch	mm	Inch	mm	Inch	mm	Inch	
75	37.500	1.4764	38.712	1.5241	56.250	2.2146	58.067	2.2861	75
76	38.000	1.4961	39.220	1.5441	57.000	2.2441	58.830	2.3161	76
77	38.500	1.5157	39.712	1.5635	57.750	2.2736	59.568	2.3452	77
78	39.000	1.5354	40.220	1.5835	58.500	2.3031	60.331	2.3752	78
79	39.500	1.5551	40.713	1.6029	59.250	2.3327	61.069	2.4043	79
80	40.000	1.5748	41.221	1.6229	60.000	2.3622	61.831	2.4343	80
81	40.500	1.5945	41.713	1.6422	60.750	2.3917	62.570	2.4634	81
82	41.000	1.6142	42.221	1.6622	61.500	2.4213	63.331	2.4934	82
83	41.500	1.6339	42.714	1.6816	62.250	2.4508	64.070	2.5225	83
84	42.000	1.6535	43.221	1.7016	63.000	2.4803	64.832	2.5524	84
85	42.500	1.6732	43.714	1.7210	63.750	2.5098	65.571	2.5815	85
86	43.000	1.6929	44.221	1.7410	64.500	2.5394	66.332	2.6115	86
87	43.500	1.7126	44.714	1.7604	65.250	2.5689	67.072	2.6406	87
88	44.000	1.7323	45.222	1.7804	66.000	2.5984	67.832	2.6706	88
89	44.500	1.7520	45.715	1.7998	66.750	2.6280	68.572	2.6997	89
90	45.000	1.7717	46.222	1.8198	67.500	2.6575	69.333	2.7296	90
91	45.500	1.7913	46.715	1.8392	68.250	2.6870	70.073	2.7588	91
92	46.000	1.8110	47.222	1.8591	69.000	2.7165	70.833	2.7887	92
93	46.500	1.8307	47.715	1.8786	69.750	2.7461	71.573	2.8178	93
94	47.000	1.8504	48.222	1.8985	70.500	2.7756	72.333	2.8478	94
95	47.500	1.8701	48.716	1.9179	71.250	2.8051	73.074	2.8769	95
96	48.000	1.8898	49.222	1.9379	72.000	2.8346	73.834	2.9068	96
97	48.500	1.9094	49.716	1.9573	72.750	2.8642	74.574	2.9360	97
98	49.000	1.9291	50.223	1.9773	73.500	2.8937	75.334	2.9659	98
99	49.500	1.9488	50.716	1.9967	74.250	2.9232	76.075	2.9951	99
100	50.000	1.9685	51.223	2.0166	75.000	2.9528	76.834	3.0250	100
101	50.500	1.9882	51.717	2.0361	75.750	2.9823	77.575	3.0541	101
102	51.000	2.0079	52.223	2.0560	76.500	3.0118	78.334	3.0840	102
103	51.500	2.0276	52.717	2.0755	77.250	3.0413	79.076	3.1132	103
104	52.000	2.0472	53.223	2.0954	78.000	3.0709	79.835	3.1431	104
105	52.500	2.0669	53.717	2.1149	78.750	3.1004	80.576	3.1723	105
106	53.000	2.0866	54.223	2.1348	79.500	3.1299	81.335	3.2022	106
107	53.500	2.1063	54.718	2.1542	80.250	3.1594	82.076	3.2314	107
108	54.000	2.1260	55.223	2.1742	81.000	3.1890	82.835	3.2612	108
109	54.500	2.1457	55.718	2.1936	81.750	3.2185	83.577	3.2904	109
110	55.000	2.1654	56.224	2.2135	82.500	3.2480	84.335	3.3203	110
111	55.500	2.1850	56.718	2.2330	83.250	3.2776	85.077	3.3495	111
112	56.000	2.2047	57.224	2.2529	84.000	3.3071	85.836	3.3794	112
113	56.500	2.2244	57.718	2.2724	84.750	3.3366	86.578	3.4086	113
114	57.000	2.2441	58.224	2.2923	85.500	3.3661	87.336	3.4384	114
115	57.500	2.2638	58.719	2.3118	86.250	3.3957	88.078	3.4676	115
116	58.000	2.2835	59.224	2.3317	87.000	3.4252	88.836	3.4975	116
117	58.500	2.3031	59.719	2.3511	87.750	3.4547	89.578	3.5267	117
118	59.000	2.3228	60.224	2.3710	88.500	3.4843	90.336	3.5565	118
119	59.500	2.3425	60.719	2.3905	89.250	3.5138	91.078	3.5858	119
120	60.000	2.3622	61.224	2.4104	90.000	3.5433	91.836	3.6156	120
121	60.500	2.3819	61.719	2.4299	90.750	3.5728	92.579	3.6448	121
122	61.000	2.4016	62.224	2.4498	91.500	3.6024	93.337	3.6747	122
123	61.500	2.4213	62.719	2.4693	92.250	3.6319	94.079	3.7039	123
124	62.000	2.4409	63.225	2.4892	93.000	3.6614	94.837	3.7337	124
125	62.500	2.4606	63.720	2.5086	93.750	3.6909	95.579	3.7630	125
126	63.000	2.4803	64.225	2.5285	94.500	3.7205	96.337	3.7928	126
127	63.500	2.5000	64.720	2.5480	95.250	3.7500	97.080	3.8220	127
128	64.000	2.5197	65.225	2.5679	96.000	3.7795	97.837	3.8519	128
129	64.500	2.5394	65.720	2.5874	96.750	3.8091	98.580	3.8811	129
130	65.000	2.5591	66.225	2.6073	97.500	3.8386	99.337	3.9109	130
131	65.500	2.5787	66.720	2.6268	98.250	3.8681	100.080	3.9402	131
132	66.000	2.5984	67.225	2.6467	99.000	3.8976	100.837	3.9700	132
133	66.500	2.6181	67.720	2.6662	99.750	3.9272	101.581	3.9992	133
134	67.000	2.6378	68.225	2.6860	100.500	3.9567	102.338	4.0290	134
135	67.500	2.6575	68.721	2.7055	101.250	3.9862	103.081	4.0583	135
136	68.000	2.6772	69.225	2.7254	102.000	4.0157	103.838	4.0881	136
137	68.500	2.6969	69.721	2.7449	102.750	4.0453	104.581	4.1174	137
138	69.000	2.7165	70.225	2.7648	103.500	4.0748	105.338	4.1472	138
139	69.500	2.7362	70.721	2.7843	104.250	4.1043	106.081	4.1764	139
140	70.000	2.7559	71.225	2.8041	105.000	4.1339	106.838	4.2062	140
141	70.500	2.7756	71.721	2.8237	105.750	4.1634	107.582	4.2355	141
142	71.000	2.7953	72.225	2.8435	106.500	4.1929	108.338	4.2653	142
143	71.500	2.8150	72.721	2.8630	107.250	4.2224	109.082	4.2946	143
144	72.000	2.8346	73.226	2.8829	108.000	4.2520	109.838	4.3243	144

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TABLE 10-30 (Cont.) **METRIC GEAR OVER PINS MEASUREMENT**
Pitch Diameter and Measurement Over Wires for External, Module Type Gears, 20-Degree Pressure Angle

No. of Teeth	Module 0.50 Wire Size = 0.8640mm; 0.0340 Inch				Module 0.75 Wire Size = 1.2960mm; 0.0510 Inch				No. of Teeth
	Pitch Diameter		Meas. Over Wire		Pitch Diameter		Meas. Over Wire		
	mm	Inch	mm	Inch	mm	Inch	mm	Inch	
145	72.500	2.8543	73.721	2.9024	108.750	4.2815	110.582	4.3536	145
146	73.000	2.8740	74.226	2.9223	109.500	4.3110	111.338	4.3834	146
147	73.500	2.8937	74.721	2.9418	110.250	4.3406	112.082	4.4127	147
148	74.000	2.9134	75.226	2.9616	111.000	4.3701	112.839	4.4425	148
149	74.500	2.9331	75.722	2.9812	111.750	4.3996	113.582	4.4718	149
150	75.000	2.9528	76.226	3.0010	112.500	4.4291	114.339	4.5015	150
151	75.500	2.9724	76.722	3.0205	113.250	4.4587	115.083	4.5308	151
152	76.000	2.9921	77.226	3.0404	114.000	4.4882	115.839	4.5606	152
153	76.500	3.0118	77.722	3.0599	114.750	4.5177	116.583	4.5899	153
154	77.000	3.0315	78.226	3.0798	115.500	4.5472	117.339	4.6196	154
155	77.500	3.0512	78.722	3.0993	116.250	4.5768	118.083	4.6489	155
156	78.000	3.0709	79.226	3.1191	117.000	4.6063	118.839	4.6787	156
157	78.500	3.0906	79.722	3.1387	117.750	4.6358	119.583	4.7080	157
158	79.000	3.1102	80.226	3.1585	118.500	4.6654	120.339	4.7378	158
159	79.500	3.1299	80.722	3.1780	119.250	4.6949	121.083	4.7671	159
160	80.000	3.1496	81.226	3.1979	120.000	4.7244	121.839	4.7968	160
161	80.500	3.1693	81.722	3.2174	120.750	4.7539	122.584	4.8261	161
162	81.000	3.1890	82.226	3.2373	121.500	4.7835	123.339	4.8559	162
163	81.500	3.2087	82.722	3.2568	122.250	4.8130	124.084	4.8852	163
164	82.000	3.2283	83.226	3.2766	123.000	4.8425	124.840	4.9149	164
165	82.500	3.2480	83.723	3.2962	123.750	4.8720	125.584	4.9443	165
166	83.000	3.2677	84.226	3.3160	124.500	4.9016	126.340	4.9740	166
167	83.500	3.2874	84.723	3.3355	125.250	4.9311	127.084	5.0033	167
168	84.000	3.3071	85.226	3.3554	126.000	4.9606	127.840	5.0331	168
169	84.500	3.3268	85.723	3.3749	126.750	4.9902	128.584	5.0624	169
170	85.000	3.3465	86.227	3.3947	127.500	5.0197	129.340	5.0921	170
171	85.500	3.3661	86.723	3.4143	128.250	5.0492	130.084	5.1214	171
172	86.000	3.3858	87.227	3.4341	129.000	5.0787	130.840	5.1512	172
173	86.500	3.4055	87.723	3.4537	129.750	5.1083	131.585	5.1805	173
174	87.000	3.4252	88.227	3.4735	130.500	5.1378	132.340	5.2102	174
175	87.500	3.4449	88.723	3.4930	131.250	5.1673	133.085	5.2396	175
176	88.000	3.4646	89.227	3.5129	132.000	5.1969	133.840	5.2693	176
177	88.500	3.4843	89.723	3.5324	132.750	5.2264	134.585	5.2986	177
178	89.000	3.5039	90.227	3.5522	133.500	5.2559	135.340	5.3284	178
179	89.500	3.5236	90.723	3.5718	134.250	5.2854	136.085	5.3577	179
180	90.000	3.5433	91.227	3.5916	135.000	5.3150	136.840	5.3874	180
181	90.500	3.5630	91.723	3.6112	135.750	5.3445	137.585	5.4167	181
182	91.000	3.5827	92.227	3.6310	136.500	5.3740	138.340	5.4465	182
183	91.500	3.6024	92.724	3.6505	137.250	5.4035	139.085	5.4758	183
184	92.000	3.6220	93.227	3.6704	138.000	5.4331	139.840	5.5055	184
185	92.500	3.6417	93.724	3.6899	138.750	5.4626	140.585	5.5349	185
186	93.000	3.6614	94.227	3.7097	139.500	5.4921	141.340	5.5646	186
187	93.500	3.6811	94.724	3.7293	140.250	5.5217	142.086	5.5939	187
188	94.000	3.7008	95.227	3.7491	141.000	5.5512	142.841	5.6236	188
189	94.500	3.7205	95.724	3.7687	141.750	5.5807	143.586	5.6530	189
190	95.000	3.7402	96.227	3.7885	142.500	5.6102	144.341	5.6827	190
191	95.500	3.7598	96.727	3.8082	143.250	5.6398	145.091	5.7122	191
192	96.000	3.7795	97.227	3.8278	144.000	5.6693	145.841	5.7418	192
193	96.500	3.7992	97.727	3.8475	144.750	5.6988	146.591	5.7713	193
194	97.000	3.8189	98.227	3.8672	145.500	5.7283	147.341	5.8008	194
195	97.500	3.8386	98.727	3.8869	146.250	5.7579	148.091	5.8303	195
196	98.000	3.8583	99.227	3.9066	147.000	5.7874	148.841	5.8599	196
197	98.500	3.8780	99.727	3.9263	147.750	5.8169	149.591	5.8894	197
198	99.000	3.8976	100.227	3.9460	148.500	5.8465	150.341	5.9189	198
199	99.500	3.9173	100.727	3.9656	149.250	5.8760	151.091	5.9485	199
200	100.000	3.9370	101.227	3.9853	150.000	5.9055	151.841	5.9780	200
201	100.500	3.9567	101.724	4.0049	150.750	5.9350	152.587	6.0073	201
202	101.000	3.9764	102.224	4.0246	151.500	5.9646	153.337	6.0369	202
203	101.500	3.9961	102.724	4.0443	152.250	5.9941	154.087	6.0664	203
204	102.000	4.0157	103.224	4.0640	153.000	6.0236	154.837	6.0959	204
205	102.500	4.0354	103.725	4.0837	153.750	6.0531	155.587	6.1255	205
240	120.000	4.7244	121.228	4.7728	180.000	7.0866	181.842	7.1591	240
280	140.000	5.5118	141.229	5.5602	210.000	8.2677	211.843	8.3403	280
300	150.000	5.9055	151.229	5.9539	225.000	8.8583	226.843	8.9308	300
340	170.000	6.6929	171.229	6.7413	255.000	10.0394	256.844	10.1120	340
380	190.000	7.4803	191.230	7.5287	285.000	11.2205	286.844	11.2931	380
400	200.000	7.8740	201.230	7.9224	300.000	11.8110	301.845	11.8836	400
440	220.000	8.6614	221.230	8.7098	330.000	12.9921	331.845	13.0648	440
480	240.000	9.4488	241.230	9.4973	360.000	14.1732	361.845	14.2459	480
500	250.000	9.8425	251.230	9.8910	375.000	14.7638	376.845	14.8364	500

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TABLE 10-30 (Cont.) **METRIC GEAR OVER PINS MEASUREMENT**
Pitch Diameter and Measurement Over Wires for External, Module Type Gears, 20-Degree Pressure Angle

No. of Teeth	Module 0.80 Wire Size = 1.3824mm; 0.0544 Inch				Module 1.00 Wire Size = 1.7280mm; 0.0680 Inch				No. of Teeth
	Pitch Diameter		Meas. Over Wire		Pitch Diameter		Meas. Over Wire		
	mm	Inch	mm	Inch	mm	Inch	mm	Inch	
5	4.000	0.1575			5.000	0.1969			5
6	4.800	0.1890			6.000	0.2362			6
7	5.600	0.2205			7.000	0.2756			7
8	6.400	0.2520			8.000	0.3150			8
9	7.200	0.2835			9.000	0.3543			9
10	8.000	0.3150			10.000	0.3937			10
11	8.800	0.3465			11.000	0.4331			11
12	9.600	0.3780			12.000	0.4724			12
13	10.400	0.4094			13.000	0.5118			13
14	11.200	0.4409			14.000	0.5512			14
15	12.000	0.4724			15.000	0.5906			15
16	12.800	0.5039			16.000	0.6299			16
17	13.600	0.5354			17.000	0.6693			17
18	14.400	0.5669	16.307	0.6420	18.000	0.7087	20.384	0.8025	18
19	15.200	0.5984	17.056	0.6715	19.000	0.7480	21.320	0.8394	19
20	16.000	0.6299	17.912	0.7052	20.000	0.7874	22.390	0.8815	20
21	16.800	0.6614	18.666	0.7349	21.000	0.8268	23.332	0.9186	21
22	17.600	0.6929	19.516	0.7684	22.000	0.8661	24.395	0.9604	22
23	18.400	0.7244	20.274	0.7982	23.000	0.9055	25.342	0.9977	23
24	19.200	0.7559	21.120	0.8315	24.000	0.9449	26.400	1.0394	24
25	20.000	0.7874	21.881	0.8615	25.000	0.9843	27.351	1.0768	25
26	20.800	0.8189	22.723	0.8946	26.000	1.0236	28.404	1.1183	26
27	21.600	0.8504	23.487	0.9247	27.000	1.0630	29.359	1.1559	27
28	22.400	0.8819	24.326	0.9577	28.000	1.1024	30.407	1.1971	28
29	23.200	0.9134	25.092	0.9879	29.000	1.1417	31.365	1.2349	29
30	24.000	0.9449	25.928	1.0208	30.000	1.1811	32.410	1.2760	30
31	24.800	0.9764	26.697	1.0511	31.000	1.2205	33.371	1.3138	31
32	25.600	1.0079	27.530	1.0839	32.000	1.2598	34.413	1.3548	32
33	26.400	1.0394	28.301	1.1142	33.000	1.2992	35.376	1.3928	33
34	27.200	1.0709	29.132	1.1469	34.000	1.3386	36.415	1.4337	34
35	28.000	1.1024	29.905	1.1773	35.000	1.3780	37.381	1.4717	35
36	28.800	1.1339	30.734	1.2100	36.000	1.4173	38.418	1.5125	36
37	29.600	1.1654	31.508	1.2405	37.000	1.4567	39.385	1.5506	37
38	30.400	1.1969	32.336	1.2731	38.000	1.4961	40.420	1.5913	38
39	31.200	1.2283	33.111	1.3036	39.000	1.5354	41.389	1.6295	39
40	32.000	1.2598	33.937	1.3361	40.000	1.5748	42.422	1.6701	40
41	32.800	1.2913	34.714	1.3667	41.000	1.6142	43.392	1.7083	41
42	33.600	1.3228	35.539	1.3992	42.000	1.6535	44.423	1.7490	42
43	34.400	1.3543	36.316	1.4298	43.000	1.6929	45.395	1.7872	43
44	35.200	1.3858	37.140	1.4622	44.000	1.7323	46.425	1.8278	44
45	36.000	1.4173	37.918	1.4929	45.000	1.7717	47.398	1.8661	45
46	36.800	1.4488	38.741	1.5252	46.000	1.8110	48.426	1.9066	46
47	37.600	1.4803	39.521	1.5559	47.000	1.8504	49.401	1.9449	47
48	38.400	1.5118	40.342	1.5883	48.000	1.8898	50.428	1.9854	48
49	39.200	1.5433	41.122	1.6190	49.000	1.9291	51.403	2.0237	49
50	40.000	1.5748	41.943	1.6513	50.000	1.9685	52.429	2.0641	50
51	40.800	1.6063	42.724	1.6821	51.000	2.0079	53.405	2.1026	51
52	41.600	1.6378	43.544	1.7143	52.000	2.0472	54.430	2.1429	52
53	42.400	1.6693	44.326	1.7451	53.000	2.0866	55.407	2.1814	53
54	43.200	1.7008	45.145	1.7774	54.000	2.1260	56.431	2.2217	54
55	44.000	1.7323	45.927	1.8082	55.000	2.1654	57.409	2.2602	55
56	44.800	1.7638	46.746	1.8404	56.000	2.2047	58.432	2.3005	56
57	45.600	1.7953	47.529	1.8712	57.000	2.2441	59.411	2.3390	57
58	46.400	1.8268	48.347	1.9034	58.000	2.2835	60.433	2.3793	58
59	47.200	1.8583	49.130	1.9343	59.000	2.3228	61.413	2.4178	59
60	48.000	1.8898	49.948	1.9664	60.000	2.3622	62.434	2.4580	60
61	48.800	1.9213	50.732	1.9973	61.000	2.4016	63.414	2.4966	61
62	49.600	1.9528	51.548	2.0295	62.000	2.4409	64.435	2.5368	62
63	50.400	1.9843	52.333	2.0603	63.000	2.4803	65.416	2.5754	63
64	51.200	2.0157	53.149	2.0925	64.000	2.5197	66.436	2.6156	64
65	52.000	2.0472	53.934	2.1234	65.000	2.5591	67.417	2.6542	65
66	52.800	2.0787	54.750	2.1555	66.000	2.5984	68.437	2.6944	66
67	53.600	2.1102	55.535	2.1864	67.000	2.6378	69.419	2.7330	67
68	54.400	2.1417	56.350	2.2185	68.000	2.6772	70.438	2.7731	68
69	55.200	2.1732	57.136	2.2494	69.000	2.7165	71.420	2.8118	69
70	56.000	2.2047	57.951	2.2815	70.000	2.7559	72.438	2.8519	70
71	56.800	2.2362	58.737	2.3125	71.000	2.7953	73.421	2.8906	71
72	57.600	2.2677	59.551	2.3445	72.000	2.8346	74.439	2.9307	72
73	58.400	2.2992	60.338	2.3755	73.000	2.8740	75.422	2.9694	73
74	59.200	2.3307	61.152	2.4075	74.000	2.9134	76.440	3.0094	74

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TABLE 10-30 (Cont.) **METRIC GEAR OVER PINS MEASUREMENT**

Pitch Diameter and Measurement Over Wires for External, Module Type Gears, 20-Degree Pressure Angle

No. of Teeth	Module 0.80 Wire Size = 1.3824mm; 0.0544 Inch				Module 1.00 Wire Size = 1.7280mm; 0.0680 Inch				No. of Teeth
	Pitch Diameter		Meas. Over Wire		Pitch Diameter		Meas. Over Wire		
	mm	Inch	mm	Inch	mm	Inch	mm	Inch	
75	60.000	2.3622	61.939	2.4385	75.000	2.9528	77.423	3.0482	75
76	60.800	2.3937	62.752	2.4706	76.000	2.9921	78.440	3.0882	76
77	61.600	2.4252	63.539	2.5015	77.000	3.0315	79.424	3.1269	77
78	62.400	2.4567	64.353	2.5336	78.000	3.0709	80.441	3.1670	78
79	63.200	2.4882	65.140	2.5646	79.000	3.1102	81.425	3.2057	79
80	64.000	2.5197	65.953	2.5966	80.000	3.1496	82.441	3.2457	80
81	64.800	2.5512	66.741	2.6276	81.000	3.1890	83.426	3.2845	81
82	65.600	2.5827	67.553	2.6596	82.000	3.2283	84.442	3.3245	82
83	66.400	2.6142	68.342	2.6906	83.000	3.2677	85.427	3.3633	83
84	67.200	2.6457	69.154	2.7226	84.000	3.3071	86.442	3.4032	84
85	68.000	2.6772	69.942	2.7536	85.000	3.3465	87.428	3.4420	85
86	68.800	2.7087	70.754	2.7856	86.000	3.3858	88.443	3.4820	86
87	69.600	2.7402	71.543	2.8167	87.000	3.4252	89.429	3.5208	87
88	70.400	2.7717	72.355	2.8486	88.000	3.4646	90.443	3.5608	88
89	71.200	2.8031	73.144	2.8797	89.000	3.5039	91.429	3.5996	89
90	72.000	2.8346	73.955	2.9116	90.000	3.5433	92.444	3.6395	90
91	72.800	2.8661	74.744	2.9427	91.000	3.5827	93.430	3.6784	91
92	73.600	2.8976	75.555	2.9746	92.000	3.6220	94.444	3.7183	92
93	74.400	2.9291	76.345	3.0057	93.000	3.6614	95.431	3.7571	93
94	75.200	2.9606	77.156	3.0376	94.000	3.7008	96.444	3.7970	94
95	76.000	2.9921	77.945	3.0687	95.000	3.7402	97.432	3.8359	95
96	76.800	3.0236	78.756	3.1006	96.000	3.7795	98.445	3.8758	96
97	77.600	3.0551	79.546	3.1317	97.000	3.8189	99.432	3.9147	97
98	78.400	3.0866	80.356	3.1636	98.000	3.8583	100.445	3.9545	98
99	79.200	3.1181	81.146	3.1947	99.000	3.8976	101.433	3.9934	99
100	80.000	3.1496	81.956	3.2266	100.000	3.9370	102.446	4.0333	100
101	80.800	3.1811	82.747	3.2577	101.000	3.9764	103.433	4.0722	101
102	81.600	3.2126	83.557	3.2896	102.000	4.0157	104.446	4.1120	102
103	82.400	3.2441	84.347	3.3208	103.000	4.0551	105.434	4.1509	103
104	83.200	3.2756	85.157	3.3526	104.000	4.0945	106.446	4.1908	104
105	84.000	3.3071	85.948	3.3838	105.000	4.1339	107.435	4.2297	105
106	84.800	3.3386	86.757	3.4156	106.000	4.1732	108.447	4.2696	106
107	85.600	3.3701	87.548	3.4468	107.000	4.2126	109.435	4.3085	107
108	86.400	3.4016	88.358	3.4786	108.000	4.2520	110.447	4.3483	108
109	87.200	3.4331	89.149	3.5098	109.000	4.2913	111.436	4.3872	109
110	88.000	3.4646	89.958	3.5416	110.000	4.3307	112.447	4.4271	110
111	88.800	3.4961	90.749	3.5728	111.000	4.3701	113.436	4.4660	111
112	89.600	3.5276	91.558	3.6046	112.000	4.4094	114.447	4.5058	112
113	90.400	3.5591	92.349	3.6358	113.000	4.4488	115.437	4.5448	113
114	91.200	3.5906	93.158	3.6676	114.000	4.4882	116.448	4.5846	114
115	92.000	3.6220	93.950	3.6988	115.000	4.5276	117.437	4.6235	115
116	92.800	3.6535	94.758	3.7306	116.000	4.5669	118.448	4.6633	116
117	93.600	3.6850	95.550	3.7618	117.000	4.6063	119.438	4.7023	117
118	94.400	3.7165	96.359	3.7937	118.000	4.6457	120.448	4.7421	118
119	95.200	3.7480	97.150	3.8248	119.000	4.6850	121.438	4.7810	119
120	96.000	3.7795	97.959	3.8566	120.000	4.7244	122.449	4.8208	120
121	96.800	3.8110	98.751	3.8878	121.000	4.7638	123.438	4.8598	121
122	97.600	3.8425	99.559	3.9197	122.000	4.8031	124.449	4.8996	122
123	98.400	3.8740	100.351	3.9508	123.000	4.8425	125.439	4.9385	123
124	99.200	3.9055	101.159	3.9826	124.000	4.8819	126.449	4.9783	124
125	100.000	3.9370	101.951	4.0138	125.000	4.9213	127.439	5.0173	125
126	100.800	3.9685	102.759	4.0456	126.000	4.9606	128.449	5.0571	126
127	101.600	4.0000	103.552	4.0768	127.000	5.0000	129.440	5.0960	127
128	102.400	4.0315	104.360	4.1086	128.000	5.0394	130.450	5.1358	128
129	103.200	4.0630	105.152	4.1398	129.000	5.0787	131.440	5.1748	129
130	104.000	4.0945	105.960	4.1716	130.000	5.1181	132.450	5.2146	130
131	104.800	4.1260	106.752	4.2028	131.000	5.1575	133.440	5.2536	131
132	105.600	4.1575	107.560	4.2346	132.000	5.1969	134.450	5.2933	132
133	106.400	4.1890	108.353	4.2659	133.000	5.2362	135.441	5.3323	133
134	107.200	4.2205	109.160	4.2976	134.000	5.2756	136.450	5.3721	134
135	108.000	4.2520	109.953	4.3289	135.000	5.3150	137.441	5.4111	135
136	108.800	4.2835	110.760	4.3606	136.000	5.3543	138.450	5.4508	136
137	109.600	4.3150	111.553	4.3919	137.000	5.3937	139.441	5.4898	137
138	110.400	4.3465	112.360	4.4236	138.000	5.4331	140.451	5.5296	138
139	111.200	4.3780	113.153	4.4549	139.000	5.4724	141.442	5.5686	139
140	112.000	4.4094	113.961	4.4866	140.000	5.5118	142.451	5.6083	140
141	112.800	4.4409	114.754	4.5179	141.000	5.5512	143.442	5.6473	141
142	113.600	4.4724	115.561	4.5496	142.000	5.5906	144.451	5.6870	142
143	114.400	4.5039	116.354	4.5809	143.000	5.6299	145.442	5.7261	143
144	115.200	4.5354	117.161	4.6126	144.000	5.6693	146.451	5.7658	144

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I R T 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 A

TABLE 10-30 (Cont.) **METRIC GEAR OVER PINS MEASUREMENT**

Pitch Diameter and Measurement Over Wires for External, Module Type Gears, 20-Degree Pressure Angle

No. of Teeth	Module 0.80 Wire Size = 1.3824mm; 0.0544 Inch				Module 1.00 Wire Size = 1.7280mm; 0.0680 Inch				No. of Teeth
	Pitch Diameter		Meas. Over Wire		Pitch Diameter		Meas. Over Wire		
	mm	Inch	mm	Inch	mm	Inch	mm	Inch	
145	116.000	4.5669	117.954	4.6439	145.000	5.7087	147.443	5.8048	145
146	116.800	4.5984	118.761	4.6756	146.000	5.7480	148.451	5.8445	146
147	117.600	4.6299	119.554	4.7069	147.000	5.7874	149.443	5.8836	147
148	118.400	4.6614	120.361	4.7386	148.000	5.8268	150.451	5.9233	148
149	119.200	4.6929	121.155	4.7699	149.000	5.8661	151.443	5.9623	149
150	120.000	4.7244	121.961	4.8016	150.000	5.9055	152.452	6.0020	150
151	120.800	4.7559	122.755	4.8329	151.000	5.9449	153.443	6.0411	151
152	121.600	4.7874	123.561	4.8646	152.000	5.9843	154.452	6.0808	152
153	122.400	4.8189	124.355	4.8959	153.000	6.0236	155.444	6.1198	153
154	123.200	4.8504	125.162	4.9276	154.000	6.0630	156.452	6.1595	154
155	124.000	4.8819	125.955	4.9589	155.000	6.1024	157.444	6.1986	155
156	124.800	4.9134	126.762	4.9906	156.000	6.1417	158.452	6.2383	156
157	125.600	4.9449	127.555	5.0219	157.000	6.1811	159.444	6.2773	157
158	126.400	4.9764	128.362	5.0536	158.000	6.2205	160.452	6.3170	158
159	127.200	5.0079	129.156	5.0849	159.000	6.2598	161.444	6.3561	159
160	128.000	5.0394	129.962	5.1166	160.000	6.2992	162.452	6.3958	160
161	128.800	5.0709	130.756	5.1479	161.000	6.3386	163.445	6.4348	161
162	129.600	5.1024	131.562	5.1796	162.000	6.3780	164.453	6.4745	162
163	130.400	5.1339	132.356	5.2109	163.000	6.4173	165.445	6.5136	163
164	131.200	5.1654	133.162	5.2426	164.000	6.4567	166.453	6.5533	164
165	132.000	5.1969	133.956	5.2739	165.000	6.4961	167.445	6.5923	165
166	132.800	5.2283	134.762	5.3056	166.000	6.5354	168.453	6.6320	166
167	133.600	5.2598	135.556	5.3369	167.000	6.5748	169.445	6.6711	167
168	134.400	5.2913	136.362	5.3686	168.000	6.6142	170.453	6.7107	168
169	135.200	5.3228	137.157	5.3999	169.000	6.6535	171.446	6.7498	169
170	136.000	5.3543	137.962	5.4316	170.000	6.6929	172.453	6.7895	170
171	136.800	5.3858	138.757	5.4629	171.000	6.7323	173.446	6.8286	171
172	137.600	5.4173	139.563	5.4946	172.000	6.7717	174.453	6.8682	172
173	138.400	5.4488	140.357	5.5259	173.000	6.8110	175.446	6.9073	173
174	139.200	5.4803	141.163	5.5576	174.000	6.8504	176.453	6.9470	174
175	140.000	5.5118	141.957	5.5889	175.000	6.8898	177.446	6.9861	175
176	140.800	5.5433	142.763	5.6206	176.000	6.9291	178.453	7.0257	176
177	141.600	5.5748	143.557	5.6519	177.000	6.9685	179.446	7.0648	177
178	142.400	5.6063	144.363	5.6836	178.000	7.0079	180.454	7.1045	178
179	143.200	5.6378	145.157	5.7149	179.000	7.0472	181.447	7.1436	179
180	144.000	5.6693	145.963	5.7466	180.000	7.0866	182.454	7.1832	180
181	144.800	5.7008	146.758	5.7779	181.000	7.1260	183.447	7.2223	181
182	145.600	5.7323	147.563	5.8096	182.000	7.1654	184.454	7.2620	182
183	146.400	5.7638	148.358	5.8409	183.000	7.2047	185.447	7.3011	183
184	147.200	5.7953	149.163	5.8726	184.000	7.2441	186.454	7.3407	184
185	148.000	5.8268	149.958	5.9039	185.000	7.2835	187.447	7.3798	185
186	148.800	5.8583	150.763	5.9356	186.000	7.3228	188.454	7.4194	186
187	149.600	5.8898	151.558	5.9668	187.000	7.3622	189.447	7.4586	187
188	150.400	5.9213	152.363	5.9986	188.000	7.4016	190.454	7.4982	188
189	151.200	5.9528	153.158	6.0298	189.000	7.4409	191.448	7.5373	189
190	152.000	5.9843	153.963	6.0615	190.000	7.4803	192.454	7.5769	190
191	152.800	6.0157	154.763	6.0930	191.000	7.5197	193.454	7.6163	191
192	153.600	6.0472	155.563	6.1245	192.000	7.5591	194.454	7.6557	192
193	154.400	6.0787	156.364	6.1560	193.000	7.5984	195.454	7.6951	193
194	155.200	6.1102	157.164	6.1875	194.000	7.6378	196.454	7.7344	194
195	156.000	6.1417	157.964	6.2190	195.000	7.6772	197.454	7.7738	195
196	156.800	6.1732	158.764	6.2505	196.000	7.7165	198.455	7.8132	196
197	157.600	6.2047	159.564	6.2820	197.000	7.7559	199.455	7.8525	197
198	158.400	6.2362	160.364	6.3135	198.000	7.7953	200.455	7.8919	198
199	159.200	6.2677	161.164	6.3450	199.000	7.8346	201.455	7.9313	199
200	160.000	6.2992	161.964	6.3765	200.000	7.8740	202.455	7.9707	200
201	160.800	6.3307	162.759	6.4078	201.000	7.9134	203.449	8.0098	201
202	161.600	6.3622	163.559	6.4393	202.000	7.9528	204.449	8.0492	202
203	162.400	6.3937	164.359	6.4708	203.000	7.9921	205.449	8.0885	203
204	163.200	6.4252	165.159	6.5023	204.000	8.0315	206.449	8.1279	204
205	164.000	6.4567	165.959	6.5338	205.000	8.0709	207.449	8.1673	205
240	192.000	7.5591	193.965	7.6364	240.000	9.4488	242.456	9.5455	240
280	224.000	8.8189	225.966	8.8963	280.000	11.0236	282.457	11.1204	280
300	240.000	9.4488	241.966	9.5262	300.000	11.8110	302.458	11.9078	300
340	272.000	10.7087	273.967	10.7861	340.000	13.3858	342.459	13.4826	340
380	304.000	11.9685	305.967	12.0460	380.000	14.9606	382.459	15.0575	380
400	320.000	12.5984	321.968	12.6759	400.000	15.7480	402.460	15.8449	400
440	352.000	13.8583	353.968	13.9357	440.000	17.3228	442.460	17.4197	440
480	384.000	15.1181	385.968	15.1956	480.000	18.8976	482.460	18.9945	480
500	400.000	15.7480	401.968	15.8255	500.000	19.6850	502.461	19.7819	500

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SECTION 11 CONTACT RATIO

To assure continuous smooth tooth action, as one pair of teeth ceases action a succeeding pair of teeth must already have come into engagement. It is desirable to have as much overlap as is possible. A measure of this overlap action is the contact ratio. This is a ratio of the length of the line-of-action to the base pitch. **Figure 11-1** shows the geometry for a spur gear pair, which is the simplest case, and is representative of the concept for all gear types. The length-of-action is determined from the intersection of the line-of-action and the outside radii. The ratio of the length-of-action to the base pitch is determined from:

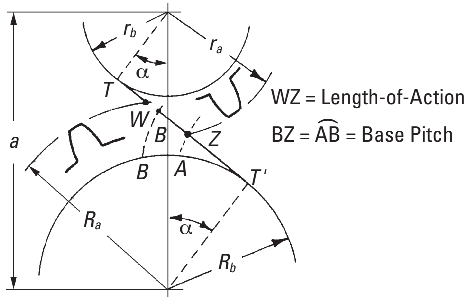


Fig. 11-1 Geometry of Contact Ratio

$$\varepsilon_\gamma = \frac{\sqrt{(R_a^2 - R_b^2)} + \sqrt{(r_a^2 - r_b^2)} - a \sin\alpha}{\pi m \cos\alpha} \qquad \textbf{(11-1)}$$

It is good practice to maintain a contact ratio of 1.2 or greater. Under no circumstances should the ratio drop below 1.1, calculated for all tolerances at their worst case values.

A contact ratio between 1 and 2 means that part of the time two pairs of teeth are in contact and during the remaining time one pair is in contact. A ratio between 2 and 3 means 2 or 3 pairs of teeth are always in contact. Such a high ratio is generally not obtained with external spur gears, but can be developed in the meshing of internal gears, helical gears, or specially designed nonstandard external spur gears.

When considering all types of gears, contact ratio is composed of two components:

1. Radial contact ratio (plane of rotation perpendicular to axes), ε_α
2. Overlap contact ratio (axial), ε_β

The sum is the total contact ratio, ε_γ.

The overlap contact ratio component exists only in gear pairs that have helical or spiral tooth forms.

11.1 Radial Contact Ratio Of Spur And Helical Gears, ε_α

The equations for radial (or plane of rotation) contact ratio for spur and helical gears are given in **Table 11-1**, with reference to **Figure 11-2**.

When the contact ratio is inadequate, there are three means to increase it. These are somewhat obvious from examination of **Equation (11-1)**.

1. Decrease the pressure angle. This makes a longer line-of-action as it extends through the region between the two outside radii.
2. Increase the number of teeth. As the number of teeth increases and the pitch diameter grows, again there is a longer line-of-action in the region between the outside radii.
3. Increase working tooth depth. This can be done by adding addendum to the tooth and thus increase the outside radius. However, this requires a larger dedendum, and requires a special tooth design.

Table 11-1 Equations of Radial Contact Ratio on Parallel Axes Gear, ε_α

Type of Gear Mesh		Formula of Radial Contact Ratio, ε_α
Spur Pair	Gear ① Gear ②	$\dfrac{\sqrt{\left(\frac{d_{a1}}{2}\right)^2-\left(\frac{d_{b1}}{2}\right)^2}+\sqrt{\left(\frac{d_{a2}}{2}\right)^2-\left(\frac{d_{b2}}{2}\right)^2}-a_x \sin\alpha_w}{\pi m \cos\alpha}$
Spur Gear and Rack	Gear ① Rack ②	$\dfrac{\sqrt{\left(\frac{d_{a1}}{2}\right)^2-\left(\frac{d_{b1}}{2}\right)^2}+\frac{h_{a2}-x_1 m}{\sin\alpha}-\frac{d_1}{2}\sin\alpha}{\pi m \cos\alpha}$
External and Internal Spur	External Gear ① Internal Gear ②	$\dfrac{\sqrt{\left(\frac{d_{a1}}{2}\right)^2-\left(\frac{d_{b1}}{2}\right)^2}-\sqrt{\left(\frac{d_{a2}}{2}\right)^2-\left(\frac{d_{b2}}{2}\right)^2}+a_x \sin\alpha_w}{\pi m \cos\alpha}$
Helical Pair	Gear ① Gear ②	$\dfrac{\sqrt{\left(\frac{d_{a1}}{2}\right)^2-\left(\frac{d_{b1}}{2}\right)^2}+\sqrt{\left(\frac{d_{a2}}{2}\right)^2-\left(\frac{d_{b2}}{2}\right)^2}-a_x \sin\alpha_{wt}}{\pi m_t \cos\alpha_t}$

An example of helical gear:

m_n = 3	α_n = 20°	β = 30°	z_1 = 12
z_2 = 60	x_1 = +0.09809	x_2 = 0	a_x = 125
α_t = 22.79588°	α_{wt} = 23.1126°	m_t = 3.46410	d_{a1} = 48.153
d_{a2} = 213.842	d_{b1} = 38.322	d_{b2} = 191.611	ε_α = 1.2939

Note that in **Table 11-1** only the radial or circular (plane of rotation) contact ratio is considered. This is true of both the spur and helical gear equations. However, for helical gears this is only one component of two. For the helical gear's total contact ratio, ε_γ, the overlap (axial) contact ratio, ε_β, must be added. See **Paragraph 11.4**.

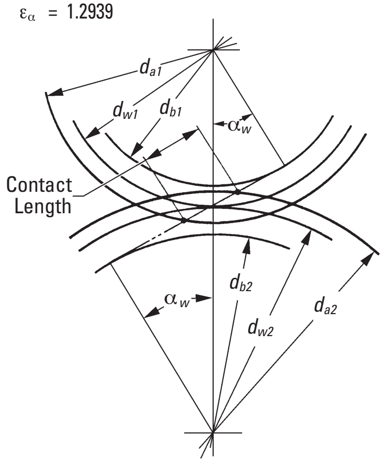


Fig. 11-2 Radial Contact Ratio of Parallel Axes Gear ε_α

11.2 Contact Ratio Of Bevel Gears, ε_α

The contact ratio of a bevel gear pair can be derived from consideration of the equivalent spur gears, when viewed from the back cone. See **Figure 8-8**.

With this approach, the mesh can be treated as spur gears. **Table 11-2** presents equations calculating the contact ratio.

An example of spiral bevel gear (see **Table 11-2**):

m = 3	α_n = 20°	β = 35°	z_1 = 20
z_2 = 40	α_t = 23.95680°	d_1 = 60	d_2 = 120
R_{v1} = 33.54102	R_{v2} = 134.16408	R_{vb1} = 30.65152	R_{vb2} = 122.60610
h_{a1} = 3.4275	h_{a2} = 1.6725	R_{va1} = 36.9685	R_{va2} = 135.83658
ε_α = 1.2825			

Table 11-2 Equations for Contact Ratio for a Bevel Gear Pair

Item	Symbol	Equation for Contact Ratio	
Back Cone Distance	R_v	$\dfrac{d}{2\cos\delta}$	
Base Circle Radius of an Equivalent Spur Gear	R_{vb}	Straight Bevel Gear $R_v\cos\alpha$	Spiral Bevel Gear $R_v\cos\alpha_t$
Outside Radius of an Equivalent Spur Gear	R_{va}	$R_v + h_a$	
Contact Ratio	ε_α	Straight Bevel Gear $\dfrac{\sqrt{R_{va1}^2 - R_{vb1}^2} + \sqrt{R_{va2}^2 - R_{vb2}^2} - (R_{v1} + R_v)\sin\alpha}{\pi m\cos\alpha}$	
		Spiral Bevel Gear $\dfrac{\sqrt{R_{va1}^2 - R_{vb1}^2} + \sqrt{R_{va2}^2 - R_{vb2}^2} - (R_{v1} + R_{v2})\sin\alpha_t}{\pi m\cos\alpha_t}$	

11.3 Contact Ratio For Nonparallel And Nonintersecting Axes Pairs, ε

This group pertains to screw gearing and worm gearing. The equations are approximations by considering the worm and worm gear mesh in the plane perpendicular to worm gear axis and likening it to spur gear and rack mesh. **Table 11-3** presents these equations.

Table 11-3 Equations for Contact Ratio of Nonparallel and Nonintersecting Meshes

Type of Gear Mesh	Equation of Contact Ratio, ε
Screw Gear ① Screw Gear ②	$\dfrac{\sqrt{\left(\dfrac{d_{a1}}{2}\right)^2 - \left(\dfrac{d_{b1}}{2}\right)^2} + \sqrt{\left(\dfrac{d_{a2}}{2}\right)^2 - \left(\dfrac{d_{b2}}{2}\right)^2} - \dfrac{a - \dfrac{d_{b1}\cos\alpha_{t1}}{2} - \dfrac{d_{b1}\cos\alpha_{t2}}{2}}{\sin\alpha_n}}{\pi m_n\cos\alpha_n}$
Worm ① Worm Gear ②	$\dfrac{\dfrac{h_{a1} - x_{x2}m_x}{\sin\alpha_x} + \sqrt{\left(\dfrac{d_{th}}{2}\right)^2 - \left(\dfrac{d_{b2}}{2}\right)^2} - \dfrac{d_2}{2}\sin\alpha_x}{\pi m_x\cos\alpha_x}$

Example of worm mesh:

$m_x = 3$	$\alpha_n = 20°$	$z_w = 2$	$z_2 = 30$
$d_1 = 44$	$d_2 = 90$	$\gamma = 7.76517°$	$\alpha_x = 20.17024°$
$h_{a1} = 3$	$d_{th} = 96$	$d_{b2} = 84.48050$	$\varepsilon = 1.8066$

11.4 Axial (Overlap) Contact Ratio, ε_β

Helical gears and spiral bevel gears have an overlap of tooth action in the axial direction. This overlap adds to the contact ratio. This is in contrast to spur gears which have no tooth action in the axial direction.

Thus, for the same tooth proportions in the plane of rotation, helical and spiral bevel gears offer a significant increase in contact ratio. The magnitude of axial contact ratio is a direct function of the gear width, as illustrated in **Figure 11-3**. Equations for calculating axial contact ratio are presented in **Table 11-4**.

It is obvious that contact ratio can be increased by either increasing the gear width or increasing the helix angle.

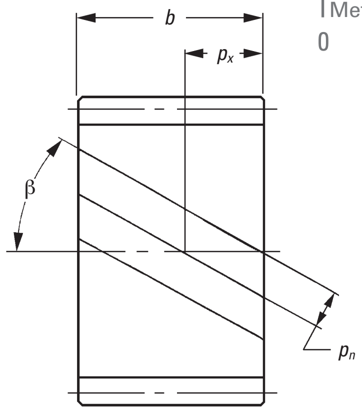


Fig. 11-3 Axial (Overlap) Contact Ratio

Table 11-4 Equations for Axial Contact Ratio of Helical and Spiral Bevel Gears, ε_β

Type of Gear	Equation of Contact Ratio	Example
Helical Gear	$\dfrac{b \sin \beta}{\pi m_n}$	$b = 50$, $\beta = 30°$, $m_n = 3$ $\varepsilon_\beta = 2.6525$
Spiral Bevel Gear	$\dfrac{R_e}{R_e - 0.5b} \quad \dfrac{b \tan \beta_m}{\pi m}$	From **Table 8-6**: $R_e = 67.08204$, $b = 20$, $\beta_m = 35°$, m = 3, $\varepsilon_\beta = 1.7462$

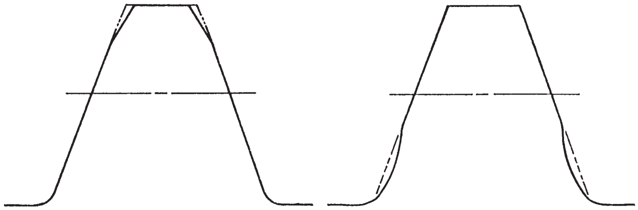
NOTE: The module *m* in spiral bevel gear equation is the normal module.

SECTION 12 GEAR TOOTH MODIFICATIONS

Intentional deviations from the involute tooth profile are used to avoid excessive tooth load deflection interference and thereby enhances load capacity. Also, the elimination of tip interference reduces meshing noise. Other modifications can accommodate assembly misalignment and thus preserve load capacity.

12.1 Tooth Tip Relief

There are two types of tooth tip relief. One modifies the addendum, and the other the dedendum. See **Figure 12-1**. Addendum relief is much more popular than dedendum modification.



(a) Addendum Tip Relief **(b)** Dedendum Modification

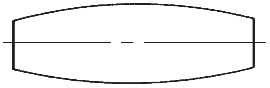
Fig. 12-1 Tip Relief

12.2 Crowning And Side Relieving

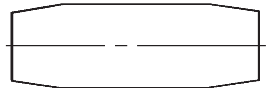
Crowning and side relieving are tooth surface modifications in the axial direction. See **Figure 12-2**.

Crowning is the removal of a slight amount of tooth from the center on out to reach edge, making the tooth surface slightly convex. This method allows the gear to maintain contact in the central region of the tooth and permits avoidance of edge contact with consequent lower load capacity. Crowning also allows a greater tolerance in the misalignment of gears in their assembly, maintaining central contact.

Relieving is a chamfering of the tooth surface. It is similar to crowning except that it is a simpler process and only an approximation to crowning. It is not as effective as crowning.



(a) Crowning



(b) Side Relieving

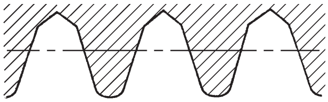
Fig. 12-2 Crowning and Relieving

12.3 Topping And Semitopping

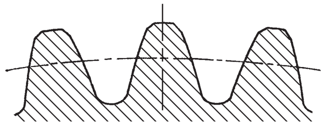
In topping, often referred to as top hobbing, the top or outside diameter of the gear is cut simultaneously with the generation of the teeth. An advantage is that there will be no burrs on the tooth top. Also, the outside diameter is highly concentric with the pitch circle. This permits secondary machining operations using this diameter for nesting.

Semitopping is the chamfering of the tooth's top corner, which is accomplished simultaneously with tooth generation. **Figure 12-3** shows a semitopping cutter and the resultant generated semitopped gear. Such a tooth tends to prevent corner damage. Also, it has no burr. The magnitude of semitopping should not go beyond a proper limit as otherwise it would significantly shorten the addendum and contact ratio. **Figure 12-4** specifies a recommended magnitude of semitopping.

Both modifications require special generating tools. They are independent modifications but, if desired, can be applied simultaneously.



(a) Teeth Form of Semitopping Cutter



(b) Semitopped Teeth Form

Fig. 12-3 Semitopping Cutter and the Gear Profile Generated

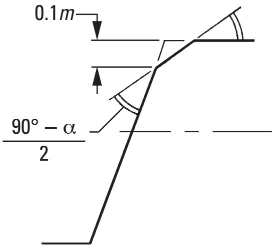


Fig. 12-4 Recommended Magnitude of Semitopping

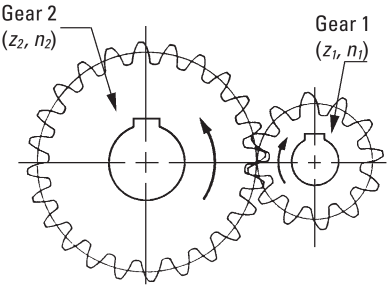
SECTION 13 GEAR TRAINS

The objective of gears is to provide a desired motion, either rotation or linear. This is accomplished through either a simple gear pair or a more involved and complex system of several gear meshes. Also, related to this is the desired speed, direction of rotation and the shaft arrangement.

13.1 Single-Stage Gear Train

A meshed gear is the basic form of a single-stage gear train. It consists of z_1 and z_2 numbers of teeth on the driver and driven gears, and their respective rotations, n_1 & n_2. The speed ratio is then:

$$\text{speed ratio} = \frac{z_1}{z_2} = \frac{n_2}{n_1} \tag{13-1}$$

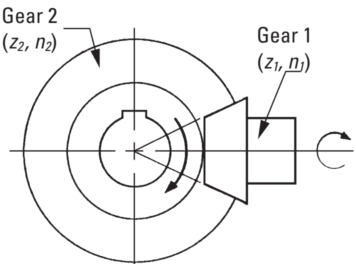


(a) A Pair of Spur Gears

13.1.1 Types Of Single-Stage Gear Trains

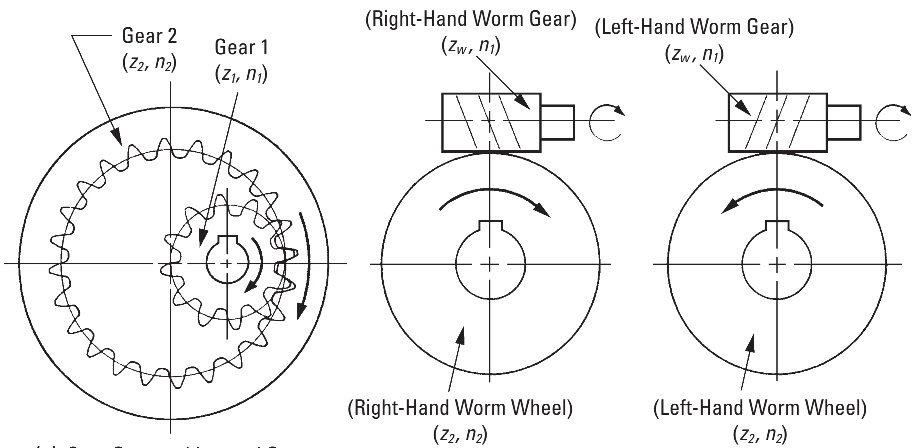
Gear trains can be classified into three types:

1. Speed ratio > 1, increasing: $n_1 < n_2$
2. Speed ratio = 1, equal speeds: $n_1 = n_2$
3. Speed ratio < 1, reducing: $n_1 > n_2$



(b) Bevel Gears

Figure 13-1 illustrates four basic types. For the very common cases of spur and bevel meshes, **Figures 13-1(a)** and **13-1(b)**, the direction of rotation of driver and driven gears are reversed. In the case of an internal gear mesh, **Figure 13-1(c)**, both gears have the same direction of rotation. In the case of a worm mesh, **Figure 13-1(d)**, the rotation direction of z_2 is determined by its helix hand.



(c) Spur Gear and Internal Gear
(d) Worm Mesh

Fig. 13-1 Single-Stage Gear Trains

In addition to these four basic forms, the combination of a rack and gear can be considered a specific type. The displacement of a rack, l, for rotation θ of the mating gear is:

$$l = \frac{\pi m z_1 \theta}{360} \tag{13-2}$$

where:

πm is the standard circular pitch
z_1 is the number of teeth of the gear

13.2 Two-Stage Gear Train

A two-stage gear train uses two single-stages in a series. **Figure 13-2** represents the basic form of an external gear two-stage gear train.

Let the first gear in the first stage be the driver. Then the speed ratio of the two-stage train is:

$$\text{Speed Ratio} = \frac{z_1}{z_2}\ \frac{z_3}{z_4} = \frac{n_2}{n_1}\ \frac{n_4}{n_3} \tag{13-3}$$

In this arrangement, $n_2 = n_3$

In the two-stage gear train, **Figure 13-2**, gear 1 rotates in the same direction as gear 4. If gears 2 and 3 have the same number of teeth, then the train simplifies as in **Figure 13-3**. In this arrangement, gear 2 is known as an idler, which has no effect on the gear ratio. The speed ratio is then:

$$\text{Speed Ratio} = \frac{z_1}{z_2}\ \frac{z_2}{z_3} = \frac{z_1}{z_3} \tag{13-4}$$

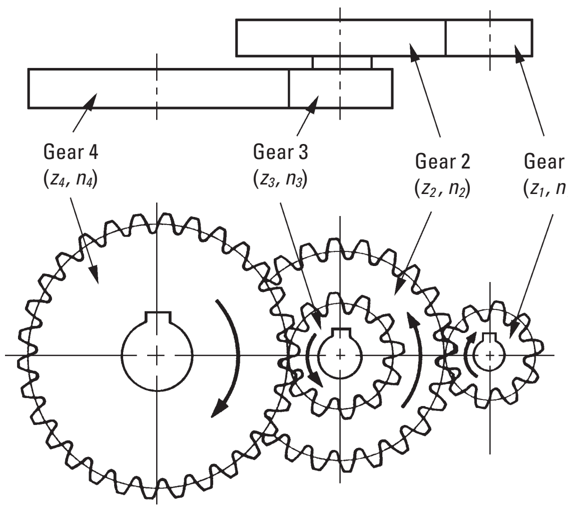


Fig. 13-2 Two-Stage Gear Train

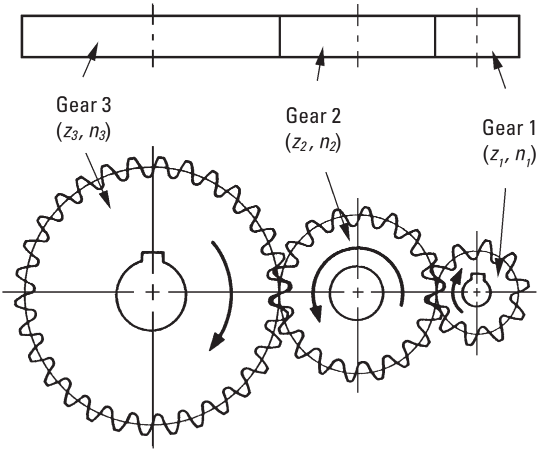


Fig. 13-3 Single-Stage Gear Train with an Idler

13.3 Planetary Gear System

The basic form of a planetary gear system is shown in **Figure 13-4**. It consists of a ***Sun Gear (A)***, ***Planet Gears (B)***, ***Internal Gear (C)*** and ***Carrier (D)***. The input and output axes of a planetary gear system are on a same line. Usually, it uses two or more planet gears to balance the load evenly. It is compact in space, but complex in structure. Planetary gear systems need a high-quality manufacturing process. The load division between planet gears, the interference of the internal gear, the balance and vibration of the rotating carrier, and the hazard of jamming, etc. are inherent problems to be solved.

Figure 13-4 is a so called 2K-H type planetary gear system. The sun gear, internal gear, and the carrier have a common axis.

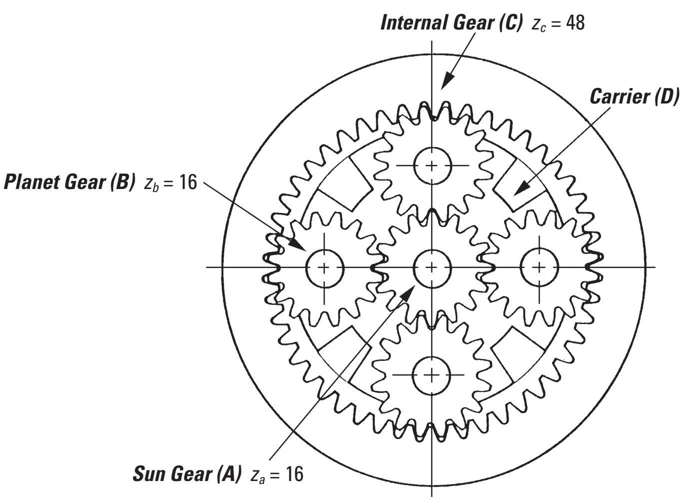


Fig. 13-4 An Example of a Planetary Gear System

13.3.1 Relationship Among The Gears In A Planetary Gear System

In order to determine the relationship among the numbers of teeth of the sun gear A, (z_a), the planet gears B, (z_b), and the internal gear C, (z_c), and the number of planet gears, N, in the system, the parameters must satisfy the following three conditions:

Condition No. 1: $z_c = z_a + 2\,z_b$ **(13-5)**

This is the condition necessary for the center distances of the gears to match. Since the equation is true only for the standard gear system, it is possible to vary the numbers of teeth by using profile shifted gear designs.

To use profile shifted gears, it is necessary to match the center distance between the sun A and planet B gears, a_{x1}, and the center distance between the planet B and internal C gears, a_{x2}.

$$a_{x1} = a_{x2} \qquad \textbf{(13-6)}$$

Condition No. 2: $\dfrac{(z_a + z_c)}{N} = \text{integer}$ **(13-7)**

This is the condition necessary for placing planet gears evenly spaced around the sun gear. If an uneven placement of planet gears is desired, then **Equation (13-8)** must be satisfied.

$$\frac{(z_a + z_c)\,\theta}{180} = \text{integer} \qquad \textbf{(13-8)}$$

where:

θ = half the angle between adjacent planet gears

Condition No. 3:

$$z_b + 2 < (z_a + z_b)\,\sin\left(\frac{180}{N}\right) \qquad \textbf{(13-9)}$$

Satisfying this condition insures that adjacent planet gears can operate without interfering with each other. This is the condition that must be met for standard gear design with equal placement of planet gears. For other conditions, the system must satisfy the relationship:

$$d_{ab} < 2\,a_x \sin\theta \qquad \textbf{(13-10)}$$

where:

d_{ab} = outside diameter of the planet gears

a_x = center distance between the sun and planet gears

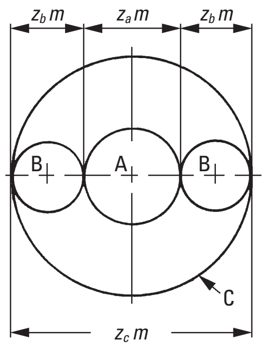


Fig. 13-5(a) Condition No. 1 of Planetary Gear System

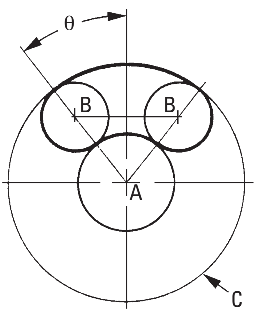


Fig. 13-5(b) Condition No. 2 of Planetary Gear System

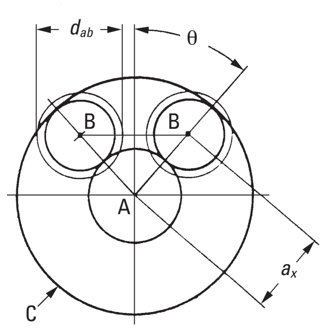


Fig. 13-5(c) Condition No. 3 of Planetary Gear System

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Besides the above three basic conditions, there can be an interference problem between the internal gear C and the planet gears B. See **SECTION 5** that discusses more about this problem.

13.3.2 Speed Ratio Of Planetary Gear System

In a planetary gear system, the speed ratio and the direction of rotation would be changed according to which member is fixed. **Figures 13-6(a)**, **13-6(b)** and **13-6(c)** contain three typical types of planetary gear mechanisms, depending upon which member is locked.

(a) Planetary Type

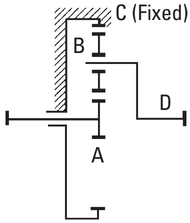

Fig. 13-6(a) Planetary Type Planetary Gear Mechanism

In this type, the internal gear is fixed. The input is the sun gear and the output is carrier D. The speed ratio is calculated as in **Table 13-1**.

Table 13-1 Equations of Speed Ratio for a Planetary Type

No.	Description	Sun Gear A z_a	Planet Gear B z_b	Internal Gear C z_c	Carrier D
1	Rotate sun gear A once while holding carrier	+1	$-\frac{z_a}{z_b}$	$-\frac{z_a}{z_c}$	0
2	System is fixed as a whole while rotating $+(z_a/z_c)$	$+\frac{z_a}{z_c}$	$+\frac{z_a}{z_c}$	$+\frac{z_a}{z_c}$	$+\frac{z_a}{z_c}$
3	Sum of 1 and 2	$1+\frac{z_a}{z_c}$	$\frac{z_a}{z_c}-\frac{z_a}{z_b}$	0 (fixed)	$+\frac{z_a}{z_c}$

$$\text{Speed Ratio} = \frac{\frac{z_a}{z_c}}{1+\frac{z_a}{z_c}} = \frac{1}{\frac{z_c}{z_a}+1} \qquad \textbf{(13-11)}$$

Note that the direction of rotation of input and output axes are the same.

Example: $z_a = 16$, $z_b = 16$, $z_c = 48$, then speed ratio = 1/4.

(b) Solar Type

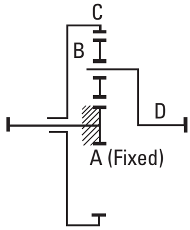

Fig. 13-6(b) Solar Type Planetary Gear Mechanism

In this type, the sun gear is fixed. The internal gear C is the input, and carrier D axis is the output. The speed ratio is calculated as in **Table 13-2**, on the following page.

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Table 13-2 Equations of Speed Ratio for a Solar Type

No.	Description	Sun Gear A z_a	Planet Gear B z_b	Internal Gear C z_c	Carrier D
1	Rotate sun gear A once while holding carrier	+1	$-\frac{z_a}{z_b}$	$-\frac{z_a}{z_c}$	0
2	System is fixed as a whole while rotating $+(z_a/z_c)$	–1	–1	–1	–1
3	Sum of 1 and 2	0 (fixed)	$-\frac{z_a}{z_b}-1$	$-\frac{z_a}{z_c}-1$	–1

$$\text{Speed Ratio} = \frac{-1}{-\frac{z_a}{z_c}-1} = \frac{1}{\frac{z_a}{z_c}+1} \qquad \textbf{(13-12)}$$

Note that the directions of rotation of input and output axes are the same.
Example: $z_a = 16$, $z_b = 16$, $z_c = 48$, then the speed ratio = 1/1.3333333.

(c) Star Type

This is the type in which Carrier D is fixed.The planet gears B rotate only on fixed axes. In a strict definition, this train loses the features of a planetary system and it becomes an ordinary gear train. The sun gear is an input axis and the internal gear is the output. The speed ratio is:

$$\text{Speed Ratio} = -\frac{z_a}{z_c} \qquad \textbf{(13-13)}$$

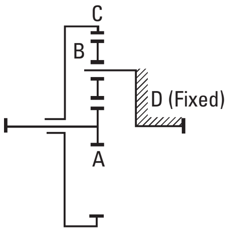

Fig. 13-6(c) Star Type Planetary Gear Mechanism

Referring to **Figure 13-6(c)**, the planet gears are merely idlers. Input and output axes have opposite rotations.
Example: $z_a = 16$, $z_b = 16$, $z_c = 48$;
then speed ratio = –1/3.

13.4 Constrained Gear System

A planetary gear system which has four gears, as in **Figure 13-5**, is an example of a constrained gear system. It is a closed loop system in which the power is transmitted from the driving gear through other gears and eventually to the driven gear. A closed loop gear system will not work if the gears do not meet specific conditions.

Let z_1, z_2 and z_3 be the numbers of gear teeth, as in **Figure 13-7**. Meshing cannot function if the length of the heavy line (belt) does not divide evenly by circular pitch. **Equation (13-14)** defines this condition.

$$\frac{z_1\theta_1}{180} + \frac{z_2(180+\theta_1+\theta_2)}{180} + \frac{z_3\theta_2}{180} = \text{integer} \qquad \textbf{(13-14)}$$

where θ_1 and θ_2 are in degrees.

Figure 13-8 shows a constrained gear system in which a rack is meshed. The heavy line in **Figure 13-8** corresponds to the belt in **Figure 13-7**. If the length of the belt cannot be evenly divided by circular pitch then the system does not work. It is described by **Equation (13-15)**.

$$\frac{z_1\theta_1}{180} + \frac{z_2(180+\theta_1)}{180} + \frac{a}{\pi m} = \text{integer} \quad \textbf{(13-15)}$$

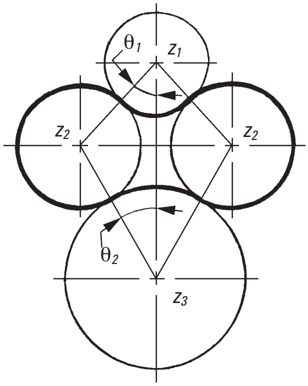

Fig. 13-7 Constrained Gear System

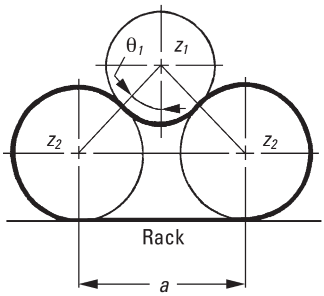

Fig. 13-8 Constrained Gear System Containing a Rack

SECTION 14 BACKLASH

Up to this point the discussion has implied that there is no backlash. If the gears are of standard tooth proportion design and operate on standard center distance they would function ideally with neither backlash nor jamming.

Backlash is provided for a variety of reasons and cannot be designated without consideration of machining conditions. The general purpose of backlash is to prevent gears from jamming by making contact on both sides of their teeth simultaneously. A small amount of backlash is also desirable to provide for lubricant space and differential expansion between the gear components and the housing. Any error in machining which tends to increase the possibility of jamming makes it necessary to increase the amount of backlash by at least as much as the possible cumulative errors. Consequently, the smaller the amount of backlash, the more accurate must be the machining of the gears. Runout of both gears, errors in profile, pitch, tooth thickness, helix angle and center distance – all are factors to consider in the specification of the amount of backlash. On the other hand, excessive backlash is objectionable, particularly if the drive is frequently reversing or if there is an overrunning load. The amount of backlash must not be excessive for the requirements of the job, but it should be sufficient so that machining costs are not higher than necessary.

In order to obtain the amount of backlash desired, it is necessary to decrease tooth thickness. See **Figure 14-1**. This decrease must almost always be greater than the desired backlash because of the errors in manufacturing and assembling. Since the amount of the decrease in tooth thickness depends upon the accuracy of machining, the allowance for a specified backlash will vary according to the manufacturing conditions.

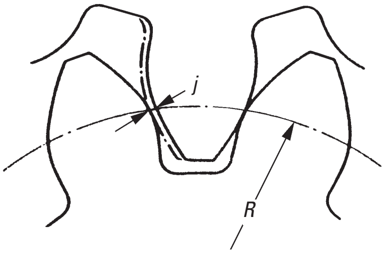

Figure 14-1 Backlash, (j) Between Two Gears

It is customary to make half of the allowance for backlash on the tooth thickness of each gear of a pair, although there are exceptions. For example, on pinions having very low numbers of teeth, it is desirable to provide all of the allowance on the mating gear so as not to weaken the pinion teeth.

In spur and helical gearing, backlash allowance is usually obtained by sinking the hob deeper into the blank than the theoretically standard depth. Further, it is true that any increase or decrease in center distance of two gears in any mesh will cause an increase or decrease in backlash. Thus, this is an alternate way of designing backlash into the system.

In the following, we give the fundamental equations for the determination of backlash in a single gear mesh. For the determination of backlash in gear trains, it is necessary to sum the backlash of each mated gear pair. However, to obtain the total backlash for a series of meshes, it is necessary to take into account the gear ratio of each mesh relative to a chosen reference shaft in the gear train. For details, see Reference 10 at the end of the technical section.

14.1 Definition Of Backlash

Backlash is defined in **Figure 14-2(a)** as the excess thickness of tooth space over the thickness of the mating tooth. There are two basic ways in which backlash arises: tooth thickness is below the zero backlash value; and the operating center distance is greater than the zero backlash value.

If the tooth thickness of either or both mating gears is less than the zero backlash value, the amount of backlash introduced in the mesh is simply this numerical difference:

$$j = s_{std} - s_{act} = \Delta s \qquad \textbf{(14-1)}$$

Linear Backlash = $j = s_s - s_2$

Angular Backlash of

Gear $= j_{\theta 1} = \dfrac{j}{R}$

Pinion $= j_{\theta 2} = \dfrac{j}{r}$

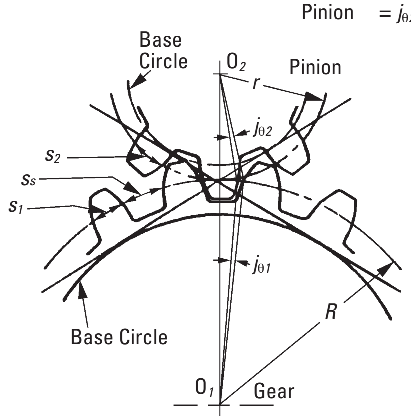

Fig. 14-2(a) Geometrical Definition of Angular Backlash

where:

j = linear backlash measured along the pitch circle (**Figure 14-2(b)**)

s_{std} = no backlash tooth thickness on the operating pitch circle, which is the standard tooth thickness for ideal gears

s_{act} = actual tooth thickness

When the center distance is increased by a relatively small amount, Δa, a backlash space develops between mating teeth, as in **Figure 14-3**. The relationship between center distance increase and linear backlash j_n along the line-of-action is:

$$j_n = 2\,\Delta a \sin\alpha \qquad \text{(14-2)}$$

Backlash, Along Line-of-Action = $j_n = j \cos\alpha$

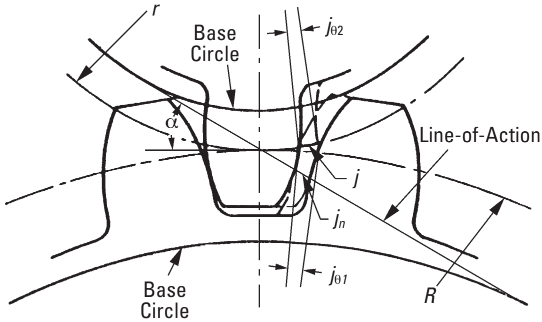

Fig. 14-2(b) Geometrical Definition of Linear Backlash

(a) Gear Teeth in Tight Mesh
No Backlash

(b) Gear Mesh with Backlash
Due to Δa

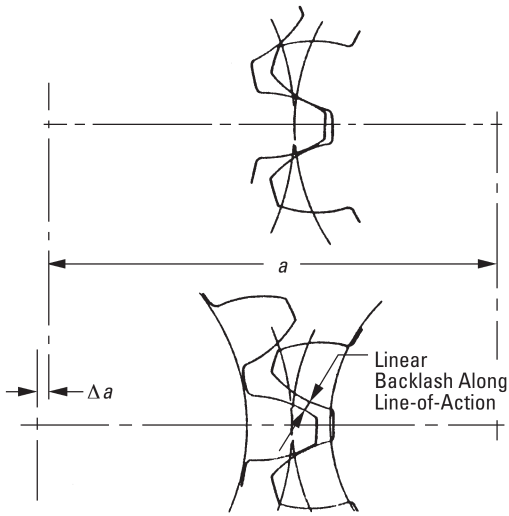

Figure 14-3 Backlash Caused by Opening of Center Distance

This measure along the line-of-action is useful when inserting a feeler gage between teeth to measure backlash. The equivalent linear backlash measured along the pitch circle is given by:

$$j = 2\,\Delta a \tan\alpha \qquad \textbf{(14-3a)}$$

where:

Δa = change in center distance

α = pressure angle

Hence, an approximate relationship between center distance change and change in backlash is:

$$\Delta a = 1.933\,\Delta j \text{ for 14.5° pressure angle gears} \qquad \textbf{(14-3b)}$$

$$\Delta a = 1.374\,\Delta j \text{ for 20° pressure angle gears} \qquad \textbf{(14-3c)}$$

Although these are approximate relationships, they are adequate for most uses. Their derivation, limitations, and correction factors are detailed in Reference 10.

Note that backlash due to center distance opening is dependent upon the tangent function of the pressure angle. Thus, 20° gears have 41% more backlash than 14.5° gears, and this constitutes one of the few advantages of the lower pressure angle.

Equations (14-3) are a useful relationship, particularly for converting to angular backlash. Also, for fine pitch gears the use of feeler gages for measurement is impractical, whereas an indicator at the pitch line gives a direct measure. The two linear backlashes are related by:

$$j = \frac{j_n}{\cos\alpha} \qquad \textbf{(14-4)}$$

The angular backlash at the gear shaft is usually the critical factor in the gear application. As seen from **Figure 14-2(a)**, this is related to the gear's pitch radius as follows:

$$j_\theta = 3440\,\frac{j}{R_1} \text{ (arc minutes)} \qquad \textbf{(14-5)}$$

Obviously, angular backlash is inversely proportional to gear radius. Also, since the two meshing gears are usually of different pitch diameters, the linear backlash of the measure converts to different angular values for each gear. Thus, an angular backlash must be specified with reference to a particular shaft or gear center.

Details of backlash calculations and formulas for various gear types are given in the following sections.

14.2 Backlash Relationships

Expanding upon the previous definition, there are several kinds of backlash: circular backlash j_t, normal backlash j_n, center backlash j_r and angular backlash j_θ (°), see **Figure 14-4**.

Table 14-1 reveals relationships among circular backlash j_t, normal backlash j_n and center backlash j_r. In this definition, j_r is equivalent to change in center distance, Δa, in **Section 14.1**.

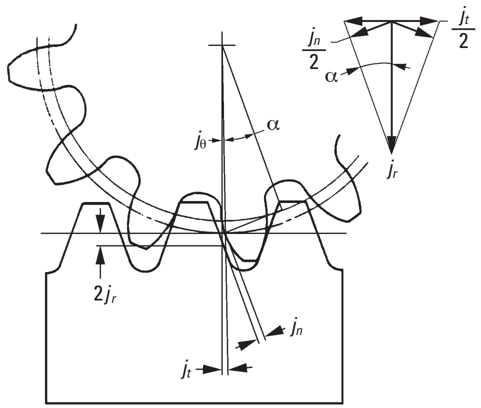

Fig. 14-4 Kinds of Backlash and Their Direction

Table 14-1 The Relationships among the Backlashes

No.	Type of Gear Meshes	The Relation between Circular Backlash j_t and Normal Backlash j_n	The Relation between Circular Backlash j_t and Center Backlash j_r
1	Spur Gear	$j_n = j_t \cos\alpha$	$j_r = \dfrac{j_t}{2\tan\alpha}$
2	Helical Gear	$j_{nn} = j_{tt} \cos\alpha_n \cos\beta$	$j_r = \dfrac{j_{tt}}{2\tan\alpha_t}$
3	Straight Bevel Gear	$j_n = j_t \cos\alpha$	$j_r = \dfrac{j_t}{2\tan\alpha\sin\delta}$
4	Spiral Bevel Gear	$j_{nn} = j_{tt} \cos\alpha_n \cos\beta_m$	$j_r = \dfrac{j_{tt}}{2\tan\alpha_t\sin\delta}$
5	Worm Worm Gear	$j_{nn} = j_{tt1} \cos\alpha_n \cos\gamma$ $j_{nn} = j_{tt2} \cos\alpha_n \cos\gamma$	$j_r = \dfrac{j_{tt2}}{2\tan\alpha_x}$

Circular backlash j_t has a relation with angular backlash j_θ, as follows:

$$j_\theta = j_t \frac{360}{\pi d} \text{ (degrees)} \qquad \textbf{(14-6)}$$

14.2.1 Backlash Of A Spur Gear Mesh

From **Figure 14-4** we can derive backlash of spur mesh as:

$$\left.\begin{aligned} j_n &= j_t \cos\alpha \\ j_r &= \frac{j_t}{2\tan\alpha} \end{aligned}\right\} \qquad \textbf{(14-7)}$$

14.2.2 Backlash Of Helical Gear Mesh

The helical gear has two kinds of backlash when referring to the tooth space. There is a cross section in the normal direction of the tooth surface *n*, and a cross section in the radial direction perpendicular to the axis, *t*.

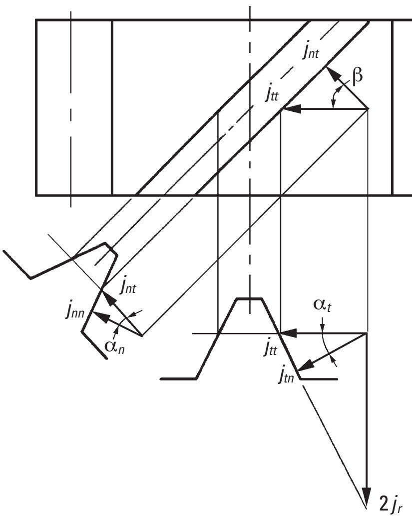

Fig. 14-5 Backlash of Helical Gear Mesh

j_{nn} = backlash in the direction normal to the tooth surface

j_{nt} = backlash in the circular direction in the cross section normal to the tooth

j_{tn} = backlash in the direction normal to the tooth surface in the cross section perpendicular to the axis

j_{tt} = backlash in the circular direction perpendicular to the axis

These backlashes have relations as follows:

In the plane normal to the tooth:

$$j_{nn} = j_{nt} \cos \alpha_n \qquad \textbf{(14-8)}$$

On the pitch surface:

$$j_{nt} = j_{tt} \cos \beta \qquad \textbf{(14-9)}$$

In the plane perpendicular to the axis:

$$\left.\begin{aligned} j_{tn} &= j_{tt} \cos \alpha_t \\ j_r &= \frac{j_{tt}}{2 \tan \alpha_t} \end{aligned}\right\} \qquad \textbf{(14-10)}$$

14.2.3 Backlash Of Straight Bevel Gear Mesh

Figure 14-6 expresses backlash for a straight bevel gear mesh.

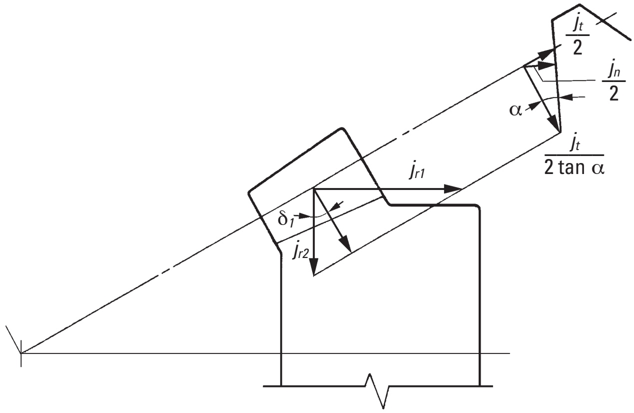

Fig. 14-6 Backlash of Straight Bevel Gear Mesh

In the cross section perpendicular to the tooth of a straight bevel gear, circular backlash at pitch line j_t, normal backlash j_n and radial backlash j_r' have the following relationships:

$$\left.\begin{aligned} j_n &= j_t \cos\alpha \\ j_r' &= \frac{j_t}{2\tan\alpha} \end{aligned}\right\} \qquad (14\text{-}11)$$

The radial backlash in the plane of axes can be broken down into the components in the direction of bevel pinion center axis, j_{r1}, and in the direction of bevel gear center axis, j_{r2}.

$$\left.\begin{aligned} j_{r1} &= \frac{j_t}{2\tan\alpha\sin\delta_1} \\ j_{r2} &= \frac{j_t}{2\tan\alpha\cos\delta_1} \end{aligned}\right\} \qquad (14\text{-}12)$$

14.2.4 Backlash Of A Spiral Bevel Gear Mesh

Figure 14-7 delineates backlash for a spiral bevel gear mesh.

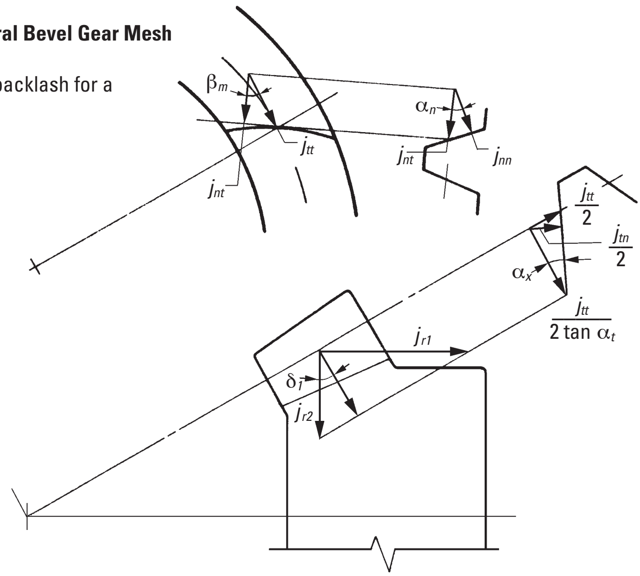

Fig. 14-7 Backlash of Spiral Bevel Gear Mesh

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In the tooth space cross section normal to the tooth:

$$j_{nn} = j_{nt} \cos \alpha_n \tag{14-13}$$

On the pitch surface:

$$j_{nt} = j_{tt} \cos \beta_m \tag{14-14}$$

In the plane perpendicular to the generatrix of the pitch cone:

$$\left.\begin{aligned} j_{tn} &= j_{tt} \cos \alpha_t \\ j_r' &= \frac{j_{tt}}{2 \tan \alpha_t} \end{aligned}\right\} \tag{14-15}$$

The radial backlash in the plane of axes can be broken down into the components in the direction of bevel pinion center axis, j_{r1}, and in the direction of bevel gear center axis, j_{r2}.

$$\left.\begin{aligned} j_{r1} &= \frac{j_{tt}}{2 \tan \alpha_t \sin \delta_1} \\ j_{r2} &= \frac{j_{tt}}{2 \tan \alpha_t \cos \delta_1} \end{aligned}\right\} \tag{14-16}$$

14.2.5 Backlash Of Worm Gear Mesh

Figure 14-8 expresses backlash for a worm gear mesh.
On the pitch surface of a worm:

$$\left.\begin{aligned} j_{nt} &= j_{tt1} \sin \gamma \\ j_{nt} &= j_{tt2} \cos \gamma \\ \tan \gamma &= \frac{j_{tt2}}{j_{tt1}} \end{aligned}\right\} \tag{14-17}$$

In the cross section of a worm perpendicular to its axis:

$$\left.\begin{aligned} j_{tn1} &= j_{tt1} \cos \alpha_t \\ j_r &= \frac{j_{tt1}}{2 \tan \alpha_t} \end{aligned}\right\} \tag{14-18}$$

In the plane perpendicular to the axis of the worm gear:

$$\left.\begin{aligned} j_{tn2} &= j_{tt2} \cos \alpha_x \\ j_r &= \frac{j_{tt2}}{2 \tan \alpha_x} \end{aligned}\right\} \tag{14-19}$$

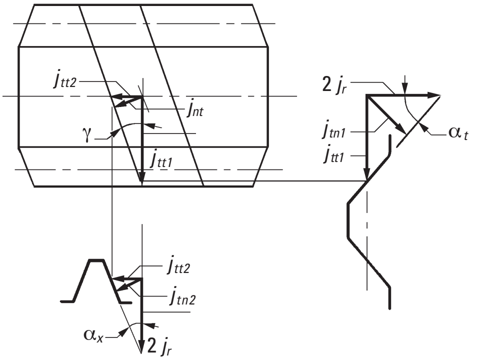

Fig. 14-8 Backlash of Worm Gear Mesh

14.3 Tooth Thickness And Backlash

There are two ways to produce backlash. One is to enlarge the center distance. The other is to reduce the tooth thickness. The latter is much more popular than the former. We are going to discuss more about the way of reducing the tooth thickness. In **SECTION 10**, we have discussed the standard tooth thickness s. In the meshing of a pair of gears, if the tooth thickness of pinion and gear were reduced by Δs_1 and Δs_2, they would generate a backlash of $\Delta s_1 + \Delta s_2$ in the direction of the pitch circle.

Let the magnitude of Δs_1, Δs_2 be 0.1. We know that $\alpha = 20°$, then:

$$j_t = \Delta s_1 + \Delta s_2 = 0.1 + 0.1 = 0.2$$

We can convert it into the backlash on normal direction:

$$j_n = j_t \cos\alpha = 0.2 \cos 20° = 0.1879$$

Let the backlash on the center distance direction be j_r, then:

$$j_r = \frac{j_t}{2\tan\alpha} = \frac{0.2}{2\tan 20°} = 0.2747$$

They express the relationship among several kinds of backlashes. In application, one should consult the JIS standard.

There are two JIS standards for backlash – one is JIS B 1703-76 for spur gears and helical gears, and the other is JIS B 1705-73 for bevel gears. All these standards regulate the standard backlashes in the direction of the pitch circle j_t or j_{tt}. These standards can be applied directly, but the backlash beyond the standards may also be used for special purposes. When writing tooth thicknesses on a drawing, it is necessary to specify, in addition, the tolerances on the thicknesses as well as the backlash. For example:

Circular tooth thickness $3.141^{-0.050}_{-0.100}$

Backlash 0.100 ... 0.200

14.4 Gear Train And Backlash

The discussions so far involved a single pair of gears. Now, we are going to discuss two stage gear trains and their backlash. In a two stage gear train, as **Figure 14-9** shows, j_1 and j_4 represent the backlashes of first stage gear train and second stage gear train respectively.

If number one gear were fixed, then the accumulated backlash on number four gear j_{tT4} would be as follows:

$$j_{tT4} = j_1 \frac{d_3}{d_2} + j_4 \qquad \textbf{(14-20)}$$

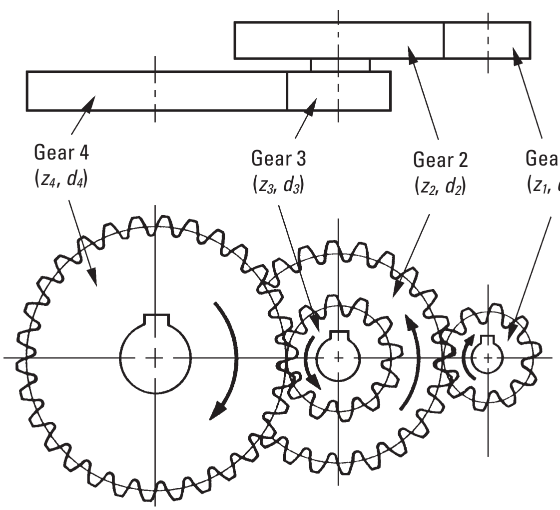

Fig. 14-9 Overall Accumulated Backlash of Two Stage Gear Train

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14
15
A

This accumulated backlash can be converted into rotation in degrees:

$$j_\theta = j_{tT4}\frac{360}{\pi d_4} \quad \text{(degrees)} \tag{14-21}$$

The reverse case is to fix number four gear and to examine the accumulated backlash on number one gear j_{tT1}.

$$j_{tT1} = j_4\frac{d_2}{d_3} + j_1 \tag{14-22}$$

This accumulated backlash can be converted into rotation in degrees:

$$j_\theta = j_{tT1}\frac{360}{\pi d_1} \quad \text{(degrees)} \tag{14-23}$$

14.5 Methods Of Controlling Backlash

In order to meet special needs, precision gears are used more frequently than ever before. Reducing backlash becomes an important issue. There are two methods of reducing or eliminating backlash – one a static, and the other a dynamic method.

The static method concerns means of assembling gears and then making proper adjustments to achieve the desired low backlash. The dynamic method introduces an external force which continually eliminates all backlash regardless of rotational position.

14.5.1 Static Method

This involves adjustment of either the gear's effective tooth thickness or the mesh center distance. These two independent adjustments can be used to produce four possible combinations as shown in **Table 14-2**.

Table 14-2

		Center Distance	
		Fixed	Adjustable
Gear Size	Fixed	I	III
	Adjustable	II	IV

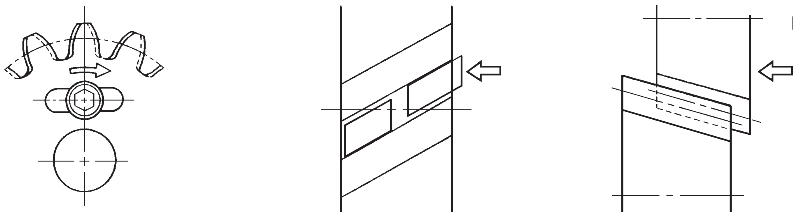
Case I

By design, center distance and tooth thickness are such that they yield the proper amount of desired minimum backlash. Center distance and tooth thickness size are fixed at correct values and require precision manufacturing.

Case II

With gears mounted on fixed centers, adjustment is made to the effective tooth thickness by axial movement or other means. Three main methods are:

1. Two identical gears are mounted so that one can be rotated relative to the other and fixed. See **Figure 14-10a**. In this way, the effective tooth thickness can be adjusted to yield the desired low backlash.
2. A gear with a helix angle such as a helical gear is made in two half thicknesses. One is shifted axially such that each makes contact with the mating gear on the opposite sides of the tooth. See **Figure 14-10b**.
3. The backlash of cone shaped gears, such as bevel and tapered tooth spur gears, can be adjusted with axial positioning. A duplex lead worm can be adjusted similarly. See **Figure 14-10c**.



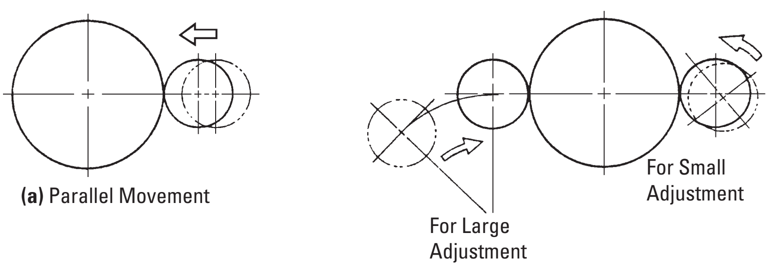
(a) Rotary Adjustment **(b)** Parallel Adjustment **(c)** Axial Adjustment

Fig. 14-10 Ways of Reducing Backlash in Case II

Case III

Center distance adjustment of backlash can be accomplished in two ways:

1. Linear Movement – **Figure 14-11a** shows adjustment along the line-of-centers in a straight or parallel axes manner. After setting to the desired value of backlash, the centers are locked in place.
2. Rotary Movement – **Figure 14-11b** shows an alternate way of achieving center distance adjustment by rotation of one of the gear centers by means of a swing arm on an eccentric bushing. Again, once the desired backlash setting is found, the positioning arm is locked.



(a) Parallel Movement

(b) Rotary Movement

Fig. 14-11 Ways of Decreasing Backlash in Case III

Case IV

Adjustment of both center distance and tooth thickness is theoretically valid, but is not the usual practice. This would call for needless fabrication expense.

14.5.2 Dynamic Methods

Dynamic methods relate to the static techniques. However, they involve a forced adjustment of either the effective tooth thickness or the center distance.

1. Backlash Removal by Forced Tooth Contact

This is derived from static Case II. Referring to **Figure 14-10a** , a forcing spring rotates the two gear halves apart. This results in an effective tooth thickness that continually fills the entire tooth space in all mesh positions.

2. Backlash Removal by Forced Center Distance Closing

This is derived from static Case III. A spring force is applied to close the center distance; in one case as a linear force along the line-of-centers, and in the other case as a torque applied to the swing arm.

In all of these dynamic methods, the applied external force should be known and properly specified. The theoretical relationship of the forces involved is as follows:

$$F > F_1 + F_2 \qquad \textbf{(14-24)}$$

where:

F_1 = Transmission Load on Tooth Surface
F_2 = Friction Force on Tooth Surface

If $F < F_1 + F_2$, then it would be impossible to remove backlash. But if F is excessively greater than a proper level, the tooth surfaces would be needlessly loaded and could lead to premature wear and shortened life. Thus, in designing such gears, consideration must be given to not only the needed transmission load, but also the forces acting upon the tooth surfaces caused by the spring load. It is important to appreciate that the spring loading must be set to accommodate the largest expected transmission force, F_1, and this maximum spring force is applied to the tooth surfaces continually and irrespective of the load being driven.

3. Duplex Lead Worm

A duplex lead worm mesh is a special design in which backlash can be adjusted by shifting the worm axially. It is useful for worm drives in high precision turntables and hobbing machines. **Figure 14-12** presents the basic concept of a duplex lead worm.

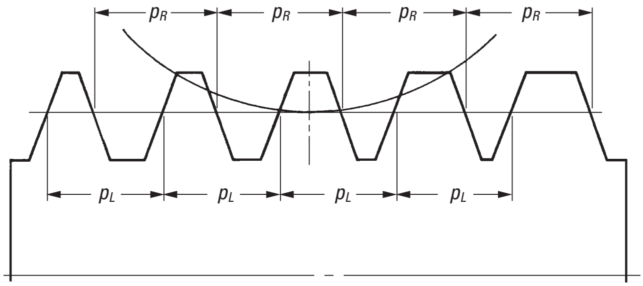


Fig. 14-12 Basic Concepts of Duplex Lead Worm

The lead or pitch, p_L and p_R, on the two sides of the worm thread are not identical. The example in **Figure 14-12** shows the case when $p_R > p_L$. To produce such a worm requires a special dual lead hob.

The intent of **Figure 14-12** is to indicate that the worm tooth thickness is progressively bigger towards the right end. Thus, it is convenient to adjust backlash by simply moving the duplex worm in the axial direction.

SECTION 15 GEAR ACCURACY

Gears are one of the basic elements used to transmit power and position. As designers, we desire them to meet various demands:

1. Minimum size.
2. Maximum power capability.
3. Minimum noise (silent operation).
4. Accurate rotation/position.

To meet various levels of these demands requires appropriate degrees of gear accuracy. This involves several gear features.

15.1 Accuracy Of Spur And Helical Gears

This discussion of spur and helical gear accuracy is based upon JIS B 1702 standard. This specification describes 9 grades of gear accuracy – grouped from 0 through 8 – and four types of pitch errors:

Single pitch error.
Pitch variation error.
Accumulated pitch error.
Normal pitch error.

Single pitch error, pitch variation and accumulated pitch errors are closely related with each other.

15.1.1 Pitch Errors of Gear Teeth

1. Single Pitch Error (f_{pt})
   The deviation between actual measured pitch value between any adjacent tooth surface and theoretical circular pitch.
2. Pitch Variation Error (f_{pu})
   Actual pitch variation between any two adjacent teeth. In the ideal case, the pitch variation error will be zero.
3. Accumulated Pitch Error (F_p)
   Difference between theoretical summation over any number of teeth interval, and summation of actual pitch measurement over the same interval.
4. Normal Pitch Error (f_{pb})
   It is the difference between theoretical normal pitch and its actual measured value.

The major element to influence the pitch errors is the runout of gear flank groove.

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Table 15-1 contains the ranges of allowable pitch errors of spur gears and helical gears for each precision grade, as specified in JIS B 1702-1976.

Table 15-1 The Allowable Single Pitch Error, Accumulated Pitch Error and Normal Pitch Error, μm

Grade	Single Pitch Error f_{pt}	Accumulated Pitch Error F_p	Normal Pitch Error f_{pb}
JIS 0	$0.5W + 1.4$	$2.0W + 5.6$	$0.9W' + 1.4$
1	$0.71W + 2.0$	$2.8W + 8.0$	$1.25W' + 2.0$
2	$1.0W + 2.8$	$4.0W + 11.2$	$1.8W' + 2.8$
3	$1.4W + 4.0$	$5.6W + 16.0$	$2.5W' + 4.0$
4	$2.0W + 5.6$	$8.0W + 22.4$	$4.0W' + 6.3$
5	$2.8W + 8.0$	$11.2W + 31.5$	$6.3W' + 10.0$
6	$4.0W + 11.2$	$16.0W + 45.0$	$10.0W' + 16.0$
7	$8.0W + 22.4$	$32.0W + 90.0$	$20.0W' + 32.0$
8	$16.0W + 45.0$	$64.0W + 180.0$	$40.0W' + 64.0$

In the above table, W and W' are the tolerance units defined as:

$$W = \sqrt[3]{d} + 0.65m \; (\mu m) \tag{15-1}$$

$$W' = 0.56W + 0.25m \; (\mu m) \tag{15-2}$$

The value of allowable pitch variation error is k times the single pitch error. **Table 15-2** expresses the formula of the allowable pitch variation error.

Table 15-2 The Allowable Pitch Variation Error, μm

Single Pitch Error, f_{pt}	Pitch Variation Error, f_{pu}
less than 5	$1.00f_{pt}$
5 or more, but less than 10	$1.06f_{pt}$
10 or more, but less than 20	$1.12f_{pt}$
20 or more, but less than 30	$1.18f_{pt}$
30 or more, but less than 50	$1.25f_{pt}$
50 or more, but less than 70	$1.32f_{pt}$
70 or more, but less than 100	$1.40f_{pt}$
100 or more, but less than 150	$1.50f_{pt}$
more than 150	$1.60f_{pt}$

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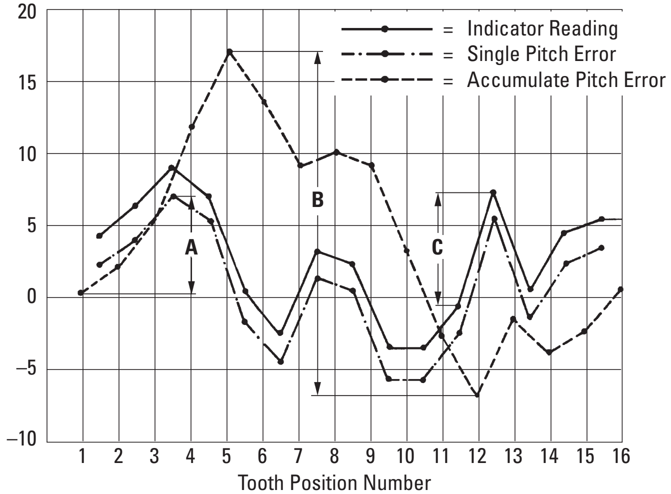
Figure 15-1 is an example of pitch errors derived from data measurements made with a dial indicator on a 15 tooth gear. Pitch differences were measured between adjacent teeth and are plotted in the figure. From that plot, single pitch, pitch variation and accumulated pitch errors are extracted and plotted.



NOTE: **A** = Max. Single Pitch Error
B = Max. Accumulated Error
C = Max. Pitch Variation Error

Fig. 15-1 Examples of Pitch Errors for a 15 Tooth Gear

15.1.2 Tooth Profile Error, f_f

Tooth profile error is the summation of deviation between actual tooth profile and correct involute curve which passes through the pitch point measured perpendicular to the actual profile. The measured band is the actual effective working surface of the gear. However, the tooth modification area is not considered as part of profile error.

15.1.3 Runout Error Of Gear Teeth, F_r

This error defines the runout of the pitch circle. It is the error in radial position of the teeth. Most often it is measured by indicating the position of a pin or ball inserted in each tooth space around the gear and taking the largest difference. Alternately, particularly for fine pitch gears, the gear is rolled with a master gear on a variable center distance fixture, which records the change in the center distance as the measure of teeth or pitch circle runout. Runout causes a number of problems, one of which is noise. The source of this error is most often insufficient accuracy and ruggedness of the cutting arbor and tooling system.

15.1.4 Lead Error, f_β

Lead error is the deviation of the actual advance of the tooth profile from the ideal value or position. Lead error results in poor tooth contact, particularly concentrating contact to the tip area. Modifications, such as tooth crowning and relieving can alleviate this error to some degree.

Shown in **Figure 15-2** (on the following page) is an example of a chart measuring tooth profile error and lead error using a Zeiss UMC 550 tester.

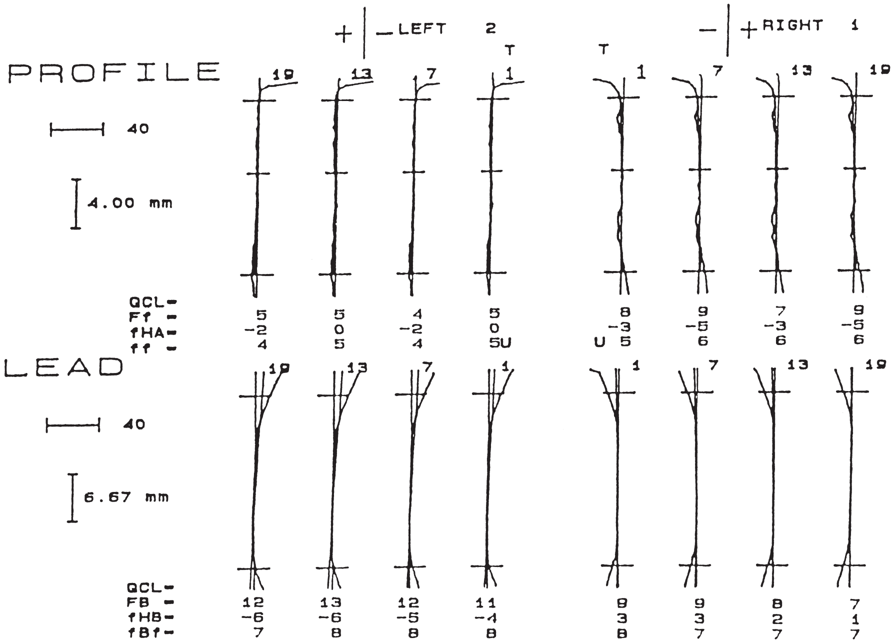


Fig. 15-2 A Sample Chart of Profile and Lead Error Measurement

Table 15-3 The Value of Allowable Tooth Profile Error, Runout Error and Lead Error, μm

Grade	Tooth Profile Error f_f	Runout Error of Gear Groove F_r	Lead Error F
JIS 0	$0.71m + 2.24$	$1.4W + 4.0$	$0.63 (0.1b + 10)$
1	$1.0m + 3.15$	$2.0W + 5.6$	$0.71 (0.1b + 10)$
2	$1.4m + 4.5$	$2.8W + 8.0$	$0.80 (0.1b + 10)$
3	$2.0m + 6.3$	$4.0W + 11.2$	$1.00 (0.1b + 10)$
4	$2.8m + 9.0$	$5.6W + 16.0$	$1.25 (0.1b + 10)$
5	$4.0m + 12.5$	$8.0W + 22.4$	$1.60 (0.1b + 10)$
6	$5.6m + 18.0$	$11.2W + 31.5$	$2.00 (0.1b + 10)$
7	$8.0m + 25.0$	$22.4W + 63.0$	$2.50 (0.1b + 10)$
8	$11.2m + 35.5$	$45.0W + 125.0$	$3.15 (0.1b + 10)$

where: W = Tolerance unit = $\sqrt[3]{d} + 0.65m$ (μm)
b = Tooth width (mm)
m = Module (mm)

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15.1.5. Outside Diameter Runout and Lateral Runout

To produce a high precision gear requires starting with an accurate gear blank. Two criteria are very important:

1. Outside diameter (OD) runout.
2. Lateral (side face) runout.

The lateral runout has a large impact on the gear tooth accuracy. Generally, the permissible runout error is related to the gear size. **Table 15-4** presents equations for allowable values of OD runout and lateral runout.

Table 15-4 The Value of Allowable OD and Lateral Runout, μm

Grade	OD Runout	Lateral Runout
JIS 0	$0.5j$	$0.71q$
1	$0.71j$	$1.0q$
2	$1.0j$	$1.4q$
3	$1.4j$	$2.0q$
4	$2.0j$	$2.8q$
5	$2.8j$	$4.0q$
6	$4.0j$	$5.6q$
7	$8.0j$	$11.2q$
8	$16.0j$	$22.4q$

where: $j = 1.1\sqrt[3]{d_a} + 5.5$

d_a = Outside diameter (mm)

$$q = \frac{6d}{b + 50} + 3$$

d = Pitch diameter (mm)

b = Tooth width (mm)

15.2 Accuracy Of Bevel Gears

JIS B 1704 regulates the specification of a bevel gear's accuracy. It also groups bevel gears into 9 grades, from 0 to 8.

There are 4 types of allowable errors:
1. Single Pitch Error.
2. Pitch Variation Error.
3. Accumulated Pitch Error.
4. Runout Error of Teeth (pitch circle).

These are similar to the spur gear errors.

1. Single Pitch Error, (f_{pt})
   The deviation between actual measured pitch value between any adjacent teeth and the theoretical circular pitch at the central cone distance.
2. Pitch Variation Error, (f_{pu})
   Absolute pitch variation between any two adjacent teeth at the central cone distance.
3. Accumulated Pitch Error, (F_p)
   Difference between theoretical pitch sum of any teeth interval, and the summation of actual measured pitches for the same teeth interval at the central cone distance.
4. Runout Error of Teeth, (F_r)
   This is the maximum amount of tooth runout in the radial direction, measured by indicating a pin or ball placed between two teeth at the central cone distance. It is the pitch cone runout.

Table 15-5 presents equations for allowable values of these various errors.

Table 15-5 Equations for Allowable Single Pitch Error, Accumulated Pitch Error and Pitch Cone Runout Error, μm

Grade	Single Pitch Error f_{pf}	Accumulated Pitch Error F_p	Runout Error of Pitch Cone F_r
JIS 0	$0.4W + 2.65$	$1.6W + 10.6$	$2.36\sqrt{d}$
1	$0.63W + 5.0$	$2.5W + 20.0$	$3.6\sqrt{d}$
2	$1.0W + 9.5$	$4.0W + 38.0$	$5.3\sqrt{d}$
3	$1.6W + 18.0$	$6.4W + 72.0$	$8.0\sqrt{d}$
4	$2.5W + 33.5$	$10.0W + 134.0$	$12.0\sqrt{d}$
5	$4.0W + 63.0$	—	$18.0\sqrt{d}$
6	$6.3W + 118.0$	—	$27.0\sqrt{d}$
7	—	—	$60.0\sqrt{d}$
8	—	—	$130.0\sqrt{d}$

where: W = Tolerance unit = $\sqrt[3]{d} + 0.65m$ (μm),
d = Pitch diameter (mm)

The equations of allowable pitch variations are in **Table 15-6**.

Table 15-6 The Formula of Allowable Pitch Variation Error (μm)

Single Pitch Error, f_{pt}	Pitch Variation Error, f_{pu}
Less than 70	$1.3f_{pt}$
70 or more, but less than 100	$1.4f_{pt}$
100 or more, but less than 150	$1.5f_{ptt}$
More than 150	$1.6f_{pt}$

The equations of allowable pitch variations are in **Table 15-6**.

Besides the above errors, there are seven specifications for bevel gear blank dimensions and angles, plus an eighth that concerns the cut gear set:

1. The tolerance of the blank outside diameter and the crown to back surface distance.
2. The tolerance of the outer cone angle of the gear blank.
3. The tolerance of the cone surface runout of the gear blank.
4. The tolerance of the side surface runout of the gear blank.
5. The feeler gauge size to check the flatness of blank back surface.
6. The tolerance of the shaft runout of the gear blank.
7. The tolerance of the shaft bore dimension deviation of the gear blank.
8. The contact band of the tooth mesh.

Item 8 relates to cutting of the two mating gears' teeth. The meshing tooth contact area must be full and even across the profiles. This is an important criterion that supersedes all other blank requirements.

15.3 Running (Dynamic) Gear Testing

An alternate simple means of testing the general accuracy of a gear is to rotate it with a mate, preferably of known high quality, and measure characteristics during rotation. This kind of tester can be either single contact (fixed center distance method) or dual (variable center distance method). This refers to action on one side or simultaneously on both sides of the tooth. This is also commonly referred to as single and double flank testing. Because of simplicity, dual contact testing is more popular than single contact. JGMA has a specification on accuracy of running tests.

1. Dual Contact (Double Flank) Testing

In this technique, the gear is forced meshed with a master gear such that there is intimate tooth contact on both sides and, therefore, no backlash. The contact is forced by a loading spring. As the gears rotate, there is variation of center distance due to various errors, most notably runout. This variation is measured and is a criterion of gear quality. A full rotation presents the total gear error, while rotation through one pitch is a tooth-to-tooth error. **Figure 15-3** presents a typical plot for such a test.

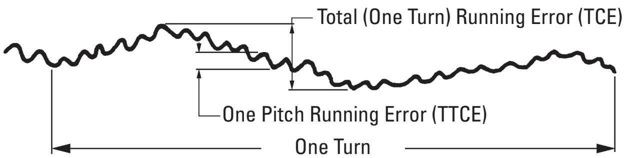

Fig. 15-3 Example of Dual Contact Running Testing Report

For American engineers, this measurement test is identical to what AGMA designates as Total Composite Tolerance (or error) and Tooth-to-Tooth Composite Tolerance. Both of these parameters are also referred to in American publications as "errors", which they truly are. Tolerance is a design value which is an inaccurate description of the parameter, since it is an error.

Allowable errors per JGMA 116-01 are presented on the next page, in **Table 15-7**.

2. Single Contact Testing

In this test, the gear is mated with a master gear on a fixed center distance and set in such a way that only one tooth side makes contact. The gears are rotated through this single flank contact action, and the angular transmission error of the driven gear is measured. This is a tedious testing method and is seldom used except for inspection of the very highest precision gears.

Table 15-7 Allowable Values of Running Errors, μm

Grade	Tooth-to-Tooth Composite Error	Total Composite Error
0	$1.12m + 3.55$	$(1.4W + 4.0) + 0.5\ (1.12m + 3.55)$
1	$1.6m + 5.0$	$(2.0W + 5.6) + 0.5\ (1.6m + 5.0)$
2	$2.24m + 7.1$	$(2.8W + 8.0) + 0.5\ (2.24m + 7.1)$
3	$3.15m + 10.0$	$(4.0W + 11.2) + 0.5\ (3.15m + 10.0)$
4	$4.5m + 14.0$	$(5.6W + 16.0) + 0.5\ (4.5m + 14.0)$
5	$6.3m + 20.0$	$(8.0W + 22.4) + 0.5\ (6.3m + 20.0)$
6	$9.0m + 28.0$	$(11.2W + 31.5) + 0.5\ (9.0m + 28.0)$
7	$12.5m + 40.0$	$(22.4W + 63.0) + 0.5\ (12.5m + 40.0)$
8	$18.0m + 56.0$	$(45.0W + 125.0) + 0.5\ (18.0m + 56.0)$

where: W = Tolerance unit $= \sqrt[3]{d} + 0.65m\ (\mu m)$
d = Pitch diameter (mm)
m = Module

SECTION 16 GEAR FORCES

In designing a gear, it is important to analyze the magnitude and direction of the forces acting upon the gear teeth, shaft, bearings, etc. In analyzing these forces, an idealized assumption is made that the tooth forces are acting upon the central part of the tooth flank.

Table 16-1 Forces Acting Upon a Gear

Types of Gears		Tangential Force, F_u	Axial Force, F_a	Radial Force, F_r
Spur Gear		$F_u = \dfrac{2000\,T}{d}$	———	$F_u \tan\alpha$
Helical Gear			$F_u \tan\beta$	$F_u \dfrac{\tan\alpha_n}{\cos\beta}$
Straight Bevel Gear		$F_u = \dfrac{2000\,T}{d_m}$ d_m is the central pitch diameter $d_m = d - b\sin\delta$	$F_u \tan\alpha\sin\delta$	$F_u \tan\alpha\cos\delta$
Spiral Bevel Gear			When convex surface is working:	
			$\dfrac{F_u}{\cos\beta_m}(\tan\alpha_n\sin\delta - \sin\beta_m\cos\delta)$	$\dfrac{F_u}{\cos\beta_m}(\tan\alpha_n\cos\delta + \sin\beta_m\sin\delta)$
			When concave surface is working:	
			$\dfrac{F_u}{\cos\beta_m}(\tan\alpha_n\sin\delta + \sin\beta_m\cos\delta)$	$\dfrac{F_u}{\cos\beta_m}(\tan\alpha_n\cos\delta - \sin\beta_m\sin\delta)$
Worm Drive	Worm (Driver)	$F_u = \dfrac{2000\,T_1}{d_1}$	$F_u \dfrac{\cos\alpha_n\cos\gamma - \mu\sin\gamma}{\cos\alpha_n\sin\gamma + \mu\cos\gamma}$	$F_u \dfrac{\sin\alpha_n}{\cos\alpha_n\sin\gamma + \mu\cos\gamma}$
	Wheel (Driven)	$F_u \dfrac{\cos\alpha_n\cos\gamma - \mu\sin\gamma}{\cos\alpha_n\cos\gamma + \mu\cos\gamma}$	F_u	
Screw Gear $\left(\begin{matrix}\Sigma = 90^\circ \\ \beta = 45^\circ\end{matrix}\right)$	Driver Gear	$F_u = \dfrac{2000\,T_1}{d_1}$	$F_u \dfrac{\cos\alpha_n\sin\beta - \mu\cos\beta}{\cos\alpha_n\cos\beta + \mu\sin\beta}$	$F_u \dfrac{\sin\alpha_n}{\cos\alpha_n\cos\beta + \mu\sin\beta}$
	Driven Gear	$F_u \dfrac{\cos\alpha_n\sin\beta - \mu\cos\beta}{\cos\alpha_n\cos\beta + \mu\sin\beta}$	F_u	

16.1 Forces In A Spur Gear Mesh

The spur gear's transmission force F_n, which is normal to the tooth surface, as in **Figure 16-1**, can be resolved into a tangential component, F_u, and a radial component, F_r. Refer to **Equation (16-1)**.

The direction of the forces acting on the gears are shown in **Figure 16-2**. The tangential component of the drive gear, F_{u1}, is equal to the driven gear's tangential component, F_{u2}, but the directions are opposite. Similarly, the same is true of the radial components.

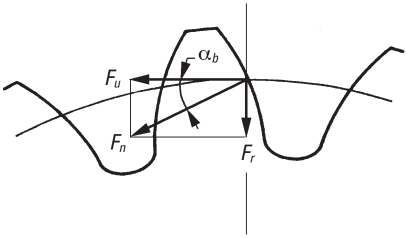

Fig. 16-1 Forces Acting on a Spur Gear Mesh

$$\left.\begin{aligned} F_u &= F_n \cos \alpha_b \\ F_r &= F_n \sin \alpha_b \end{aligned}\right\} \qquad \textbf{(16-1)}$$

16.2 Forces In A Helical Gear Mesh

The helical gear's transmission force, F_n, which is normal to the tooth surface, can be resolved into a tangential component, F_1, and a radial component, F_r.

$$\left.\begin{aligned} F_1 &= F_n \cos \alpha_n \\ F_r &= F_n \sin \alpha_n \end{aligned}\right\} \qquad \textbf{(16-2)}$$

The tangential component, F_1, can be further resolved into circular subcomponent, F_u, and axial thrust subcomponent, F_a.

$$\left.\begin{aligned} F_u &= F_1 \cos \beta \\ F_a &= F_1 \sin \beta \end{aligned}\right\} \qquad \textbf{(16-3)}$$

Substituting and manipulating the above equations result in:

$$\left.\begin{aligned} F_a &= F_u \tan \beta \\ F_r &= F_u \frac{\tan \alpha_n}{\cos \beta} \end{aligned}\right\} \qquad \textbf{(16-4)}$$

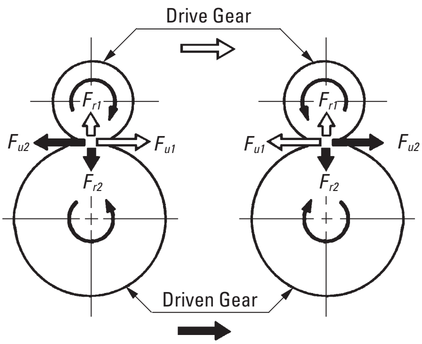

Fig. 16-2 Directions of Forces Acting on a Spur Gear Mesh

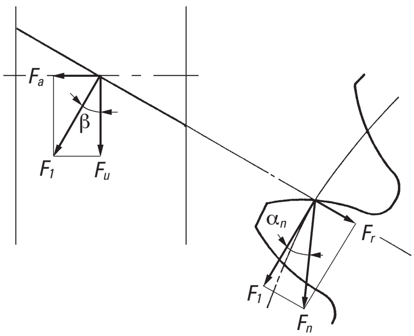

Fig. 16-3 Forces Acting on a Helical Gear Mesh

The directions of forces acting on a helical gear mesh are shown in **Figure 16-4**. The axial thrust sub-component from drive gear, F_{a1}, equals the driven gear's, F_{a2}, but their directions are opposite. Again, this case is the same as tangential components F_{u1}, F_{u2} and radial components F_{r1}, F_{r2}.

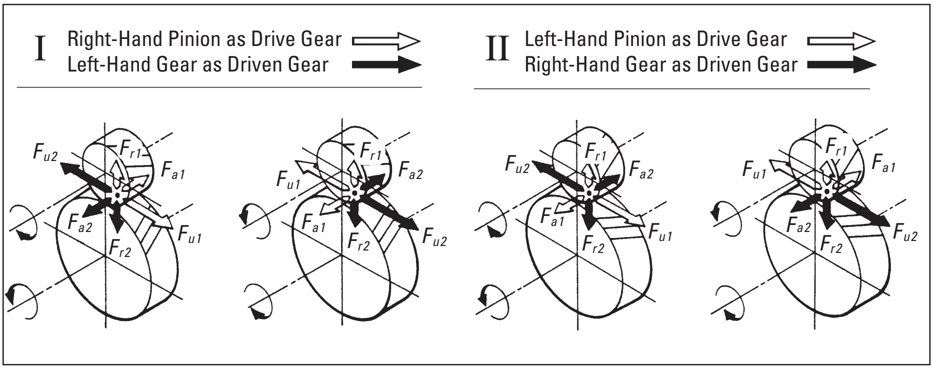

Fig. 16-4 Directions of Forces Acting on a Helical Gear Mesh

16.3 Forces On A Straight Bevel Gear Mesh

The forces acting on a straight bevel gear are shown in **Figure 16-5**. The force which is normal to the central part of the tooth face, F_n, can be split into tangential component, F_u, and radial component, F_1, in the normal plane of the tooth.

$$\left.\begin{aligned} F_u &= F_n \cos\alpha \\ F_1 &= F_n \sin\alpha \end{aligned}\right\} \qquad \textbf{(16-5)}$$

Again, the radial component, F_1, can be divided into an axial force, F_a, and a radial force, F_r, perpendicular to the axis.

$$\left.\begin{aligned} F_a &= F_1 \sin\delta \\ F_r &= F_1 \cos\delta \end{aligned}\right\} \qquad \textbf{(16-6)}$$

And the following can be derived:

$$\left.\begin{aligned} F_a &= F_u \tan\alpha_n \sin\delta \\ F_r &= F_u \tan\alpha_n \cos\delta \end{aligned}\right\} \qquad \textbf{(16-7)}$$

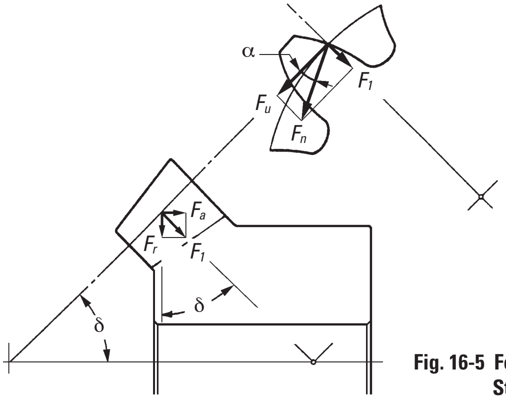

Fig. 16-5 Forces Acting on a Straight Bevel Gear Mesh

Let a pair of straight bevel gears with a shaft angle $\Sigma = 90°$, a pressure angle $\alpha_n = 20°$ and tangential force, F_u, to the central part of tooth face be 100. Axial force, F_a, and radial force, F_r, will be as presented in **Table 16-2**.

Table 16-2 Values of Axial Force, F_a, and Radial Force, F_r

(1) Pinion

Forces on the Gear Tooth	Ratio of Numbers of Teeth $\frac{z_2}{z_1}$						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Axial Force / Radial Force	$\frac{25.7}{25.7}$	$\frac{20.2}{30.3}$	$\frac{16.3}{32.6}$	$\frac{13.5}{33.8}$	$\frac{11.5}{34.5}$	$\frac{8.8}{35.3}$	$\frac{7.1}{35.7}$

(2) Gear

Forces on the Gear Tooth	Ratio of Numbers of Teeth $\frac{z_2}{z_1}$						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Axial Force / Radial Force	$\frac{25.7}{25.7}$	$\frac{30.3}{20.2}$	$\frac{32.6}{16.3}$	$\frac{33.8}{13.5}$	$\frac{34.5}{11.5}$	$\frac{35.3}{8.8}$	$\frac{35.7}{7.1}$

Figure 16-6 contains the directions of forces acting on a straight bevel gear mesh. In the meshing of a pair of straight bevel gears with shaft angle $\Sigma = 90°$, all the forces have relations as per **Equations (16-8)**.

$$\left.\begin{aligned} F_{u1} &= F_{u2} \\ F_{r1} &= F_{a2} \\ F_{a1} &= F_{r2} \end{aligned}\right\} \qquad \textbf{(16-8)}$$

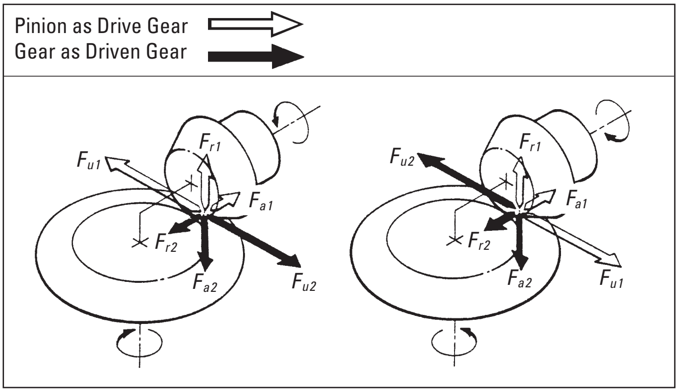

Fig. 16-6 Directions of Forces Acting on a Straight Bevel Gear Mesh

I R T 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 A

16.4 Forces In A Spiral Bevel Gear Mesh

Spiral gear teeth have convex and concave sides. Depending on which surface the force is acting on, the direction and magnitude changes. They differ depending upon which is the driver and which is the driven. **Figure 16-7** presents the profile orientations of right- and left-hand spiral teeth. If the profile of the driving gear is convex, then the profile of the driven gear must be concave. **Table 16-3** presents the concave/convex relationships.

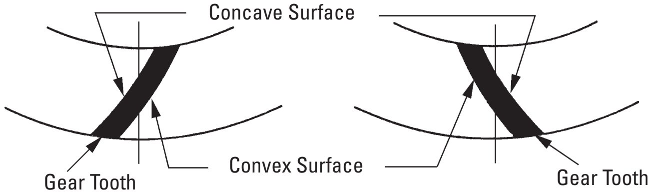

Fig. 16-7 Convex Surface and Concave Surface of a Spiral Bevel Gear

Table 16-3 Concave and Convex Sides of a Spiral Bevel Gear Mesh

Right-Hand Gear as Drive Gear

Rotational Direction of Drive Gear	Meshing Tooth Face	
	Right-Hand Drive Gear	**Left-Hand Driven Gear**
Clockwise	Convex	Concave
Counterclockwise	Concave	Convex

Left-Hand Gear as Drive Gear

Rotational Direction of Drive Gear	Meshing Tooth Face	
	Left-Hand Drive Gear	**Right-Hand Driven Gear**
Clockwise	Concave	Convex
Counterclockwise	Convex	Concave

NOTE: The rotational direction of a bevel gear is defined as the direction one sees viewed along the axis from the back cone to the apex.

16.4.1 Tooth Forces On A Convex Side Profile

The transmission force, F_n, can be resolved into components F_1 and F_t as:

$$\left.\begin{aligned} F_1 &= F_n \cos \alpha_n \\ F_t &= F_n \sin \alpha_n \end{aligned}\right\} \quad (16\text{-}9)$$

Then F_1 can be resolved into components F_u and F_s :

$$\left.\begin{aligned} F_u &= F_1 \cos \beta_m \\ F_s &= F_1 \sin \beta_m \end{aligned}\right\} \quad (16\text{-}10)$$

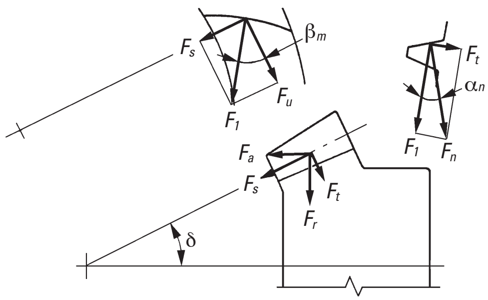

Fig. 16-8 When Meshing on the Convex Side of Tooth Face

On the axial surface, F_t and F_s can be resolved into axial and radial subcomponents.

$$\left.\begin{aligned} F_a &= F_t \sin\delta - F_s \cos\delta \\ F_r &= F_t \cos\delta + F_s \sin\delta \end{aligned}\right\} \qquad \textbf{(16-11)}$$

By substitution and manipulation, we obtain:

$$\left.\begin{aligned} F_a &= \frac{F_u}{\cos\beta_m}(\tan\alpha_n \sin\delta - \sin\beta_m \cos\delta) \\ F_r &= \frac{F_u}{\cos\beta_m}(\tan\alpha_n \cos\delta + \sin\beta \sin\delta) \end{aligned}\right\} \qquad \textbf{(16-12)}$$

16.4.2 Tooth Forces On A Concave Side Profile

On the surface which is normal to the tooth profile at the central portion of the tooth, the transmission force, F_n, can be split into F_1 and F_t as (see **Figure 16-9**):

$$\left.\begin{aligned} F_1 &= F_n \cos\alpha_n \\ F_t &= F_n \sin\alpha_n \end{aligned}\right\} \qquad \textbf{(16-13)}$$

And F_1 can be separated into components F_u and F_s on the pitch surface:

$$\left.\begin{aligned} F_u &= F_1 \cos\beta_m \\ F_s &= F_1 \sin\beta_m \end{aligned}\right\} \qquad \textbf{(16-14)}$$

So far, the equations are identical to the convex case. However, differences exist in the signs for equation terms. On the axial surface, F_t and F_s can be resolved into axial and radial subcomponents. Note the sign differences.

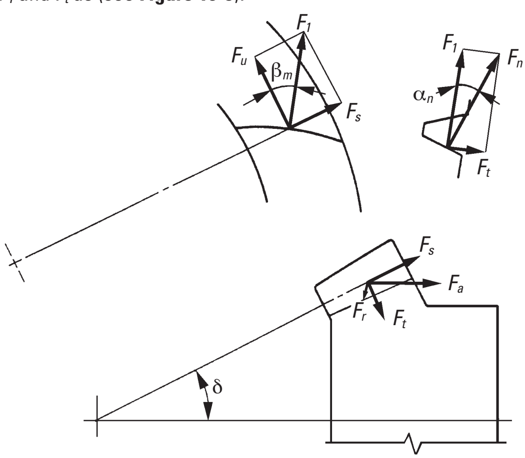

Fig. 16-9 When Meshing on the Concave Side of Tooth Face

$$\left.\begin{aligned} F_a &= F_t \sin\delta + F_s \cos\delta \\ F_r &= F_t \cos\delta - F_s \sin\delta \end{aligned}\right\} \qquad \textbf{(16-15)}$$

The above can be manipulated to yield:

$$\left.\begin{aligned} F_a &= \frac{F_u}{\cos\beta_m}(\tan\alpha_n \sin\delta + \sin\beta_m \cos\delta) \\ F_r &= \frac{F_u}{\cos\beta_m}(\tan\alpha_n \cos\delta - \sin\beta_m \sin\delta) \end{aligned}\right\} \qquad \textbf{(16-16)}$$

Let a pair of spiral bevel gears have a shaft angle $\Sigma = 90°$, a pressure angle $\alpha_n = 20°$, and a spiral angle $\beta_m = 35°$. If the tangential force, F_u, to the central portion of the tooth face is 100, the axial thrust force, F_a, and radial force, F_r, have the relationship shown in **Table 16-4**.

Table 16-4 Values of Axial Thrust Force, F_a, and Radial Force, F_r

(1) Pinion

Meshing Tooth Face	**Ratio of Number of Teeth $\frac{z_2}{z_1}$**						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Concave Side of Tooth	$\frac{80.9}{-18.1}$	$\frac{82.9}{-1.9}$	$\frac{82.5}{8.4}$	$\frac{81.5}{15.2}$	$\frac{80.5}{20.0}$	$\frac{78.7}{26.1}$	$\frac{77.4}{29.8}$
Convex Side of Tooth	$\frac{-18.1}{80.9}$	$\frac{-33.6}{75.8}$	$\frac{-42.8}{71.1}$	$\frac{-48.5}{67.3}$	$\frac{-52.4}{64.3}$	$\frac{-57.2}{60.1}$	$\frac{-59.9}{57.3}$

(2) Gear

Meshing Tooth Face	**Ratio of Number of Teeth $\frac{z_2}{z_1}$**						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Concave Side of Tooth	$\frac{80.9}{-18.1}$	$\frac{75.8}{-33.6}$	$\frac{71.1}{-42.8}$	$\frac{67.3}{-48.5}$	$\frac{64.3}{-52.4}$	$\frac{60.1}{-57.2}$	$\frac{57.3}{-59.9}$
Convex Side of Tooth	$\frac{-18.1}{80.9}$	$\frac{-1.9}{82.9}$	$\frac{8.4}{82.5}$	$\frac{15.2}{81.5}$	$\frac{20.0}{80.5}$	$\frac{26.1}{78.7}$	$\frac{29.8}{77.4}$

The value of axial force, F_a, of a spiral bevel gear, from **Table 16-4**, could become negative. At that point, there are forces tending to push the two gears together. If there is any axial play in the bearing, it may lead to the undesirable condition of the mesh having no backlash. Therefore, it is important to pay particular attention to axial plays. From **Table 16-4(2)**, we understand that axial thrust force, F_a, changes from positive to negative in the range of teeth ratio from 1.5 to 2.0 when a gear carries force on the convex side. The precise turning point of axial thrust force, F_a, is at the teeth ratio $z_1 / z_2 = 1.57357$.

Figure 16-10 describes the forces for a pair of spiral bevel gears with shaft angle Σ = 90°, pressure angle α_n = 20°, spiral angle β_m = 35° and the teeth ratio, u, ranging from 1 to 1.57357.

Σ = 90°, α_n = 20°, β_m = 35°, u < 1.57357.

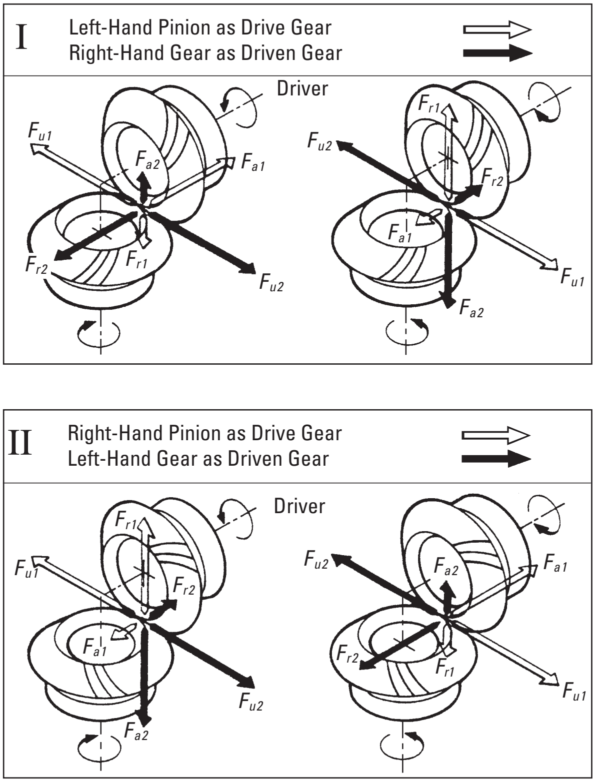

Fig. 16-10 The Direction of Forces Carried by Spiral Bevel Gears (1)

Figure 16-11 expresses the forces of another pair of spiral bevel gears taken with the teeth ratio equal to or larger than 1.57357.

$\Sigma = 90°,\ \alpha_n = 20°,\ \beta_m = 35°,\ u \geq 1.57357$

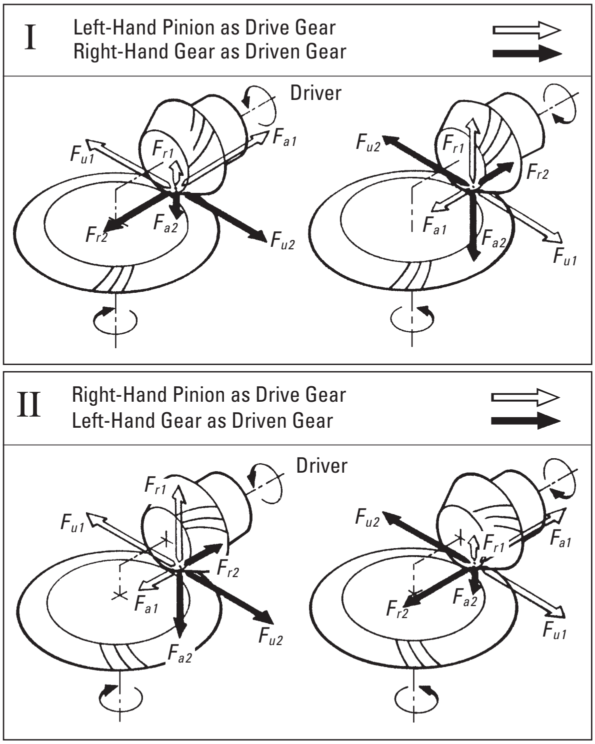

Fig. 16-11 The Direction of Forces Carried by Spiral Bevel Gears (2)

16.5 Forces In A Worm Gear Mesh

16.5.1 Worm as the Driver

For the case of a worm as the driver, **Figure 16-12**, the transmission force, F_n, which is normal to the tooth surface at the pitch circle can be resolved into components F_1 and F_{r1}.

$$\left.\begin{aligned} F_1 &= F_n \cos \alpha_n \\ F_{r1} &= F_n \sin \alpha_n \end{aligned}\right\} \qquad \textbf{(16-17)}$$

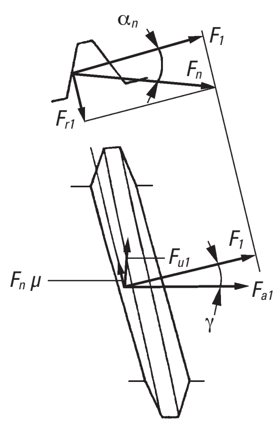

Fig. 16-12 Forces Acting on the Tooth Surface of a Worm

At the pitch surface of the worm, there is, in addition to the tangential component, F_1, a friction sliding force on the tooth surface, μF_n. These two forces can be resolved into the circular and axial directions as:

$$\left.\begin{aligned} F_{u1} &= F_1 \sin\gamma + F_n\,\mu\cos\gamma \\ F_{a1} &= F_1 \cos\gamma - F_n\,\mu\sin\gamma \end{aligned}\right\} \tag{16-18}$$

and by substitution, the result is:

$$\left.\begin{aligned} F_{u1} &= F_n\,(\cos\alpha_n \sin\gamma + \mu\cos\gamma) \\ F_{a1} &= F_n\,(\cos\alpha_n \cos\gamma - \mu\sin\gamma) \\ F_{r1} &= F_n \sin\alpha_n \end{aligned}\right\} \tag{16-19}$$

Figure 16-13 presents the direction of forces in a worm gear mesh with a shaft angle $\Sigma = 90°$. These forces relate as follows:

$$\left.\begin{aligned} F_{a1} &= F_{u2} \\ F_{u1} &= F_{a2} \\ F_{r1} &= F_{r2} \end{aligned}\right\} \tag{16-20}$$

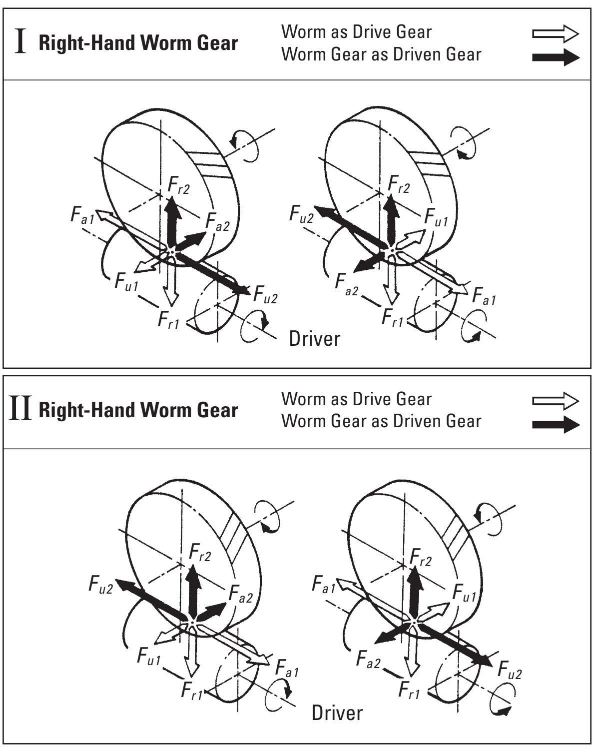

Figure 16-13 Direction of Forces in a Worm Gear Mesh

The coefficient of friction has a great effect on the transmission of a worm gear. **Equation (16-21)** presents the efficiency when the worm is the driver.

$$\eta_R = \frac{T_2}{T_1 i} = \frac{F_{u2}}{F_{u1}} \tan\gamma = \frac{\cos\alpha_n \cos\gamma - \mu \sin\gamma}{\cos\alpha_n \sin\gamma + \mu\cos\gamma} \tan\gamma \qquad \textbf{(16-21)}$$

16.5.2 Worm Gear as the Driver

For the case of a worm gear as the driver, the forces are as in **Figure 16-14** and per **Equations (16-22)**.

$$\left.\begin{aligned} F_{u2} &= F_n(\cos\alpha_n \cos\gamma + \mu\sin\gamma) \\ F_{a2} &= F_n(\cos\alpha_n \sin\gamma - \mu\cos\gamma) \\ F_{r2} &= F_n \sin\alpha_n \end{aligned}\right\} \qquad \textbf{(16-22)}$$

When the worm and worm gear are at 90° shaft angle, **Equations (16-20)** apply. Then, when the worm gear is the driver, the transmission efficiency η_I is expressed as per **Equation (16-23)**.

$$\eta_I = \frac{T_1\, i}{T_2} = \frac{F_{u1}}{F_{u2}\tan\gamma} = \frac{\cos\alpha_n \sin\gamma - \mu\cos\gamma}{\cos\alpha_n \cos\gamma + \mu\sin\gamma}\,\frac{1}{\tan\gamma} \qquad \textbf{(16-23)}$$

The equations concerning worm and worm gear forces contain the coefficient μ. This indicates the coefficient of friction is very important in the transmission of power.

16.6 Forces In A Screw Gear Mesh

The forces in a screw gear mesh are similar to those in a worm gear mesh. For screw gears that have a shaft angle $\Sigma = 90°$, merely replace the worm's lead angle γ, in **Equation (16-22)**, with the screw gear's helix angle β_1.

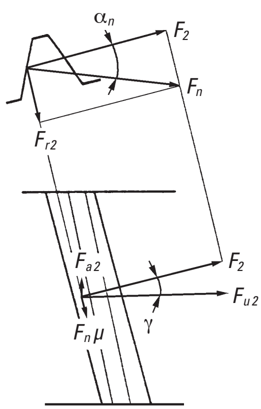

Fig. 16-14 Forces in a Worm Gear Mesh

In the general case when the shaft angle is not 90°, as in **Figure 16-15**, the driver screw gear has the same forces as for a worm mesh. These are expressed in **Equations (16-24)**.

$$\left.\begin{aligned} F_{u1} &= F_n(\cos\alpha_n\cos\beta_1 + \mu\sin\beta_1) \\ F_{a1} &= F_n(\cos\alpha_n\sin\beta_1 - \mu\cos\beta_1) \\ F_{r1} &= F_n\sin\alpha_n \end{aligned}\right\} \qquad (16\text{-}24)$$

Forces acting on the driven gear can be calculated per **Equations (16-25)**.

$$\left.\begin{aligned} F_{u2} &= F_{a1}\sin\Sigma + F_{u1}\cos\Sigma \\ F_{a2} &= F_{u1}\sin\Sigma - F_{a1}\cos\Sigma \\ F_{r2} &= F_{r1} \end{aligned}\right\} \qquad (16\text{-}25)$$

If the Σ term in **Equation (16-25)** is 90°, it becomes identical to **Equation (16-20)**. **Figure 16-16** presents the direction of forces in a screw gear mesh when the shaft angle $\Sigma = 90°$ and $\beta_1 = \beta_2 = 45°$.

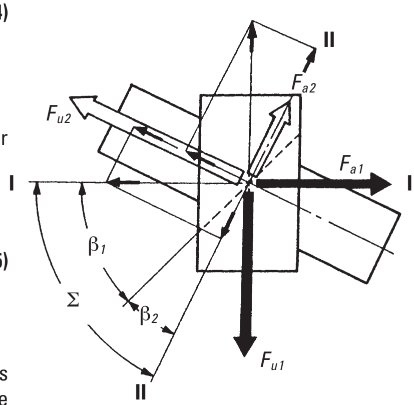

Fig. 16-15 The Forces in a Screw Gear Mesh

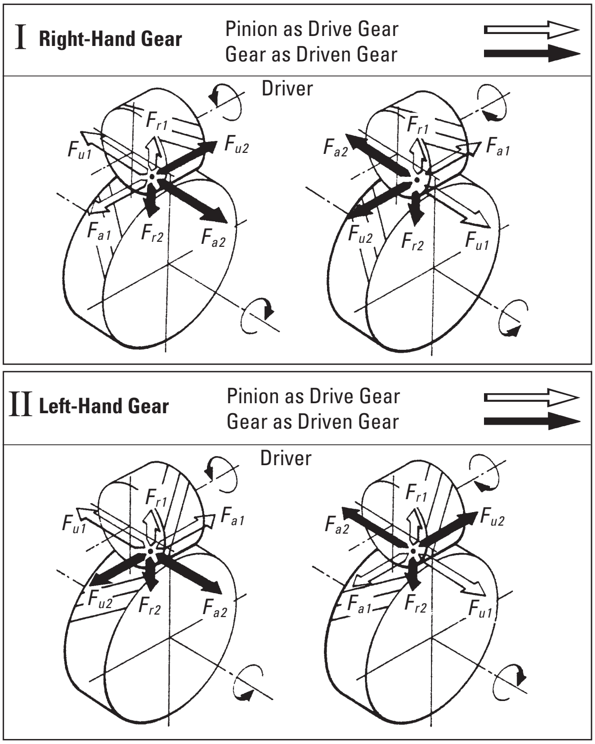

Fig. 16-16 Directions of Forces in a Screw Gear Mesh

SECTION 17 STRENGTH AND DURABILITY OF GEARS

The strength of gears is generally expressed in terms of bending strength and surface durability. These are independent criteria which can have differing criticalness, although usually both are important.

Discussions in this section are based upon equations published in the literature of the Japanese Gear Manufacturer Association (JGMA). Reference is made to the following JGMA specifications:

Specifications of JGMA:

JGMA 401-01	Bending Strength Formula of Spur Gears and Helical Gears
JGMA 402-01	Surface Durability Formula of Spur Gears and Helical Gears
JGMA 403-01	Bending Strength Formula of Bevel Gears
JGMA 404-01	Surface Durability Formula of Bevel Gears
JGMA 405-01	The Strength Formula of Worm Gears

Generally, bending strength and durability specifications are applied to spur and helical gears (including double helical and internal gears) used in industrial machines in the following range:

Module:	m	1.5 to 25 mm
Pitch Diameter:	d	25 to 3200 mm
Tangential Speed:	v	less than 25 m/sec
Rotating Speed:	n	less than 3600 rpm

Conversion Formulas: Power, Torque and Force

Gear strength and durability relate to the power and forces to be transmitted. Thus, the equations that relate tangential force at the pitch circle, F_t (kgf), power, P (kw), and torque, T (kgf • m) are basic to the calculations. The relations are as follows:

$$F_t = \frac{102\,P}{v} = \frac{1.95 \times 10^6\,P}{d_w n} = \frac{2000\,T}{d_w} \qquad \textbf{(17-1)}$$

$$P = \frac{F_t\,v}{102} = \frac{10^{-6}}{1.95} F_t\,d_w n \qquad \textbf{(17-2)}$$

$$T = \frac{F_t\,d_w}{2000} = \frac{974\,P}{n} \qquad \textbf{(17-3)}$$

where: v : Tangential Speed of Working Pitch Circle (m/sec)

$$v = \frac{d_w n}{19100}$$

d_w : Working Pitch Diameter (mm)
n : Rotating Speed (rpm)

17.1 Bending Strength Of Spur And Helical Gears

In order to confirm an acceptable safe bending strength, it is necessary to analyze the applied tangential force at the working pitch circle, F_t, vs. allowable force, $F_{t\ \lim}$. This is stated as:

$$F_t \le F_{t\ \lim} \qquad \textbf{(17-4)}$$

It should be noted that the greatest bending stress is at the root of the flank or base of the dedendum. Thus, it can be stated:

σ_F = actual stress on dedendum at root
$\sigma_{F\ \mathrm{lim}}$ = allowable stress

Then **Equation (17-4)** becomes **Equation (17-5)**

$$\sigma_F \le \sigma_{F\ \mathrm{lim}} \tag{17-5}$$

Equation (17-6) presents the calculation of $F_{t\ \mathrm{lim}}$:

$$F_{t\ \mathrm{lim}} = \sigma_{F\ \mathrm{lim}} \frac{m_n b}{Y_F Y_\varepsilon Y_\beta} \left(\frac{K_L K_{FX}}{K_V K_O} \right) \frac{1}{S_F} \quad \text{(kgf)} \tag{17-6}$$

Equation (17-6) can be converted into stress by **Equation (17-7)**:

$$\sigma_F = F_t \frac{Y_F Y_\varepsilon Y_\beta}{m_n b} \left(\frac{K_V K_O}{K_L K_{FX}} \right) S_F \quad \text{(kgf/mm}^2\text{)} \tag{17-7}$$

17.1.1 Determination of Factors in the Bending Strength Equation

If the gears in a pair have different blank widths, let the wider one be b_w and the narrower one be b_s.

And if:

$b_w - b_s \le m_n$, b_w and b_s can be put directly into **Equation (17-6)**.
$b_w - b_s > m_n$, the wider one would be changed to $b_s + m_n$ and the narrower one, b_s, would be unchanged.

17.1.2 Tooth Profile Factor, Y_F

The factor Y_F is obtainable from **Figure 17-1** based on the equivalent number of teeth, z_v, and coefficient of profile shift, x, if the gear has a standard tooth profile with 20° pressure angle, per JIS B 1701. The theoretical limit of undercut is shown. Also, for profile shifted gears the limit of too narrow (sharp) a tooth top land is given. For internal gears, obtain the factor by considering the equivalent racks.

17.1.3 Load Distribution Factor, Y_ε

Load distribution factor is the reciprocal of radial contact ratio.

$$Y_\varepsilon = \frac{1}{\varepsilon_\alpha} \tag{17-8}$$

Table 17-1 shows the radial contact ratio of a standard spur gear.

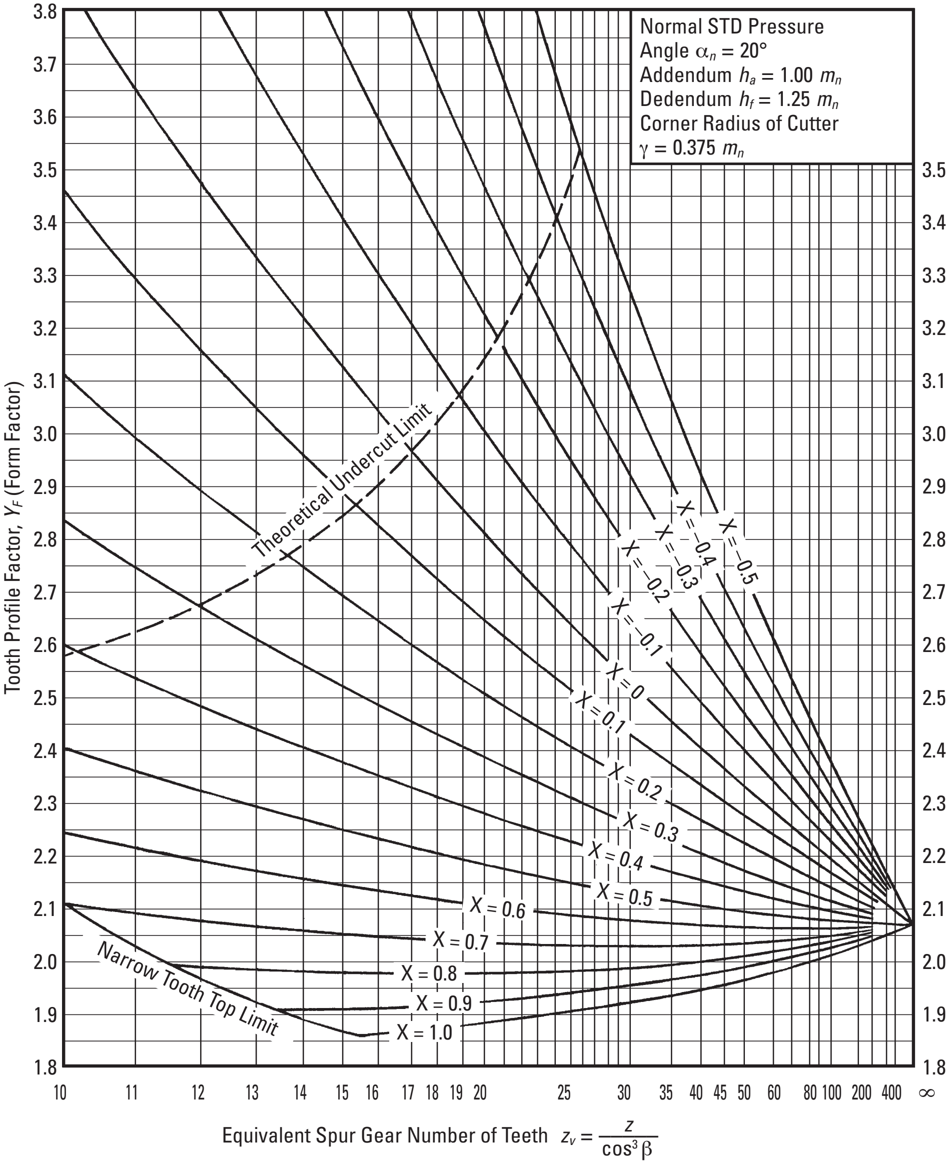

Fig. 17-1 Chart of Tooth Profile Factor, Y_F

Table 17-1 Radial Contact Ratio of Standard Spur Gears, ε_α ($\alpha = 20°$)

	12	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	110	120
12	1.420																				
15	1.451	1.481																			
20	1.489	1.519	1.557																		
25	1.516	1.547	1.584	1.612																	
30	1.537	1.567	1.605	1.633	1.654																
35	1.553	1.584	1.622	1.649	1.670	1.687															
40	1.567	1.597	1.635	1.663	1.684	1.700	1.714														
45	1.578	1.609	1.646	1.674	1.695	1.711	1.725	1.736													
50	1.588	1.618	1.656	1.683	1.704	1.721	1.734	1.745	1.755												
55	1.596	1.626	1.664	1.691	1.712	1.729	1.742	1.753	1.763	1.771											
60	1.603	1.633	1.671	1.698	1.719	1.736	1.749	1.760	1.770	1.778	1.785										
65	1.609	1.639	1.677	1.704	1.725	1.742	1.755	1.766	1.776	1.784	1.791	1.797									
70	1.614	1.645	1.682	1.710	1.731	1.747	1.761	1.772	1.781	1.789	1.796	1.802	1.808								
75	1.619	1.649	1.687	1.714	1.735	1.752	1.765	1.777	1.786	1.794	1.801	1.807	1.812	1.817							
80	1.623	1.654	1.691	1.719	1.740	1.756	1.770	1.781	1.790	1.798	1.805	1.811	1.817	1.821	1.826						
85	1.627	1.657	1.695	1.723	1.743	1.760	1.773	1.785	1.794	1.802	1.809	1.815	1.821	1.825	1.830	1.833					
90	1.630	1.661	1.699	1.726	1.747	1.764	1.777	1.788	1.798	1.806	1.813	1.819	1.824	1.829	1.833	1.837	1.840				
95	1.634	1.664	1.702	1.729	1.750	1.767	1.780	1.791	1.801	1.809	1.816	1.822	1.827	1.832	1.836	1.840	1.844	1.847			
100	1.636	1.667	1.705	1.732	1.753	1.770	1.783	1.794	1.804	1.812	1.819	1.825	1.830	1.835	1.839	1.843	1.846	1.850	1.853		
110	1.642	1.672	1.710	1.737	1.758	1.775	1.788	1.799	1.809	1.817	1.824	1.830	1.835	1.840	1.844	1.848	1.852	1.855	1.858	1.863	
120	1.646	1.676	1.714	1.742	1.762	1.779	1.792	1.804	1.813	1.821	1.828	1.834	1.840	1.844	1.849	1.852	1.856	1.859	1.862	1.867	1.871
RACK	1.701	1.731	1.769	1.797	1.817	1.834	1.847	1.859	1.868	1.876	1.883	1.889	1.894	1.899	1.903	1.907	1.911	1.914	1.917	1.992	1.926

17.1.4 Helix Angle Factor, Y_β

Helix angle factor can be obtained from **Equation (17-9)**.

$$\left.\begin{array}{l} \text{When } 0 \le \beta \le 30°\text{, then } \quad Y_\beta = 1 - \dfrac{\beta}{120} \\[2ex] \text{When } \beta > 30°\text{, then } \quad Y_\beta = 0.75 \end{array}\right\} \qquad \textbf{(17-9)}$$

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17.1.5 Life Factor, K_L

We can choose the proper life factor, K_L, from **Table 17-2**. The number of cyclic repetitions means the total loaded meshings during its lifetime.

Table 17-2 Life Factor, K_L

Number of Cyclic Repetitions	Hardness (1) HB 120 … 220	Hardness (2) Over HB 220	Gears with Carburizing Gears with Nitriding
Under 10000	1.4	1.5	1.5
Approx. 10^5	1.2	1.4	1.5
Approx. 10^6	1.1	1.1	1.1
Above 10^7	1.0	1.0	1.0

NOTES: (1) Cast iron gears apply to this column.
(2) For induction hardened gears, use the core hardness.

17.1.6 Dimension Factor of Root Stress, K_{FX}

Generally, this factor is unity.

$$K_{FX} = 1.00 \tag{17-10}$$

17.1.7 Dynamic Load Factor, K_V

Dynamic load factor can be obtained from **Table 17-3** based on the precision of the gear and its pitch line linear speed.

Table 17-3 Dynamic Load Factor, K_V

Precision Grade of Gears from JIS B 1702		Tangential Speed at Pitch Line (m/s)						
Tooth Profile								
Unmodified	Modified	Under 1	1 to less than 3	3 to less than 5	5 to less than 8	8 to less than 12	12 to less than 18	18 to less than 25
	1	—	—	1.0	1.0	1.1	1.2	1.3
1	2	—	1.0	1.05	1.1	1.2	1.3	1.5
2	3	1.0	1.1	1.15	1.2	1.3	1.5	
3	4	1.0	1.2	1.3	1.4	1.5		
4	—	1.0	1.3	1.4	1.5			
5	—	1.1	1.4	1.5				
6	—	1.2	1.5					

17.1.8 Overload Factor, K_O

Overload factor, K_O, is the quotient of actual tangential force divided by nominal tangential force, F_t. If tangential force is unknown, **Table 17-4** provides guiding values.

$$K_O = \frac{\text{Actual tangential force}}{\text{Nominal tangential force, } F_t} \qquad \textbf{(17-11)}$$

Table 17-4 Overload Factor, K_O

Impact from Prime Mover	Impact from Load Side of Machine		
	Uniform Load	**Medium Impact Load**	**Heavy Impact Load**
Uniform Load (Motor, Turbine, Hydraulic Motor)	1.0	1.25	1.75
Light Impact Load (Multicylinder Engine)	1.25	1.5	2.0
Medium Impact Load (Single Cylinder Engine)	1.5	1.75	2.25

17.1.9 Safety Factor for Bending Failure, S_F

Safety factor, S_F, is too complicated to be decided precisely. Usually, it is set to at least 1.2.

17.1.10 Allowable Bending Stress At Root, $\sigma_{F\,lim}$

For the unidirectionally loaded gear, the allowable bending stresses at the root are shown in **Tables 17-5** to **17-8**. In these tables, the value of $\sigma_{F\,lim}$ is the quotient of the tensile fatigue limit divided by the stress concentration factor 1.4. If the load is bidirectional, and both sides of the tooth are equally loaded, the value of allowable bending stress should be taken as 2/3 of the given value in the table. The core hardness means hardness at the center region of the root.

See **Table 17-5** for $\sigma_{F\,lim}$ of gears without case hardening. **Table 17-6** gives $\sigma_{F\,lim}$ of gears that are induction hardened; and **Tables 17-7** and **17-8** give the values for carburized and nitrided gears, respectively. In **Tables 17-8A** and **17-8B**, examples of calculations are given.

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Table 17-5 Gears Without Case Hardening

Material	Arrows indicate the ranges	Hardness HB	Hardness HV	Tensile Strength Lower limit kgf/mm² (Reference)	$\sigma_{F\ lim}$ kgf/mm²
Cast Steel Gear	SC37			37	10.4
	SC42			42	12.0
	SC46			46	13.2
	SC49			49	14.2
	SCC3			55	15.8
				60	17.2
Normalized Carbon Steel Gear		120	126	39	13.8
		130	136	42	14.8
		140	147	45	15.8
	S25C	150	157	48	16.8
		160	167	51	17.6
		170	178	55	18.4
	S35C	180	189	58	19.0
		190	200	61	19.5
	S43C, S48C, S53C	200	210	64	20
		210	221	68	20.5
	S58C	220	231	71	21
		230	242	74	21.5
		240	252	77	22
		250	263	81	22.5
Quenched and Tempered Carbon Steel Gear		160	167	51	18.2
		170	178	55	19.4
		180	189	58	20.2
		190	200	61	21
	S35C	200	210	64	22
		210	221	68	23
		220	231	71	23.5
		230	242	74	24
	S43C, S48C	240	252	77	24.5
		250	263	81	25
	S53C	260	273	84	25.5
		270	284	87	26
	S58C	280	295	90	26
		290	305	93	26.5
Quenched and Tempered Alloy Steel Gear		220	231	71	25
		230	242	74	26
		240	252	77	27.5
		250	263	81	28.5
	SMn443	260	273	84	29.5
		270	284	87	31
	SNC836	280	295	90	32
		290	305	93	33
	SCM435, SCM440	300	316	97	34
		310	327	100	35
	SNCM439	320	337	103	36.5
		330	347	106	37.5
		340	358	110	39
		350	369	113	40
		360	380	117	41

Table 17-6 Induction Hardened Gears

Material	Arrows indicate the ranges	Heat Treatment Before Induction Hardening	Core Hardness		Surface Hardness HV	$\sigma_{F\ lim}$ kgf/mm²
			HB	HV		
Structural Carbon Steel Hardened Throughout	S48C, S43C	Normalized	160	167	More than 550	21
			180	189	"	21
			220	231	"	21.5
			240	252	"	22
	S48C, S43C	Quenched and Tempered	200	210	More than 550	23
			210	221	"	23.5
			220	231	"	24
			230	242	"	24.5
			240	252	"	25
			250	263	"	25
Structural Alloy Steel Hardened Throughout	SMn443, SCM440, SNC836, SCM435, SNCM439	Quenched and Tempered	230	242	More than 550	27
			240	252	"	28
			250	263	"	29
			260	273	"	30
			270	284	"	31
			280	295	"	32
			290	305	"	33
			300	316	"	34
			310	327	"	35
			320	337	"	36.5
Hardened Except Root Area						75% of the above

NOTES: 1. If a gear is not quenched completely, or not evenly, or has quenching cracks, the $\sigma_{F\ lim}$ will drop dramatically.

2. If the hardness after quenching is relatively low, the value of $\sigma_{F\ lim}$ should be that given in **Table 17-5.**

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Metric
0 10

Table 17-7 Carburized Gears

Material	Arrows indicate the ranges	Core Hardness HB	Core Hardness HV	$\sigma_{F\ lim}$ kgf/mm²
Structural Carbon Steel	S15C S15CK	140	147	18.2
		150	157	19.6
		160	167	21
		170	178	22
		180	189	23
		190	200	24
Structural Alloy Steel		220	231	34
		230	242	36
		240	252	38
		250	263	39
		260	273	41
		270	284	42.5
	SNC415	280	295	44
	SCM415	290	305	45
		300	316	46
	SCM420	310	327	47
		320	337	48
	SNC815	330	347	49
	SNCM420	340	358	50
		350	369	51
		360	380	51.5
		370	390	52

Table 17-8 Nitrided Gears

Material	Surface Hardness (Reference)	Core Hardness HB	Core Hardness HV	$\sigma_{F\ lim}$ kgf/mm²
Alloy Steel except Nitriding Steel	More than HV 650	220	231	30
		240	252	33
		260	273	36
		280	295	38
		300	316	40
		320	337	42
		340	358	44
		360	380	46
Nitriding Steel SACM645	More than HV 650	220	231	32
		240	252	35
		260	273	38
		280	295	41
		300	316	44

NOTE: The above two tables apply only to those gears which have adequate depth of surface hardness. Otherwise, the gears should be rated according to **Table 17-5**.

17.1.11 Example of Bending Strength Calculation

Table 17-8A Spur Gear Design Details

No.	Item	Symbol	Unit	Pinion	Gear
1	Normal Module	m_n	mm	2	
2	Normal Pressure Angle	α_n	degree	20°	
3	Helix Angle	β		0°	
4	Number of Teeth	z		20	40
5	Center Distance	a_x	mm	60	
6	Coefficient of Profile Shift	x		+0.15	–0.15
7	Pitch Circle Diameter	d	mm	40.000	80.000
8	Working Pitch Circle Diameter	d_w		40.000	80.000
9	Tooth Width	b		20	20
10	Precision Grade			JIS 5	JIS 5
11	Manufacturing Method			Hobbing	
12	Surface Roughness			12.5 μm	
13	Revolutions per Minute	n	rpm	1500	750
14	Linear Speed	v	m/s	3.142	
15	Direction of Load			Unidirectional	
16	Duty Cycle		cycles	Over 10^7 cycles	
17	Material			SCM 415	
18	Heat Treatment			Carburizing	
19	Surface Hardness			HV 600 … 640	
20	Core Hardness			HB 260 … 280	
21	Effective Carburized Depth		mm	0.3 … 0.5	

Table 17-8B Bending Strength Factors

No.	Item	Symbol	Unit	Pinion	Gear
1	Allowable Bending Stress at Root	$\sigma_{F\ lim}$	kgf/mm²	42.5	
2	Normal Module	m_n	mm	2	
3	Tooth Width	b		20	
4	Tooth Profile Factor	Y_F		2.568	2.535
5	Load Distribution Factor	Y_ε		0.619	
6	Helix Angle Factor	Y_β		1.0	
7	Life Factor	K_L		1.0	
8	Dimension Factor of Root Stress	K_{FX}		1.0	
9	Dynamic Load Factor	K_V		1.4	
10	Overload Factor	K_O		1.0	
11	Safety Factor	S_F		1.2	
12	Allowable Tangential Force on Working Pitch Circle	$F_{t\ lim}$	kgf	636.5	644.8

17.2 Surface Strength Of Spur And Helical Gears

The following equations can be applied to both spur and helical gears, including double helical and internal gears, used in power transmission. The general range of application is:

Module:	m	1.5 to 25 mm
Pitch Circle:	d	25 to 3200 mm
Linear Speed:	v	less than 25 m/sec
Rotating Speed:	n	less than 3600 rpm

17.2.1 Conversion Formulas

To rate gears, the required transmitted power and torques must be converted to tooth forces. The same conversion formulas, **Equations (17-1)**, **(17-2)** and **(17-3)**, of **SECTION 17** (page T-150) are applicable to surface strength calculations.

17.2.2 Surface Strength Equations

As stated in **SECTION 17.1**, the tangential force, F_t, is not to exceed the allowable tangential force, $F_{t\ \lim}$. The same is true for the allowable Hertz surface stress, $\sigma_{H\ \lim}$. The Hertz stress σ_H is calculated from the tangential force, F_t. For an acceptable design, it must be less than the allowable Hertz stress $\sigma_{H\ \lim}$. That is:

$$\sigma_H \le \sigma_{H\ \lim} \tag{17-12}$$

The tangential force, $F_{t\ \lim}$, in kgf, at the standard pitch circle, can be calculated from **Equation (17-13)**.

$$F_{t\ \lim} = \sigma_{H\ \lim}{}^2\, d_1 b_H \frac{u}{u \pm 1}\left(\frac{K_{HL} Z_L Z_R Z_V Z_W K_{HX}}{Z_H Z_M Z_\varepsilon Z_\beta}\right)^2 \frac{1}{K_{H\beta} K_V K_O}\ \frac{1}{S_H{}^2} \tag{17-13}$$

The Hertz stress σ_H (kgf/mm^2) is calculated from **Equation (17-14)**, where u is the ratio of numbers of teeth in the gear pair.

$$\sigma_H = \sqrt{\frac{F_t}{d_1 b_H}\ \frac{u \pm 1}{u}}\ \frac{Z_H Z_M Z_\varepsilon Z_\beta}{K_{HL} Z_L Z_R Z_V Z_W K_{HX}}\ \sqrt{K_{H\beta} K_V K_O}\ \ S_H \tag{17-14}$$

The "+" symbol in **Equations (17-13)** and **(17-14)** applies to two external gears in mesh, whereas the "–" symbol is used for an internal gear and an external gear mesh. For the case of a rack and gear, the quantity $u/(u \pm 1)$ becomes 1.

17.2.3 Determination Of Factors In The Surface Strength Equations

17.2.3.A Effective Tooth Width, b_H (mm)

The narrower face width of the meshed gear pair is assumed to be the effective width for surface strength. However, if there are tooth modifications, such as chamfer, tip relief or crowning, an appropriate amount should be subtracted to obtain the effective tooth width.

17.2.3.B Zone Factor, Z_H

The zone factor is defined as:

$$Z_H = \sqrt{\frac{2\cos\beta_b \cos\alpha_{wt}}{\cos^2\alpha_t \sin\alpha_{wt}}} = \frac{1}{\cos\alpha_t}\sqrt{\frac{2\cos\beta_b}{\tan\alpha_{wt}}} \tag{17-15}$$

where:

$\beta_b = \tan^{-1}(\tan\beta \cos\alpha_t)$

The zone factors are presented in **Figure 17-2** for tooth profiles per JIS B 1701, specified in terms of profile shift coefficients x_1 and x_2, numbers of teeth z_1 and z_2 and helix angle β.

The "+" symbol in **Figure 17-2** applies to external gear meshes, whereas the "–" is used for internal gear and external gear meshes.

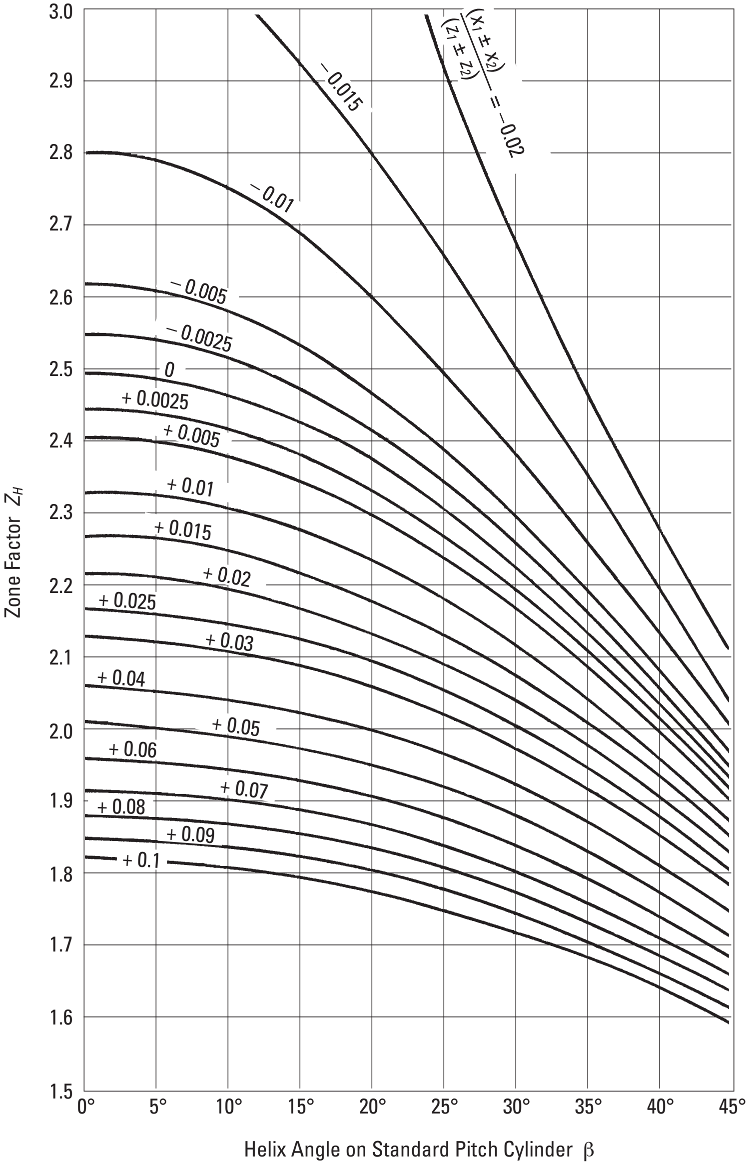


Fig. 17-2 Zone Factor Z_H

17.2.3.C Material Factor, Z_M

$$Z_M = \sqrt{\frac{1}{\pi\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)}} \qquad \text{(17-16)}$$

where:
ν = Poisson's Ratio, and E = Young's Modulus

Table 17-9 contains several combinations of material and their material factor.

Table 17-9 Material Factor, Z_M

Gear				Meshing Gear				Material Factor Z_M (kgf/mm²)$^{0.5}$
Material	Symbol	E Young's Modulus kgf/mm²	Poisson's Ratio	Material	Symbol	E Young's Modulus kgf/mm²	Poisson's Ratio	
Structural Steel	*	21000	0.3	Structural Steel	*	21000	0.3	60.6
				Cast Steel	SC	20500		60.2
				Ductile Cast Iron	FCD	17600		57.9
				Gray Cast Iron	FC	12000		51.7
Cast Steel	SC	20500		Cast Steel	SC	20500		59.9
				Ductile Cast Iron	FCD	17600		57.6
				Gray Cast Iron	FC	12000		51.5
Ductile Cast Iron	FCD	17600		Ductile Cast Iron	FCD	17600		55.5
				Gray Cast Iron	FC	12000		50.0
Gray Cast Iron	FC	12000		Gray Cast Iron	FC	12000		45.8

***NOTE:** Structural steels are S…C, SNC, SNCM, SCr, SCM, etc.

17.2.4 Contact Ratio Factor, Z_ε

This factor is fixed at 1.0 for spur gears.
For helical gear meshes, Z_ε is calculated as follows:

Helical gear:
When $\varepsilon_\beta \le 1$,

$$Z_\varepsilon = \sqrt{1 - \varepsilon_\beta + \frac{\varepsilon_\beta}{\varepsilon_\alpha}}$$

When $\varepsilon_\beta > 1$,

$$Z_\varepsilon = \sqrt{\frac{1}{\varepsilon_\alpha}} \qquad \text{(17-17)}$$

where: ε_α = Radial contact ratio
ε_β = Overlap ratio

17.2.5 Helix Angle Factor, Z_β

This is a difficult parameter to evaluate. Therefore, it is assumed to be 1.0 unless better information is available.

$$Z_\beta = 1.0 \qquad \textbf{(17-18)}$$

17.2.6 Life Factor, K_{HL}

This factor reflects the number of repetitious stress cycles. Generally, it is taken as 1.0. Also, when the number of cycles is unknown, it is assumed to be 1.0.

When the number of stress cycles is below 10 million, the values of **Table 17-10** can be applied.

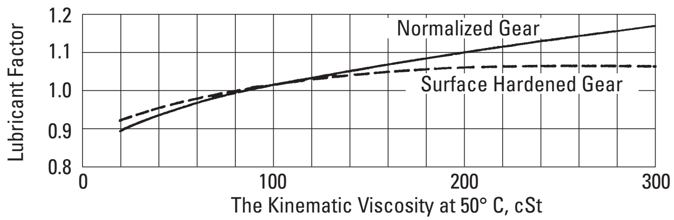
Table 17-10 Life Factor, K_{HL}

Duty Cycles	Life Factor
less than 10^5	1.5
approx. 10^5	1.3
approx. 10^6	1.15
above 10^7	1.0

NOTES:
1. The duty cycle is the meshing cycles during a lifetime.
2. Although an idler has two meshing points in one cycle, it is still regarded as one repetition.
3. For bidirectional gear drives, the larger loaded direction is taken as the number of cyclic loads.

17.2.7 Lubricant Factor, Z_L

The lubricant factor is based upon the lubricant's kinematic viscosity at 50°C. See **Figure 17-3**.



NOTE: Normalized gears include quenched and tempered gears

Fig. 17-3 Lubricant Factor, Z_L

17.2.8 Surface Roughness Factor, Z_R

This factor is obtained from **Figure 17-4** on the basis of the average roughness R_{maxm} (μm). The average roughness is calculated by **Equation (17-19)** using the surface roughness values of the pinion and gear, R_{max1} and R_{max2}, and the center distance, a, in mm.

$$R_{maxm} = \frac{R_{max1} + R_{max2}}{2} \sqrt[3]{\frac{100}{a}} \quad (\mu\text{m}) \qquad \textbf{(17-19)}$$

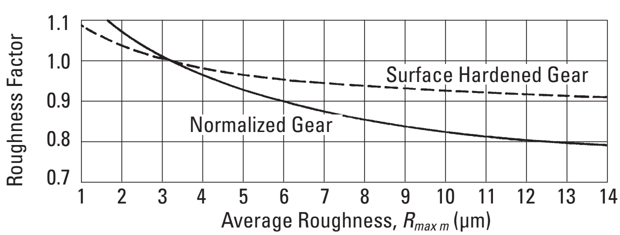
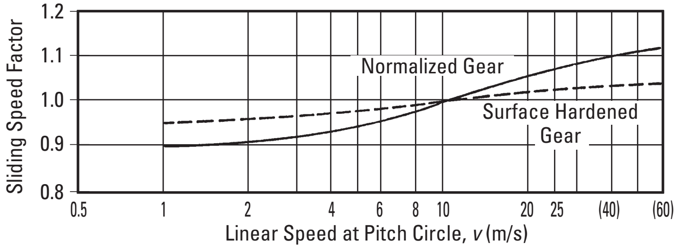


Fig. 17-4 Surface Roughness Factor, Z_R

17.2.9 Sliding Speed Factor, Z_V

This factor relates to the linear speed of the pitch line. See **Figure 17-5**.



NOTE: Normalized gears include quenched and tempered gears.

Fig. 17-5 Sliding Speed Factor, Z_V

17.2.10 Hardness Ratio Factor, Z_W

The hardness ratio factor applies only to the gear that is in mesh with a pinion which is quenched and ground. The ratio is calculated by **Equation (17-20)**.

$$Z_W = 1.2 - \frac{HB_2 - 130}{1700} \qquad \textbf{(17-20)}$$

where: HB_2 = Brinell hardness of gear range: $130 \le HB_2 \le 470$
If a gear is out of this range, the Z_W is assumed to be 1.0.

17.2.11 Dimension Factor, K_{HX}

Because the conditions affecting this parameter are often unknown, the factor is usually set at 1.0.

$$K_{HX} = 1.0 \qquad \textbf{(17-21)}$$

17.2.12 Tooth Flank Load Distribution Factor, $K_{H\beta}$

(a) When tooth contact under load is not predictable: This case relates the ratio of the gear face width to the pitch diameter, the shaft bearing mounting positions, and the shaft sturdiness. See **Table 17-11**. This attempts to take into account the case where the tooth contact under load is not good or known.

Table 17-11 Tooth Flank Load Distribution Factor for Surface Strength, $K_{H\beta}$

$\frac{b}{d_1}$	Method of Gear Shaft Support			
	Bearings on Both Ends			Bearing on One End
	Gear Equidistant from Bearings	Gear Close to One End (Rugged Shaft)	Gear Close to One End (Weak Shaft)	
0.2	1.0	1.0	1.1	1.2
0.4	1.0	1.1	1.3	1.45
0.6	1.05	1.2	1.5	1.65
0.8	1.1	1.3	1.7	1.85
1.0	1.2	1.45	1.85	2.0
1.2	1.3	1.6	2.0	2.15
1.4	1.4	1.8	2.1	——
1.6	1.5	2.05	2.2	——
1.8	1.8	——	——	——
2.0	2.1	——	——	——

NOTES:
1. The b means effective face width of spur & helical gears. For double helical gears, b is face width including central groove.
2. Tooth contact must be good under no load.
3. The values in this table are not applicable to gears with two or more mesh points, such as an idler.

(b) When tooth contact under load is good: In this case, the shafts are rugged and the bearings are in good close proximity to the gears, resulting in good contact over the full width and working depth of the tooth flanks. Then the factor is in a narrow range, as specified below:

$$K_{H\beta} = 1.0 \ldots 1.2 \qquad \textbf{(17-22)}$$

17.2.13 Dynamic Load Factor, K_V

Dynamic load factor is obtainable from **Table 17-3** according to the gear's precision grade and pitch line linear speed.

17.2.14 Overload Factor, K_o

The overload factor is obtained from either **Equation (17-11)** or from **Table 17-4**.

17.2.15 Safety Factor For Pitting, S_H

The causes of pitting involves many environmental factors and usually is difficult to precisely define. Therefore, it is advised that a factor of at least 1.15 be used.

17.2.16 Allowable Hertz Stress, $\sigma_{H\,lim}$

The values of allowable Hertz stress for various gear materials are listed in **Tables 17-12** through **17-16**. Values for hardness not listed can be estimated by interpolation. Surface hardness is defined as hardness in the pitch circle region.

Table 17-12 Gears without Case Hardening – Allowable Hertz Stress

Material	Arrows indicate the ranges	Surface Hardness HB	Surface Hardness HV	Lower Limit of Tensile Strength kgf/mm² (Reference)	$\sigma_{H\,lim}$ kgf/mm²
Cast Steel	SC37			37	34
	SC42			42	35
	SC46			46	36
	SC49			49	37
	SCC3			55	39
				60	40
Normalized Structural Steel		120	126	39	41.5
		130	136	42	42.5
		140	147	45	44
	S25C	150	157	48	45
		160	167	51	46.5
	S35C	170	178	55	47.5
		180	189	58	49
		190	200	61	50
	S43C, S48C, S53C	200	210	64	51.5
		210	221	68	52.5
	S58C	220	231	71	54
		230	242	74	55
		240	253	77	56.5
		250	263	81	57.5
Quenched and Tempered Structural Steel		160	167	51	51
		170	178	55	52.5
		180	189	58	54
		190	200	61	55.5
		200	210	64	57
	S35C	210	221	68	58.5
		220	231	71	60
		230	242	74	61
		240	252	77	62.5
	S43C, S48C	250	263	81	64
		260	273	84	65.5
	S53C	270	284	87	67
		280	295	90	68.5
	S58C	290	305	93	70
		300	316	97	71
		310	327	100	72.5
		320	337	103	74
		330	347	106	75.5
		340	358	110	77
		350	369	113	78.5

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Table 17-12 Gears without Case Hardening – Allowable Hertz Stress (continued)

Material	Arrows indicate the ranges				Surface Hardness HB	Surface Hardness HV	Lower Limit of Tensile Strength kgf/mm² (Reference)	$\sigma_{H\ lim}$ kgf/mm²
Quenched and Tempered Alloy Steel					220	231	71	70
	↑				230	242	74	71.5
					240	252	77	73
					250	263	81	74.5
					260	273	84	76
		↑			270	284	87	77.5
	SMn443		↑		280	295	90	79
				↑	290	305	93	81
		SNC836			300	316	97	82.5
					310	327	100	84
		SCM435			320	337	103	85.5
					330	347	106	87
					340	358	110	88.5
	↓		SCM440	SNCM439	350	369	113	90
		↓			360	380	117	92
					370	391	121	93.5
			↓		380	402	126	95
					390	413	130	96.5
				↓	400	424	135	98

Continued from the previous page

Table 17-13 Gears with Induction Hardening – Allowable Hertz Stress

Material		Heat Treatment before Induction Hardening	Surface Hardness HV (Quenched)	$\sigma_{H\ lim}$ kgf/mm²
Structural Carbon Steel	S43C S48C	Normalized	420	77
			440	80
			460	82
			480	85
			500	87
			520	90
			540	92
			560	93.5
			580	95
			600 and above	96
		Quenched and Tempered	500	96
			520	99
			540	101
			560	103
			580	105
			600	106.5
			620	107.5
			640	108.5
			660	109
			680 and above	109.5
Structural Alloy Steel	SMn443 SCM435 SCM440 SNC836 SNCM439	Quenched and Tempered	500	109
			520	112
			540	115
			560	117
			580	119
			600	121
			620	123
			640	124
			660	125
			680 and above	126

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Table 17-14 Carburized Gears – Allowable Hertz Stress

Material		Effective Carburized Depth	Surface Hardness HV (Quenched)	$\sigma_{H\,\lim}$ kgf/mm^2
Structural Carbon Steel	S15C S15CK	Relatively Shallow (See **Table 17-14A**, row A)	580	115
			600	117
			620	118
			640	119
			660	120
			680	120
			700	120
			720	119
			740	118
			760	117
			780	115
			800	113
Structural Alloy Steel	SCM415 SCM420 SNC420 SNC815 SNCM420	Relatively Shallow (See **Table 17-14A**, row A)	580	131
			600	134
			620	137
			640	138
			660	138
			680	138
			700	138
			720	137
			740	136
			760	134
			780	132
			800	130
		Relatively Thick (See **Table 17-14A**, row B)	580	156
			600	160
			620	164
			640	166
			660	166
			680	166
			700	164
			720	161
			740	158
			760	154
			780	150
			800	146

NOTES: 1. Gears with thin effective carburized depth have "A" row values in the **Table 17-14A**. For thicker depths, use "B" values. The effective carburized depth is defined as the depth which has the hardness greater than HV 513 or HRC50.

2. The effective carburizing depth of ground gears is defined as the residual layer depth after grinding to final dimensions.

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Table 17-14A

Module		1.5	2	3	4	5	6	8	10	15	20	25
Depth, mm	A	0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.9	1.2	1.5	1.8
	B	0.3	0.3	0.5	0.7	0.8	0.9	1.1	1.4	2.0	2.5	3.4

NOTE: For two gears with large numbers of teeth in mesh, the maximum shear stress point occurs in the inner part of the tooth beyond the carburized depth. In such a case, a larger safety factor, S_H, should be used.

Table 17-15 Gears with Nitriding – Allowable Hertz Stress

Material		Surface Hardness (Reference)	$\sigma_{H\ lim}$ kgf/mm^2	
Nitriding Steel	SACM 645 etc.	Over HV 650	Standard Processing Time	120
			Extra Long Processing Time	130 … 140

NOTE: In order to ensure the proper strength, this table applies only to those gears which have adequate depth of nitriding. Gears with insufficient nitriding or where the maximum shear stress point occurs much deeper than the nitriding depth should have a larger safety factor, S_H.

Table 17-16 Gears with Soft Nitriding(1) – Allowable Hertz Stress

Material	Nitriding Time Hours	$\sigma_{H\ lim}$ kgf/mm^2		
		Relative Radius of Curvature mm(2)		
		less than 10	10 to 20	more than 20
Structural Steel or Alloy Steel	2	100	90	80
	4	110	100	90
	6	120	110	100

NOTES: (1) Applicable to salt bath soft nitriding and gas soft nitriding gears.
(2) Relative radius of curvature is obtained from **Figure 17-6**.

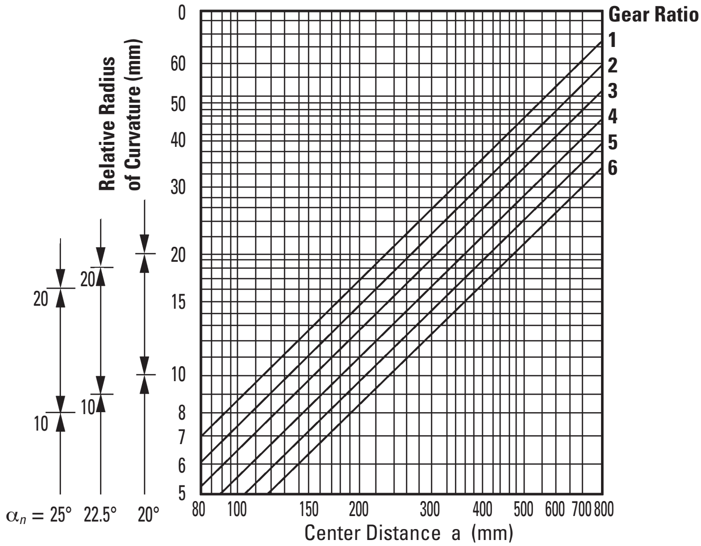

Fig. 17-6 Relative Radius of Curvature

17.2.17 Example Of Surface Strength Calculation

Table 17-16A Spur Gear Design Details

No.	Item	Symbol	Unit	Pinion	Gear
1	Normal Module	m_n	mm	2	
2	Normal Pressure Angle	α_n	degree	20°	
3	Helix Angle	β		0°	
4	Number of Teeth	z		20	40
5	Center Distance	a_x	mm	60	
6	Coefficient of Profile Shift	x		+0.15	– 0.15
7	Pitch Circle Diameter	d	mm	40.000	80.000
8	Working Pitch Circle Diameter	d_w		40.000	80.000
9	Tooth Width	b		20	20
10	Precision Grade			JIS 5	JIS 5
11	Manufacturing Method			Hobbing	
12	Surface Roughness			12.5 μm	
13	Revolutions per Minute	n	rpm	1500	750
14	Linear Speed	v	m/s	3.142	
15	Direction of Load			Unidirectional	
16	Duty Cycle		cycle	Over 107 Cycles	
17	Material			SCM 415	
18	Heat Treatment			Carburizing	
19	Surface Hardness			HV 600 … 640	
20	Core Hardness			HB 260 … 280	
21	Effective Carburized Depth		mm	0.3 … 0.5	

Table 17-16B Surface Strength Factors Calculation

No.	Item	Symbol	Unit	Pinion	Gear
1	Allowable Hertz Stress	$\delta_{H\ \mathrm{lim}}$	kgf/mm^2	164	
2	Pitch Diameter of Pinion	d_1	mm	40	
3	Effective Tooth Width	b_H		20	
4	Teeth Ratio (z_2/z_1)	u		2	
5	Zone Factor	Z_H		2.495	
6	Material Factor	Z_M	(kgf/mm^2)$^{0.5}$	60.6	
7	Contact Ratio Factor	Z_ε		1.0	
8	Helix Angle Factor	Z_β		1.0	
9	Life Factor	K_{HL}		1.0	
10	Lubricant Factor	Z_L		1.0	
11	Surface Roughness Factor	Z_R		0.90	
12	Sliding Speed Factor	Z_V		0.97	
13	Hardness Ratio Factor	Z_W		1.0	
14	Dimension Factor of Root Stress	K_{HX}		1.0	
15	Load Distribution Factor	$K_{H\beta}$		1.025	
16	Dynamic Load Factor	K_V		1.4	
17	Overload Factor	K_O		1.0	
18	Safety Factor for Pitting	S_H		1.15	
19	Allowable Tangential Force on Standard Pitch Circle	$F_{t\ \mathrm{lim}}$	kgf	251.9	

17.3 Bending Strength Of Bevel Gears

This information is valid for bevel gears which are used in power transmission in general industrial machines. The applicable ranges are:

Module:	m	1.5 to 25 mm
Pitch Diameter:	d	less than 1600 mm for straight bevel gears less than 1000 mm for spiral bevel gears
Linear Speed:	v	less than 25 m/sec
Rotating Speed:	n	less than 3600 rpm

17.3.1 Conversion Formulas

In calculating strength, tangential force at the pitch circle, F_{tm}, in kgf; power, P, in kW, and torque, T, in kgf • m, are the design criteria. Their basic relationships are expressed in **Equations (17-23)** through **(17-25)**.

$$F_{tm} = \frac{102\ P}{v_m} = \frac{1.95 \times 10^6\ P}{d_m n} = \frac{2000\ T}{d_m} \qquad \textbf{(17-23)}$$

$$P = \frac{F_{tm} v_m}{102} = 5.13 \times 10^{-7}\ F_{tm} d_m n \qquad \textbf{(17-24)}$$

$$T = \frac{F_{tm} d_m}{2000} = \frac{974\ P}{n} \qquad \textbf{(17-25)}$$

17.3.2 Bending Strength Equations

The tangential force, F_{tm}, acting at the central pitch circle should be equal to or less than the allowable tangential force, $F_{tm\ \lim}$, which is based upon the allowable bending stress $\sigma_{F\lim}$. That is:

$$F_{tm} \le F_{tm\ \lim} \tag{17-26}$$

The bending stress at the root, σ_F, which is derived from F_{tm} should be equal to or less than the allowable bending stress $\sigma_{F\ \lim}$.

$$\sigma_F \le \sigma_{F\ \lim} \tag{17-27}$$

The tangential force at the central pitch circle, $F_{tm\ \lim}$ (kgf), is obtained from **Equation (17-28)**.

$$F_{tm\ \lim} = 0.85 \cos \beta_m \sigma_{F\ \lim} \text{mb} \frac{R_a - 0.5\ b}{R_a} \frac{1}{Y_F Y_\varepsilon Y_\beta Y_C} \left(\frac{K_L K_{FX}}{K_M K_V K_O}\right) \frac{1}{K_R} \tag{17-28}$$

where:
- β_m : Central spiral angle (degrees)
- m : Radial module (mm)
- R_a : Cone distance (mm)

And the bending strength σ_F (kgf/mm^2) at the root of tooth is calculated from **Equation (17-29)**.

$$\sigma_F = F_{tm} \frac{Y_F Y_\varepsilon Y_\beta Y_C}{0.85 \cos \beta_m\ mb} \frac{R_a}{R_a - 0.5\ b} \left(\frac{K_M K_V K_O}{K_L K_{FX}}\right) K_R \tag{17-29}$$

17.3.3 Determination of Factors in Bending Strength Equations

17.3.3.A Tooth Width, *b* (mm)

The term *b* is defined as the tooth width on the pitch cone, analogous to face width of spur or helical gears. For the meshed pair, the narrower one is used for strength calculations.

17.3.3.B Tooth Profile Factor, Y_F

The tooth profile factor is a function of profile shift, in both the radial and axial directions. Using the equivalent (virtual) spur gear tooth number, the first step is to determine the radial tooth profile factor, Y_{FO}, from **Figure 17-8** for straight bevel gears and **Figure 17-9** for spiral bevel gears. Next, determine the axial shift factor, K, with **Equation (17-33)** from which the axial shift correction factor, C, can be obtained using **Figure 17-7**. Finally, calculate Y_F by **Equation (17-30)**.

$$Y_F = CY_{FO} \tag{17-30}$$

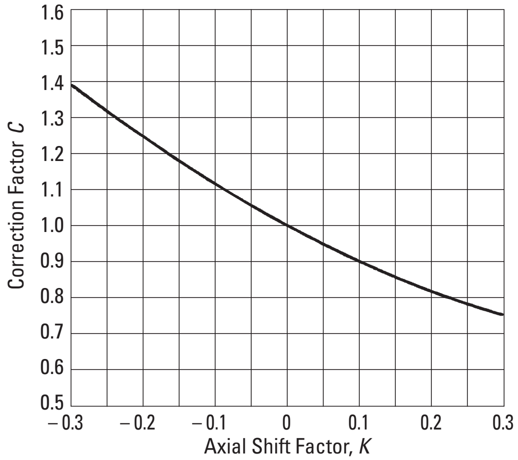

Fig. 17-7 Correction Factor for Axial Shift, *C*

Should the bevel gear pair not have any axial shift, then the coefficient C is 1, as per **Figure 17-7**. The tooth profile factor, Y_F, per **Equation (17-31)** is simply the Y_{F0}. This value is from **Figure 17-8** or **17-9**, depending upon whether it is a straight or spiral bevel gear pair. The graph entry parameter values are per **Equation (17-32)**.

$$Y_F = Y_{F0} \qquad \textbf{(17-31)}$$

$$\left.\begin{aligned} z_v &= \frac{z}{\cos\delta\cos^3\beta_m} \\ x &= \frac{h_a - h_{a0}}{m} \end{aligned}\right\} \qquad \textbf{(17-32)}$$

where:
- h_a = Addendum at outer end (mm)
- h_{a0} = Addendum of standard form (mm)
- m = Radial module (mm)

The axial shift factor, K, is computed from the formula:

$$K = \frac{1}{m}\left\{s - 0.5\,\pi m - \frac{2\,(h_a - h_{a0})\tan\alpha_n}{\cos\beta_m}\right\} \qquad \textbf{(17-33)}$$

17.3.3.C Load Distribution Factor, Y_ε

Load distribution factor is the reciprocal of radial contact ratio.

$$Y_\varepsilon = \frac{1}{\varepsilon_\alpha} \qquad \textbf{(17-34)}$$

The radial contact ratio for a straight bevel gear mesh is:

$$\varepsilon_\alpha = \frac{\sqrt{(R_{va1}{}^2 - R_{vb1}{}^2)} + \sqrt{(R_{va2}{}^2 - R_{vb2}{}^2)} - (R_{v1} + R_{v2})\sin\alpha}{\pi m\cos\alpha}$$

And the radial contact ratio for spiral bevel gear is: **(17-35)**

$$\varepsilon_\alpha = \frac{\sqrt{(R_{va1}{}^2 - R_{vb1}{}^2)} + \sqrt{(R_{va2}{}^2 - R_{vb2}{}^2)} - (R_{v1} + R_{v2})\sin\alpha_t}{\pi m\cos\alpha_t}$$

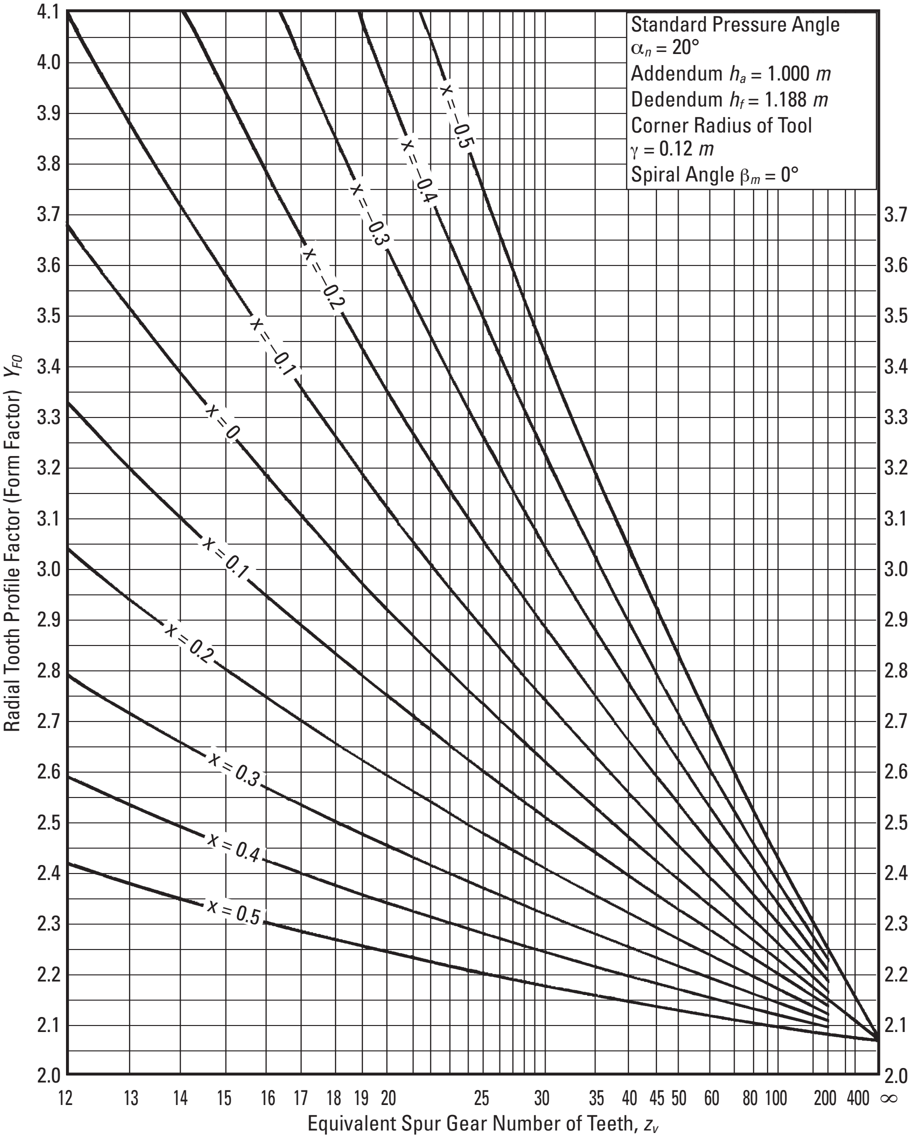

Fig. 17-8 Radial Tooth Profile Factor for Straight Bevel Gear

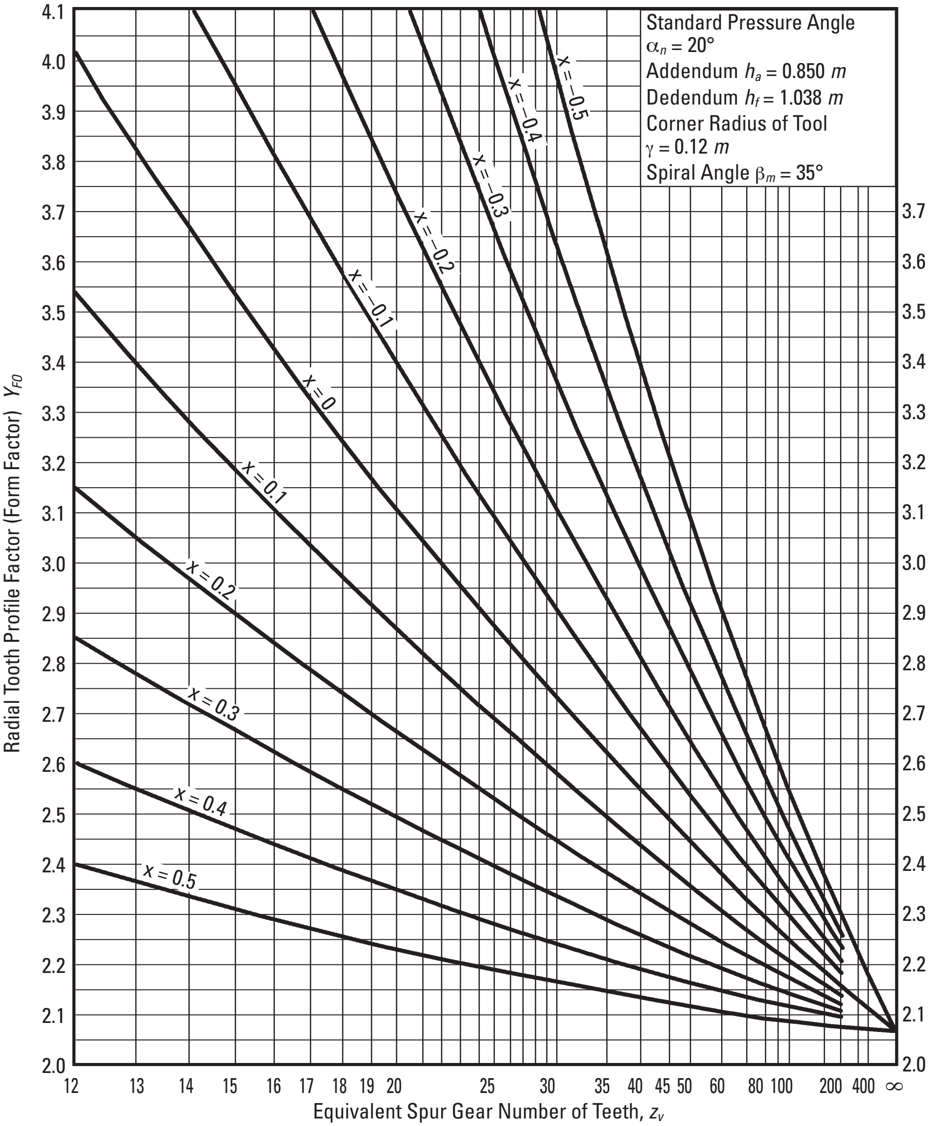

Fig. 17-9 Radial Tooth Profile Factor for Spiral Bevel Gear

See **Tables 17-17** through **17-19** for some calculating examples of radial contact ratio for various bevel gear pairs.

Table 17-17 The Radial Contact Ratio for Gleason's Straight Bevel Gear, ε_α

$\Sigma = 90°, \alpha = 20°$

z_2 \ z_1	12	15	16	18	20	25	30	36	40	45	60
12	1.514										
15	1.529	1.572									
16	1.529	1.578	1.588								
18	1.528	1.584	1.597	1.616							
20	1.525	1.584	1.599	1.624	1.640						
25	1.518	1.577	1.595	1.625	1.650	1.689					
30	1.512	1.570	1.587	1.618	1.645	1.697	1.725				
36	1.508	1.563	1.579	1.609	1.637	1.692	1.732	1.758			
40	1.506	1.559	1.575	1.605	1.632	1.688	1.730	1.763	1.775		
45	1.503	1.556	1.571	1.600	1.626	1.681	1.725	1.763	1.781	1.794	
60	1.500	1.549	1.564	1.591	1.615	1.668	1.710	1.751	1.773	1.796	1.833

Table 17-18 The Radial Contact Ratio for Standard Bevel Gear, ε_α

$\Sigma = 90°, \alpha = 20°$

z_2 \ z_1	12	15	16	18	20	25	30	36	40	45	60
12	1.514										
15	1.545	1.572									
16	1.554	1.580	1.588								
18	1.571	1.595	1.602	1.616							
20	1.585	1.608	1.615	1.628	1.640						
25	1.614	1.636	1.643	1.655	1.666	1.689					
30	1.634	1.656	1.663	1.675	1.685	1.707	1.725				
36	1.651	1.674	1.681	1.692	1.703	1.725	1.742	1.758			
40	1.659	1.683	1.689	1.702	1.712	1.734	1.751	1.767	1.775		
45	1.666	1.691	1.698	1.711	1.721	1.743	1.760	1.776	1.785	1.794	
60	1.680	1.707	1.714	1.728	1.739	1.762	1.780	1.796	1.804	1.813	1.833

Table 17-19 The Radial Contact Ratio for Gleason's Spiral Bevel Gear, ε_α

$\Sigma = 90°, \alpha_n = 20°, \beta_m = 35°$

z_2 \ z_1	12	15	16	18	20	25	30	36	40	45	60
12	1.221										
15	1.228	1.254									
16	1.227	1.258	1.264								
18	1.225	1.260	1.269	1.280							
20	1.221	1.259	1.269	1.284	1.293						
25	1.214	1.253	1.263	1.282	1.297	1.319					
30	1.209	1.246	1.257	1.276	1.293	1.323	1.338				
36	1.204	1.240	1.251	1.270	1.286	1.319	1.341	1.355			
40	1.202	1.238	1.248	1.266	1.283	1.316	1.340	1.358	1.364		
45	1.201	1.235	1.245	1.263	1.279	1.312	1.336	1.357	1.366	1.373	
60	1.197	1.230	1.239	1.256	1.271	1.303	1.327	1.349	1.361	1.373	1.392

17.3.3.D Spiral Angle Factor, Y_β

The spiral angle factor is a function of the spiral angle. The value is arbitrarily set by the following conditions:

$$\left.\begin{aligned} &\text{When } 0 \le \beta_m \le 30°, \quad && Y_\beta = 1 - \frac{\beta_m}{120} \\ &\text{When } \beta_m \ge 30°, && Y_\beta = 0.75 \end{aligned}\right\} \tag{17-36}$$

17.3.3.E Cutter Diameter Effect Factor, Y_C

This factor of cutter diameter, Y_C, can be obtained from **Table 17-20** by the value of tooth flank length, $b / \cos \beta_m$ (mm), over cutter diameter. If cutter diameter is not known, assume $Y_C = 1.00$.

Table 17-20 Cutter Diameter Effect Factor, Y_C

Types of Bevel Gears	Relative Size of Cutter Diameter			
	∞	6 Times Tooth Width	5 Times Tooth Width	4 Times Tooth Width
Straight Bevel Gears	1.15	——	——	——
Spiral and Zerol Bevel Gears	——	1.00	0.95	0.90

17.3.3.F Life Factor, K_L

We can choose a proper life factor, K_L, from **Table 17-2** similarly to calculating the bending strength of spur and helical gears.

17.3.3.G Dimension Factor Of Root Bending Stress, K_{FX}

This is a size factor that is a function of the radial module, m. Refer to **Table 17-21** for values.

Table 17-21 Dimension Factor for Bending Strength, K_{FX}

Radial Module at Outside Diameter, m	Gears Without Hardened Surface	Gears With Hardened Surface
1.5 to 5	1.0	1.0
above 5 to 7	0.99	0.98
above 7 to 9	0.98	0.96
above 9 to 11	0.97	0.94
above 11 to 13	0.96	0.92
above 13 to 15	0.94	0.90
above 15 to 17	0.93	0.88
above 17 to 19	0.92	0.86
above 19 to 22	0.90	0.83
above 22 to 25	0.88	0.80

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17.3.3.H Tooth Flank Load Distribution Factor, K_M

Tooth flank load distribution factor, K_M, is obtained from **Table 17-22** or **Table 17-23.**

Table 17-22 Tooth Flank Load Distribution, K_M, for Spiral Bevel Gears, Zerol Bevel Gears and Straight Bevel Gears with Crowning

Stiffness of Shaft, Gear Box, etc.	Both Gears Supported on Two Sides	One Gear Supported on One End	Both Gears Supported on One End
Very Stiff	1.2	1.35	1.5
Average	1.4	1.6	1.8
Somewhat Weak	1.55	1.75	2.0

Table 17-23 Tooth Flank Load Distribution Factor, K_M, for Straight Bevel Gears without Crowning

Stiffness of Shaft, Gear Box, etc.	Both Gears Supported on Two Sides	One Gear Supported on One End	Both Gears Supported on One End
Very Stiff	1.05	1.15	1.35
Average	1.6	1.8	2.1
Somewhat Weak	2.2	2.5	2.8

17.3.3.I Dynamic Load Factor, K_V

Dynamic load factor, K_V, is a function of the precision grade of the gear and the tangential speed at the outer pitch circle, as shown in **Table 17-24.**

Table 17-24 Dynamic Load Factor, K_V

Precision Grade of Gears from JIS B 1702	Tangential Speed at Outer Pitch Circle (m/s)						
	Up to 1	Above 1 to 3	Above 3 to 5	Above 5 to 8	Above 8 to 12	Above 12 to 18	Above 18 to 25
1	1.0	1.1	1.15	1.2	1.3	1.5	1.7
2	1.0	1.2	1.3	1.4	1.5	1.7	
3	1.0	1.3	1.4	1.5	1.7		
4	1.1	1.4	1.5	1.7			
5	1.2	1.5	1.7				
6	1.4	1.7					

17.3.3.K Reliability Factor, K_R

The reliability factor should be assumed to be as follows:

1. General case: $K_R = 1.2$
2. When all other factors can be determined accurately: $K_R = 1.0$
3. When all or some of the factors cannot be known with certainty: $K_R = 1.4$

17.3.3.L Allowable Bending Stress at Root, $\sigma_{F\,\mathrm{lim}}$

The allowable stress at root $\sigma_{F\mathrm{lim}}$ can be obtained from **Tables 17-5** through **17-8**, similar to the case of spur and helical gears.

17.3.4 Examples of Bevel Gear Bending Strength Calculations

Table 17-24A Gleason Straight Bevel Gear Design Details

No.	Item	Symbol	Unit	Pinion	Gear
1	Shaft Angle	Σ	degree	90°	
2	Module	m	mm	2	
3	Pressure Angle	α	degree	20°	
4	Central Spiral Angle	β_m		0°	
5	Number of Teeth	z		20	40
6	Pitch Circle Diameter	d	mm	40.000	80.000
7	Pitch Cone Angle	δ	degree	26.56505°	63.43495°
8	Cone Distance	R_e	mm	44.721	
9	Tooth Width	b		15	
10	Central Pitch Circle Diameter	d_m		33.292	66.584
11	Precision Grade			JIS 3	JIS 3
12	Manufacturing Method			Gleason No. 104	
13	Surface Roughness			12.5 μm	12.5 μm
14	Revolutions per Minute	n	rpm	1500	750
15	Linear Speed	v	m/s	3.142	
16	Direction of Load			Unidirectional	
17	Duty Cycle		cycle	More than 10^7 cycles	
18	Material			SCM 415	
19	Heat Treatment			Carburized	
20	Surface Hardness			HV 600 … 640	
21	Core Hardness			HB 260 … 280	
22	Effective Carburized Depth		mm	0.3 … 0.5	

I R T 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 A

Table 17-24B Bending Strength Factors for Gleason Straight Bevel Gear

No.	Item	Symbol	Unit	Pinion	Gear
1	Central Spiral Angle	β_m	degree	0°	
2	Allowable Bending Stress at Root	$\sigma_{F\,\lim}$	kgf/mm²	42.5	42.5
3	Module	m	mm	2	
4	Tooth Width	b	mm	15	
5	Cone Distance	R_e	mm	44.721	
6	Tooth Profile Factor	Y_F		2.369	2.387
7	Load Distribution Factor	Y_ε		0.613	
8	Spiral Angle Factor	Y_β		1.0	
9	Cutter Diameter Effect Factor	Y_C		1.15	
10	Life Factor	K_L		1.0	
11	Dimension Factor	K_{FX}		1.0	
12	Tooth Flank Load Distribution Factor	K_M		1.8	1.8
13	Dynamic Load Factor	K_V		1.4	
14	Overload Factor	K_O		1.0	
15	Reliability Factor	K_R		1.2	
16	Allowable Tangential Force at Central Pitch Circle	$F_{t\,\lim}$	kgf	178.6	177.3

17.4 Surface Strength Of Bevel Gears

This information is valid for bevel gears which are used in power transmission in general industrial machines. The applicable ranges are:

Radial Module:	m	1.5 to 25 mm
Pitch Diameter:	d	Straight bevel gear under 1600 mm
		Spiral bevel gear under 1000 mm
Linear Speed:	v	less than 25 m/sec
Rotating Speed:	n	less than 3600 rpm

17.4.1 Basic Conversion Formulas

The same formulas of **SECTION 17.3** apply. (See page **T**-171).

17.4.2 Surface Strength Equations

In order to obtain a proper surface strength, the tangential force at the central pitch circle, F_{tm}, must remain below the allowable tangential force at the central pitch circle, $F_{tm\,\lim}$, based on the allowable Hertz stress $\sigma_{H\,\lim}$.

$$F_{tm} \le F_{tm\,\lim} \qquad \textbf{(17-37)}$$

Alternately, the Hertz stress σ_H, which is derived from the tangential force at the central pitch circle must be smaller than the allowable Hertz stress $\sigma_{H\lim}$.

$$\sigma_H \le \sigma_{H\,\lim} \qquad \textbf{(17-38)}$$

The allowable tangential force at the central pitch circle, $F_{tm\ \lim}$, in kgf can be calculated from **Equation (17-39)**.

$$F_{tm\ \lim} = \left[\left(\frac{\sigma_{H\ \lim}}{Z_M}\right)^2 \frac{d_1}{\cos\delta_1} \frac{R_e - 0.5\,b}{R_e}\, b\, \frac{u^2}{u^2+1}\right] \cdot \left[\left(\frac{K_{HL} Z_L Z_R Z_V Z_W K_{HX}}{Z_H Z_\varepsilon Z_\beta}\right)^2 \frac{1}{K_{H\beta} K_V K_O} \frac{1}{C_R^2}\right] \quad \textbf{(17-39)}$$

The Hertz stress, σ_H (kgf/mm²) is calculated from **Equation (17-40)**.

$$\sigma_H = \sqrt{\frac{\cos\delta_1 F_{tm}}{d_1 b} \frac{u^2+1}{u^2} \frac{R_e}{R_e - 0.5\,b}} \cdot \left[\frac{Z_H Z_M Z_\varepsilon Z_\beta}{K_{HL} Z_L Z_R Z_V Z_W K_{HX}} \sqrt{K_{H\beta} K_V K_O}\ C_R\right] \quad \textbf{(17-40)}$$

17.4.3 Determination of Factors In Surface Strength Equations

17.4.3.A Tooth Width, b (mm)

This term is defined as the tooth width on the pitch cone. For a meshed pair, the narrower gear's "b" is used for strength calculations.

17.4.3.B Zone Factor, Z_H

The zone factor is defined as:

$$Z_H = \sqrt{\frac{2\cos\beta_b}{\sin\alpha_t \cos\alpha_t}} \quad \textbf{(17-41)}$$

where:
- β_m = Central spiral angle
- α_n = Normal pressure angle
- α_t = Central radial pressure angle $= \tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta_m}\right)$
- β_b = $\tan^{-1}(\tan\beta_m \cos\alpha_t)$

If the normal pressure angle α_n is 20°, 22.5° or 25°, the zone factor can be obtained from **Figure 17-10**.

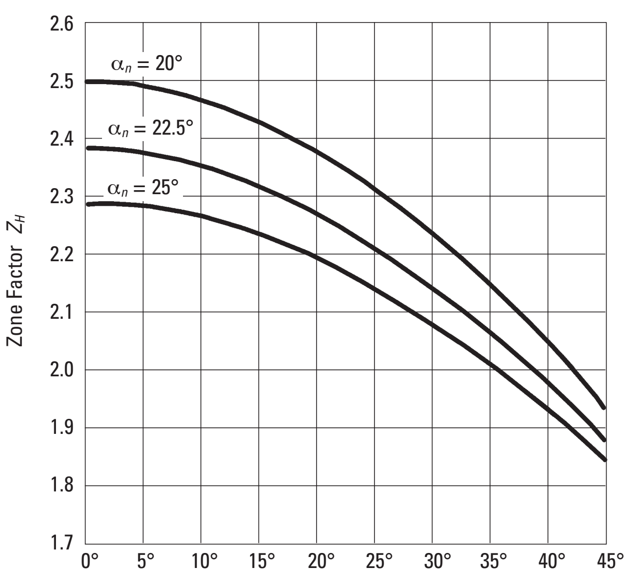

Fig. 17-10 Zone Factor, Z_H

17.4.3.C Material Factor, Z_M

The material factor, Z_M, is obtainable from **Table 17-9**.

17.4.3.D Contact Ratio Factor, Z_ε

The contact ratio factor is calculated from the equations below.

Straight bevel gear: $Z_\varepsilon = 1.0$

Spiral bevel gear:

$$\left.\begin{aligned}&\text{when } \varepsilon_\alpha \le 1,\ Z_\varepsilon = \sqrt{1 - \varepsilon_\beta + \frac{\varepsilon_\beta}{\varepsilon_\alpha}}\\ &\text{when } \varepsilon_\beta > 1,\ Z_\varepsilon = \sqrt{\frac{1}{\varepsilon_\alpha}}\end{aligned}\right\} \qquad \textbf{(17-42)}$$

where: ε_α = Radial Contact Ratio
ε_β = Overlap Ratio

17.4.3.E Spiral Angle Factor, Z_β

Little is known about these factors, so usually it is assumed to be unity.

$$Z_\beta = 1.0 \qquad \textbf{(17-43)}$$

17.4.3.F Life Factor, K_{HL}

The life factor for surface strength is obtainable from **Table 17-10**.

17.4.3.G Lubricant Factor, Z_L

The lubricant factor, Z_L, is found in **Figure 17-3**.

17.4.3.H Surface Roughness Factor, Z_R

The surface roughness factor is obtainable from **Figure 17-11** on the basis of average roughness, R_{maxm}, in μm. The average surface roughness is calculated by **Equation (17-44)** from the surface roughnesses of the pinion and gear (R_{max1} and R_{max2}), and the center distance, a, in mm.

$$R_{maxm} = \frac{R_{max1} + R_{max2}}{2}\sqrt[3]{\frac{100}{a}}\ (\mu\text{m}) \qquad \textbf{(17-44)}$$

where: $a = R_m(\sin\delta_1 + \cos\delta_1)$

$$R_m = R_e - \frac{b}{2}$$

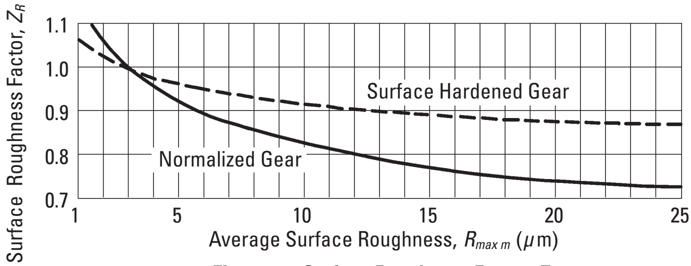

Fig. 17-11 Surface Roughness Factor, Z_R

17.4.3.I Sliding Speed Factor, Z_V

The sliding speed factor is obtained from **Figure 17-5** based on the pitch circle linear speed.

17.4.3.J Hardness Ratio Factor, Z_W

The hardness ratio factor applies only to the gear that is in mesh with a pinion which is quenched and ground. The ratio is calculated by **Equation (17-45)**.

$$Z_W = 1.2 - \frac{HB_2 - 130}{1700} \qquad \textbf{(17-45)}$$

where Brinell hardness of the gear is: $130 \le HB_2 \le 470$

If the gear's hardness is outside of this range, Z_W is assumed to be unity.

$$Z_W = 1.0 \qquad \textbf{(17-46)}$$

17.4.3.K Dimension Factor, K_{HX}

Since, often, little is known about this factor, it is assumed to be unity.

$$K_{HX} = 1.0 \qquad \textbf{(17-47)}$$

17.4.3.L Tooth Flank Load Distribution Factor, $K_{H\beta}$

Factors are listed in **Tables 17-25** and **17-26**. If the gear and pinion are unhardened, the factors are to be reduced to 90% of the values in the table.

Table 17-25 Tooth Flank Load Distribution Factor for Spiral Bevel Gears, Zerol Bevel Gears and Straight Bevel Gears with Crowning, $K_{H\beta}$

Stiffness of Shaft, Gear Box, etc.	Both Gears Supported on Two Sides	One Gear Supported on One End	Both Gears Supported on One End
Very Stiff	1.3	1.5	1.7
Average	1.6	1.85	2.1
Somewhat Weak	1.75	2.1	2.5

Table 17-26 Tooth Flank Load Distribution Factor for Straight Bevel Gear without Crowning, $K_{H\beta}$

Stiffness of Shaft, Gear Box, etc.	Both Gears Supported on Two Sides	One Gear Supported on One End	Both Gears Supported on One End
Very Stiff	1.3	1.5	1.7
Average	1.85	2.1	2.6
Somewhat Weak	2.8	3.3	3.8

17.4.3.M Dynamic Load Factor, K_V

The dynamic load factor can be obtained from **Table 17-24**.

17.4.3.N Overload Factor, K_O

The overload factor can be computed by **Equation 17-11** or found in **Table 17-4**.

17.4.3.O Reliability Factor, C_R

The general practice is to assume C_R to be at least 1.15.

17.4.3.P Allowable Hertz Stress, $\sigma_{H\ \lim}$

The values of allowable Hertz stress are given in **Tables 17-12** through **17-16**.

17.4.4 Examples Of Bevel Gear Surface Strength Calculation

Table 17-26A Gleason Straight Bevel Gear Design Details

<table>
<tr><th>No.</th><th>Item</th><th>Symbol</th><th>Unit</th><th>Pinion</th><th>Gear</th></tr>
<tr><td>1</td><td>Shaft Angle</td><td>Σ</td><td>degree</td><td colspan="2">90°</td></tr>
<tr><td>2</td><td>Module</td><td>m</td><td>mm</td><td colspan="2">2</td></tr>
<tr><td>3</td><td>Pressure Angle</td><td>α</td><td rowspan="2">degree</td><td colspan="2">20°</td></tr>
<tr><td>4</td><td>Central Spiral Angle</td><td>β_m</td><td colspan="2">0°</td></tr>
<tr><td>5</td><td>Number of Teeth</td><td>z</td><td></td><td>20</td><td>40</td></tr>
<tr><td>6</td><td>Pitch Circle Diameter</td><td>d</td><td>mm</td><td>40.000</td><td>80.000</td></tr>
<tr><td>7</td><td>Pitch Cone Angle</td><td>δ</td><td>degree</td><td>26.56505°</td><td>63.43495°</td></tr>
<tr><td>8</td><td>Cone Distance</td><td>R_e</td><td rowspan="3">mm</td><td colspan="2">44.721</td></tr>
<tr><td>9</td><td>Tooth Width</td><td>b</td><td colspan="2">15</td></tr>
<tr><td>10</td><td>Central Pitch Circle Diameter</td><td>d_m</td><td>33.292</td><td>66.584</td></tr>
<tr><td>11</td><td>Precision Grade</td><td rowspan="3"></td><td rowspan="3"></td><td>JIS 3</td><td>JIS 3</td></tr>
<tr><td>12</td><td>Manufacturing Method</td><td colspan="2">Gleason No. 104</td></tr>
<tr><td>13</td><td>Surface Roughness</td><td>12.5 μm</td><td>12.5 μm</td></tr>
<tr><td>14</td><td>Revolutions per Minute</td><td>n</td><td>rpm</td><td>1500</td><td>750</td></tr>
<tr><td>15</td><td>Linear Speed</td><td>v</td><td>m/s</td><td colspan="2">3.142</td></tr>
<tr><td>16</td><td>Direction of Load</td><td rowspan="2"></td><td></td><td colspan="2">Unidirectional</td></tr>
<tr><td>17</td><td>Duty Cycle</td><td>cycle</td><td colspan="2">Over 10^7 cycles</td></tr>
<tr><td>18</td><td>Material</td><td rowspan="5"></td><td rowspan="4"></td><td colspan="2">SCM 415</td></tr>
<tr><td>19</td><td>Heat Treatment</td><td colspan="2">Carburized</td></tr>
<tr><td>20</td><td>Surface Hardness</td><td colspan="2">HV 600 … 640</td></tr>
<tr><td>21</td><td>Core Hardness</td><td colspan="2">HB 260 … 280</td></tr>
<tr><td>22</td><td>Effective Carburized Depth</td><td>mm</td><td colspan="2">0.3 … 0.5</td></tr>
</table>

Table 17-26B Surface Strength Factors of Gleason Straight Bevel Gear

No.	Item	Symbol	Unit	Pinion	Gear
1	Allowable Hertz Stress	$\sigma_{H\ lim}$	kgf/mm^2	164	
2	Pinion's Pitch Diameter	d_1	mm	40.000	
3	Pinion's Pitch Cone Angle	δ_1	degree	26.56505°	
4	Cone Distance	R_e	mm	44.721	
5	Tooth Width	b		15	
6	Numbers of Teeth Ratio z_2 / z_1	u		2	
7	Zone Factor	Z_H		2.495	
8	Material Factor	Z_M	(kgf/mm^2)$^{0.5}$	60.6	
9	Contact Ratio Factor	Z_ε		1.0	
10	Spiral Angle Factor	Z_β		1.0	
11	Life Factor	K_{HL}		1.0	
12	Lubricant Factor	Z_L		1.0	
13	Surface Roughness Factor	Z_R		0.90	
14	Sliding Speed Factor	Z_V		0.97	
15	Hardness Ratio Factor	Z_W		1.0	
16	Dimension Factor of Root Stress	K_{HX}		1.0	
17	Load Distribution Factor	$K_{H\beta}$		2.1	
18	Dynamic Load Factor	K_V		1.4	
19	Overload Factor	K_O		1.0	
20	Reliability Factor	C_R		1.15	
21	Allowable Tangential Force on Central Pitch Circle	$F_{t\ lim}$	kgf	103.0	103.0

17.5 Strength Of Worm Gearing

This information is applicable for worm gear drives that are used to transmit power in general industrial machines with the following parameters:

Axial Module:	m_x	1 to 25 mm
Pitch Diameter of Worm Gear:	d_2	less than 900 mm
Sliding Speed:	v_s	less than 30 m/sec
Rotating Speed, Worm Gear:	n_2	less than 600 rpm

17.5.1 Basic Formulas:

Sliding Speed, v_s (m/s)

$$v_s = \frac{d_1 n_1}{19100 \cos \gamma} \qquad \textbf{(17-48)}$$

17.5.2 Torque, Tangential Force and Efficiency

(1) Worm as Driver Gear (Speed Reducing)

$$\left.\begin{aligned} T_2 &= \frac{F_t d_2}{2000} \\ T_1 &= \frac{T_2}{u\,\eta_R} = \frac{F_t d_2}{2000\,u\,\eta_R} \\ \eta_R &= \frac{\tan\gamma\left(1-\tan\gamma\,\dfrac{\mu}{\cos\alpha_n}\right)}{\tan\gamma+\dfrac{\mu}{\cos\alpha_n}} \end{aligned}\right\} \qquad \textbf{(17-49)}$$

where:
T_2 = Nominal torque of worm gear (kg • m)
T_1 = Nominal torque of worm (kgf • m)
F_t = Nominal tangential force on worm gear's pitch circle (kgf)
d_2 = Pitch diameter of worm gear (mm)
u = Teeth number ratio = z_2/z_w
η_R = Transmission efficiency, worm driving (not including bearing loss, lubricant agitation loss, etc.)
μ = Friction coefficient

(2) Worm Gear as Driver Gear (Speed Increasing)

$$\left.\begin{aligned} T_2 &= \frac{F_t d_2}{2000} \\ T_1 &= \frac{T_2\eta_I}{u} = \frac{F_t d_2 \eta_I}{2000\,u} \\ \eta_I &= \frac{\tan\gamma-\dfrac{\mu}{\cos\alpha_n}}{\tan\gamma\left(1+\tan\gamma\,\dfrac{\mu}{\cos\alpha_n}\right)} \end{aligned}\right\} \qquad \textbf{(17-50)}$$

where: η_I = Transmission efficiency, worm gear driving (not including bearing loss, lubricant agitation loss, etc.)

17.5.3 Friction Coefficient, μ

The friction factor varies as sliding speed changes. The combination of materials is important. For the case of a worm that is carburized and ground, and mated with a phosphorous bronze worm gear, see **Figure 17-12**. For some other materials, see **Table 17-27**.

For lack of data, friction coefficient of materials not listed in **Table 17-27** are very difficult to obtain. H.E. Merritt has offered some further information on this topic. See Reference 9.

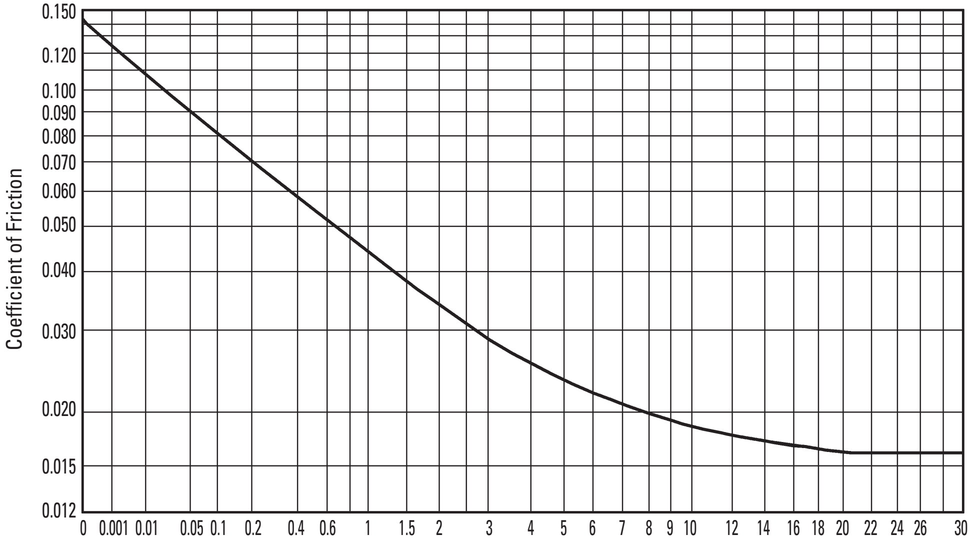

Fig. 17-12 Friction Coefficient, μ

Table 17-27 Combinations of Materials and Their Coefficients of Friction, μ

Combination of Materials	μ
Cast Iron and Phosphor Bronze	μ in **Figure 17-12** times 1.15
Cast Iron and Cast Iron	μ in **Figure 17-12** times 1.33
Quenched Steel and Aluminum Alloy	μ in **Figure 17-12** times 1.33
Steel and Steel	μ in **Figure 17-12** times 2.00

17.5.4 Surface Strength of Worm Gearing Mesh

(1) Calculation of Basic Load

Provided dimensions and materials of the worm pair are known, the allowable load is as follows:

$F_{t\ \mathrm{lim}}$ = Allowable tangential force (kgf)

$$= 3.82\ K_v K_n S_{c\,\mathrm{lim}}\ Z d_2^{0.8}\ m_x \frac{Z_L Z_M Z_R}{K_C} \qquad \textbf{(17-51)}$$

$T_{2\ \mathrm{lim}}$ = Allowable worm gear torque (kgf • m)

$$= 0.00191\ K_v K_n S_{c\,\mathrm{lim}}\ Z d_2^{1.8}\ m_x \frac{Z_L Z_M Z_R}{K_C} \qquad \textbf{(17-52)}$$

(2) Calculation of Equivalent Load

The basic load **Equations (17-51)** and **(17-52)** are applicable under the conditions of no impact and the pair can operate for 26000 hours minimum. The condition of "no impact" is defined as the starting torque which must be less than 200% of the rated torque; and the frequency of starting should be less than twice per hour.

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An equivalent load is needed to compare with the basic load in order to determine an actual design load, when the conditions deviate from the above.

Equivalent load is then converted to an equivalent tangential force, F_{te}, in kgf:

$$F_{te} = F_t K_h K_s \tag{17-53}$$

and equivalent worm gear torque, T_{2e}, in kgf • m:

$$T_{2e} = T_2 K_h K_s \tag{17-54}$$

(3) Determination of Load

Under no impact condition, to have life expectancy of 26000 hours, the following relationships must be satisfied:

$$F_t \le F_{t\,\mathrm{lim}} \quad \text{or} \quad T_2 \le T_{2\,\mathrm{lim}} \tag{17-55}$$

For all other conditions:

$$F_{te} \le F_{t\,lim} \quad \text{or} \quad T_{2e} \le T_{2\,lim} \tag{17-56}$$

NOTE: If load is variable, the maximum load should be used as the criterion.

17.5.5 Determination of Factors in Worm Gear Surface Strength Equations

17.5.5.A Tooth Width of Worm Gear, b_2 (mm)

Tooth width of worm gear is defined as in **Figure 17-13**.

17.5.5.B Zone Factor, Z

$$\left.\begin{array}{l} \text{If } b_2 < 2.3\, m_x\sqrt{Q+1}\text{, then:} \\ Z = (\text{Basic zone factor}) \times \dfrac{b_2}{2\, m_x \sqrt{Q+1}} \\ \text{If } b_2 \ge 2.3\, m_x\sqrt{Q+1}\text{, then:} \\ Z = (\text{Basic zone factor}) \times 1.15 \end{array}\right\} \tag{17-57}$$

where: Basic Zone Factor is obtained from **Table 17-28**

Q : Diameter factor $= \dfrac{d_1}{m_x}$

z_w : number of worm threads

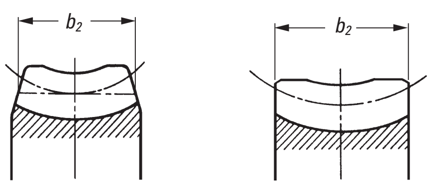

Fig. 17-13 Tooth Width of Worm Gear

Table 17-28 Basic Zone Factors

z_w \ Q	7	7.5	8	8.5	9	9.5	10	11	12	13	14	17	20
1	1.052	1.065	1.084	1.107	1.128	1.137	1.143	1.160	1.202	1.260	1.318	1.402	1.508
2	1.055	1.099	1.144	1.183	1.214	1.223	1.231	1.250	1.280	1.320	1.360	1.447	1.575
3	0.989	1.109	1.209	1.260	1.305	1.333	1.350	1.365	1.393	1.422	1.442	1.532	1.674
4	0.981	1.098	1.204	1.301	1.380	1.428	1.460	1.490	1.515	1.545	1.570	1.666	1.798

17.5.5.C Sliding Speed Factor, K_v

The sliding speed factor is obtainable from **Figure 17-14**, where the abscissa is the pitch line linear velocity.

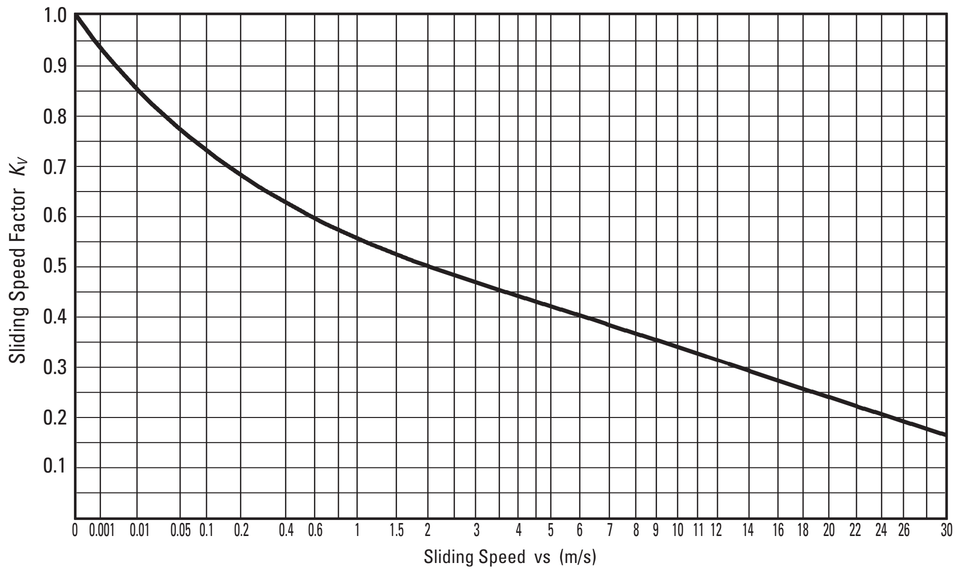

Fig. 17-14 Sliding Speed Factor, K_v

17.5.5.D Rotating Speed Factor, K_n

The rotating speed factor is presented in **Figure 17-15** as a function of the worm gear's rotating speed, n_2.

17.5.5.E Lubricant Factor, Z_L

Let Z_L = 1.0 if the lubricant is of proper viscosity and has antiscoring additives.

Some bearings in worm gear boxes may need a low viscosity lubricant. Then Z_L is to be less than 1.0. The recommended kinetic viscosity of lubricant is given in **Table 17-29**.

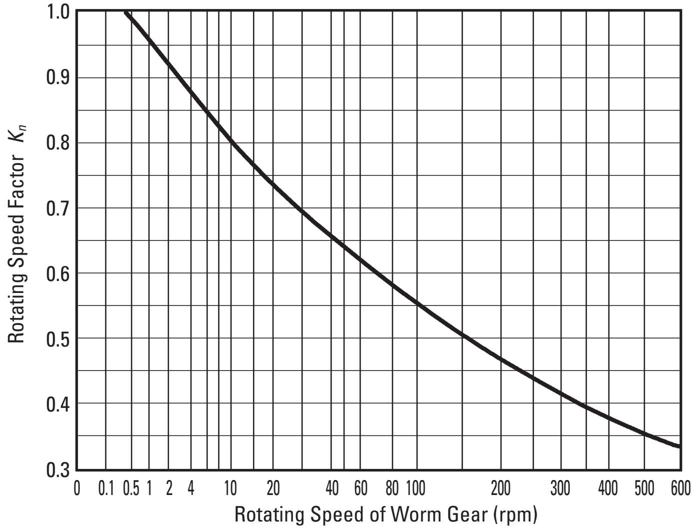

Fig. 17-15 Rotating Speed Factor, K_n

Table 17-29 Recommended Kinematic Viscosity of Lubricant

Unit: cSt/37.8°C

Operating Lubricant Temperature		Sliding Speed (m/s)		
Highest Operating Temperature	**Lubricant Temperature at Start of Operation**	**Less than 2.5**	**2.5 to 5**	**More than 5**
0°C to less than 10°C	–10°C ... 0°C	110 ... 130	110 ... 130	110 ... 130
	more than 0°C	110 ... 150	110 ... 150	110 ... 150
10°C to less than 30°C	more than 0°C	200 ... 245	150 ... 200	150 ... 200
30°C to less than 55°C	more than 0°C	350 ... 510	245 ... 350	200 ... 245
55°C to less than 80°C	more than 0°C	510 ... 780	350 ... 510	245 ... 350
80°C to less than 100°C	more than 0°C	900 ... 1100	510 ... 780	350 ... 510

17.5.5.F Lubrication Factor, Z_M

The lubrication factor, Z_M, is obtained from **Table 17-30**.

Table 17-30 Lubrication Factor, Z_M

Sliding Speed (m/s)	Less than 10	10 to 14	More than 14
Oil Bath Lubrication	1.0	0.85	——
Forced Circulation Lubrication	1.0	1.0	1.0

17.5.5.G Surface Roughness Factor, Z_R

This factor is concerned with resistance to pitting of the working surfaces of the teeth. Since there is insufficient knowledge about this phenomenon, the factor is assumed to be 1.0.

$$Z_R = 1.0 \tag{17-58}$$

It should be noted that for **Equation (17-58)** to be applicable, surfaces roughness of the worm and worm gear must be less than 3 μm and 12 μm respectively. If either is rougher, the factor is to be adjusted to a smaller value.

17.5.5.H Contact Factor, K_c

Quality of tooth contact will affect load capacity dramatically. Generally, it is difficult to define precisely, but JIS B 1741 offers guidelines depending on the class of tooth contact.

$$\left.\begin{array}{ll} \text{Class A} & K_c = 1.0 \\ \text{Class B, C} & K_c > 1.0 \end{array}\right\} \tag{17-59}$$

Table 17-31 gives the general values of K_c depending on the JIS tooth contact class.

Table 17-31 Classes of Tooth Contact and General Values of Contact Factor, K_c

Class	Proportion of Tooth Contact		K_c
	Tooth Width Direction	Tooth Height Direction	
A	More than 50% of Effective Width of Tooth	More than 40% of Effective Height of Tooth	1.0
B	More than 35% of Effective Width of Tooth	More than 30% of Effective Height of Tooth	1.3 … 1.4
C	More than 20% of Effective Width of Tooth	More than 20% of Effective Height of Tooth	1.5 … 1.7

17.5.5.I Starting Factor, K_s

This factor depends upon the magnitude of starting torque and the frequency of starts. When starting torque is less than 200% of rated torque, K_s factor is per **Table 17-32**.

Table 17-32 Starting Factor, K_s

Starting Factor	Starting Frequency per Hour			
	Less than 2	2 … 5	5 … 10	More than 10
K_s	1.0	1.07	1.13	1.18

17.5.5.J Time Factor, K_h

This factor is a function of the desired life and the impact environment. See **Table 17-33**. The expected lives in between the numbers shown in **Table 17-33** can be interpolated.

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Table 17-33 Time Factor, K_h

Impact from Prime Mover	Expected Life	K_h Impact from Load: Uniform Load	Medium Impact	Strong Impact
Uniform Load (Motor, Turbine, Hydraulic Motor)	1500 Hours	0.80	0.90	1.0
	5000 Hours	0.90	1.0	1.25
	26000 Hours*	1.0	1.25	1.50
	60000 Hours	1.25	1.50	1.75
Light Impact (Multicylinder engine)	1500 Hours	0.90	1.0	1.25
	5000 Hours	1.0	1.25	1.50
	26000 Hours*	1.25	1.50	1.75
	60000 Hours	1.50	1.75	2.0
Medium Impact (Single cylinder engine)	1500 Hours	1.0	1.25	1.50
	5000 Hours	1.25	1.50	1.75
	26000 Hours*	1.50	1.70	2.0
	60000 Hours	1.75	2.0	2.25

***NOTE:** For a machine that operates 10 hours a day, 260 days a year; this number corresponds to ten years of operating life.

17.5.5.K Allowable Stress Factor, $S_{c\ \mathrm{lim}}$

Table 17-34 presents the allowable stress factors for various material combinations. Note that the table also specifies governing limits of sliding speed, which must be adhered to if scoring is to be avoided.

Table 17-34 Allowable Stress Factor for Surface Strength, $S_{c\ \mathrm{lim}}$

Material of Worm Gear	Material of Worm	$S_{c\ \mathrm{lim}}$	Sliding Speed Limit before Scoring (m/s) *
Phosphor Bronze Centrifugal Casting	Alloy Steel Carburized & Quenched	1.55	30
	Alloy Steel HB 400	1.34	20
	Alloy Steel HB 250	1.12	10
Phosphor Bronze Chilled Casting	Alloy Steel Carburized & Quenched	1.27	30
	Alloy Steel HB 400	1.05	20
	Alloy Steel HB 250	0.88	10
Phosphor Bronze Sand Molding or Forging	Alloy Steel Carburized & Quenched	1.05	30
	Alloy Steel HB 400	0.84	20
	Alloy Steel HB 250	0.70	10
Aluminum Bronze	Alloy Steel Carburized & Quenched	0.84	20
	Alloy Steel HB 400	0.67	15
	Alloy Steel HB 250	0.56	10
Brass	Alloy Steel HB 400	0.49	8
	Alloy Steel HB 250	0.42	5
Ductile Cast Iron	Ductile Cast Iron but with a higher hardness than the worm gear	0.70	5
Cast Iron (Perlitic)	Phosphor Bronze Casting and Forging	0.63	2.5
	Cast Iron but with a higher hardness than the worm gear	0.42	2.5

***NOTE:** The value indicates the maximum sliding speed within the limit of the allowable stress factor, $S_{c\ \mathrm{lim}}$. Even when the allowable load is below the allowable stress level, if the sliding speed exceeds the indicated limit, there is danger of scoring gear surfaces.

17.5.6 Examples Of Worm Mesh Strength Calculation

Table 17-35A Worm and Worm Gear Design Details

No.	Item	Symbol	Unit	Worm	Worm Gear
1	Axial Module	m_x	mm	2	
2	Normal Pressure Angle	α_n	degree	20°	
3	No. of Threads, No. of Teeth	z_w, z_2		1	40
4	Pitch Diameter	d	mm	28	80
5	Lead Angle	γ	degree	4.08562	
6	Diameter Factor	Q		14	——
7	Tooth Width	b	mm	( )	20
8	Manufacturing Method			Grinding	Hobbing
9	Surface Roughness			3.2 μm	12.5 μm
10	Revolutions per Minute	n	rpm	1500	37.5
11	Sliding Speed	v_s	m/s	2.205	
12	Material			S45C	Al BC2
13	Heat Treatment			Induction Hardening	——
14	Surface Hardness			HS 63 … 68	——

Table 17-35B Surface Strength Factors and Allowable Force

No.	Item	Symbol	Unit	Worm Gear
1	Axial Module	m_x	mm	2
2	Worm Gear Pitch Diameter	d_2		80
3	Zone Factor	Z		1.5157
4	Sliding Speed Factor	K_v		0.49
5	Rotating Speed Factor	K_n		0.66
6	Lubricant Factor	Z_L		1.0
7	Lubrication Factor	Z_M		1.0
8	Surface Roughness Factor	Z_R		1.0
9	Contact Factor	K_C		1.0
10	Allowable Stress Factor	$S_{C\,lim}$		0.67
11	Allowable Tangential Force	$F_{t\,lim}$	kgf	83.5

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SECTION 18 DESIGN OF PLASTIC GEARS

18.1 General Considerations Of Plastic Gearing

Plastic gears are continuing to displace metal gears in a widening arena of applications. Their unique characteristics are also being enhanced with new developments, both in materials and processing. In this regard, plastics contrast somewhat dramatically with metals, in that the latter materials and processes are essentially fully developed and, therefore, are in a relatively static state of development.

Plastic gears can be produced by hobbing or shaping, similarly to metal gears or alternatively by molding. The molding process lends itself to considerably more economical means of production; therefore, a more in-depth treatment of this process will be presented in this section.

Among the characteristics responsible for the large increase in plastic gear usage, the following are probably the most significant:

1. Cost effectiveness of the injection-molding process.
2. Elimination of machining operations; capability of fabrication with inserts and integral designs.
3. Low density: lightweight, low inertia.
4. Uniformity of parts.
5. Capability to absorb shock and vibration as a result of elastic compliance.
6. Ability to operate with minimum or no lubrication, due to inherent lubricity.
7. Relatively low coefficient of friction.
8. Corrosion-resistance; elimination of plating, or protective coatings.
9. Quietness of operation.
10. Tolerances often less critical than for metal gears, due in part to their greater resilience.
11. Consistency with trend to greater use of plastic housings and other components.
12. One step production; no preliminary or secondary operations.

At the same time, the design engineer should be familiar with the limitations of plastic gears relative to metal gears. The most significant of these are the following:

1. Less load-carrying capacity, due to lower maximum allowable stress; the greater compliance of plastic gears may also produce stress concentrations.
2. Plastic gears cannot generally be molded to the same accuracy as high-precision machined metal gears.
3. Plastic gears are subject to greater dimensional instabilities, due to their larger coefficient of thermal expansion and moisture absorption.
4. Reduced ability to operate at elevated temperatures; as an approximate figure, operation is limited to less than 120°C. Also, limited cold temperature operations.
5. Initial high mold cost in developing correct tooth form and dimensions.
6. Can be negatively affected by certain chemicals and even some lubricants.
7. Improper molding tools and process can produce residual internal stresses at the tooth roots, resulting in over stressing and/or distortion with aging.
8. Costs of plastics track petrochemical pricing, and thus are more volatile and subject to increases in comparison to metals.

18.2 Properties Of Plastic Gear Materials

Popular materials for plastic gears are acetal resins such as DELRIN*, Duracon M90; nylon resins such as ZYTEL*, NYLATRON**, MC901 and acetal copolymers such as CELCON***. The physical and mechanical properties of these materials vary with regard to strength, rigidity, dimensional stability, lubrication requirements, moisture absorption, etc. Standardized tabular data is available from various manufacturers' catalogs. Manufacturers in the U.S.A. provide this information in units customarily used in the U.S.A. In general, the data is less simplified and fixed than for the metals. This is because plastics are subject to wider formulation variations and are often regarded as proprietary compounds and mixtures. **Tables 18-1** through **18-9** are representative listings of physical and mechanical properties of gear plastics taken from a variety of sources. All reprinted tables are in their original units of measure.

It is common practice to use plastics in combination with different metals and materials other than plastics. Such is the case when gears have metal hubs, inserts, rims, spokes, etc. In these cases, one must be cognizant of the fact that plastics have an order of magnitude different coefficients of thermal expansion as well as density and modulus of elasticity. For this reason, **Table 18-10** is presented.

Other properties and features that enter into consideration for gearing are given in **Table 18-11** (Wear) and **Table 18-12** (Poisson's Ratio).

Moisture has a significant impact on plastic properties as can be seen in **Tables 18-1** thru **18-5**. Ranking of plastics is given in Table 18-13. In this table, rate refers to expansion from dry to full moist condition. Thus, a 0.20% rating means a dimensional increase of 0.002 mm/mm. Note that this is only a rough guide, as exact values depend upon factors of composition and processing, both the raw material and gear molding. For example, it can be seen that the various types and grades of nylon can range from 0.07% to 2.0%.

Table 18-14 lists safe stress values for a few basic plastics and the effect of glass fiber reinforcement.

Table 18-1 Physical Properties of Plastics Used in Gears

Material	Tensile Strength (psi x 10^3)	Flexural Strength (psi x 10^3)	Compressive Modulus (psi x 10^3)	Heat Distortion Temperature (°F @ 264 psi)	Water Absorption (% in 24 hrs)	Rockwell Hardness	Mold Shrinkage (in./in.)
Acetal	8.8 – 1.0	13 – 14	410	230 – 255	0.25	M94 R120	0.022
ABS	4.5 – 8.5	5 – 13.5	120 – 200	180 – 245	0.2 – 0.5	R80 – 120	0.003 0.007
Nylon 6/6	11.2 – 13.1	14.6	400	200	1.3	R118 – 123	0.007 0.015
Nylon 6/10	7 – 8.5	10.5	400	145	0.4	R111	0.015
Polycarbonate	8 – 9.5	11 – 13	350	265 – 290	0.15	M70 R112	0.005 0.007
High Impact Polystyrene	1.9 – 4	5.5 – 12.5	300 – 500	160 – 205	0.05 – 0.10	M25 – 69	0.003 0.005
Polyurethane	4.5 – 8	7.1	85	160 – 205	0.60 – 0.80	M29 R90	0.009
Polyvinyl Chloride	6 – 9	8 – 15	300 – 400	140 – 175	0.07 – 0.40	R100 – 120	0.002 0.004
Polysulfone	10.2	15.4	370	345	0.22	M69 R120	0.0076
MoS_2 – Filled Nylon	10.2	10	350	140	0.4	D785	0.012

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* Registered trademark, E.I. du Pont de Nemours and Co., Wilmington, Delaware, 19898.
** Registered trademark, The Polymer Corporation, P.O. Box 422, Reading, Pennsylvania, 19603.
*** Registered trademark, Celanese Corporation, 26 Main St., Chatham, N.J. 07928.

Table 18-2 Property Chart for Basic Polymers for Gearing

	Water Absorp. 24hrs.	Mold Shrinkage	Tensile Strength * Yield • Break	Flexural Modulus	Izod Impact Strength Notched	Deflect. Temp. @ 264 psi	Coeff. of Linear Thermal Expan.	Specific Gravity
Units	%	in. / in.	psi	psi	lb.ft./ in.^{2}	°F	10^{-5} °F	
ASTM	**D570**	**D955**	**D638**	**D790**	**D256**	**D648**	**D696**	**D792**
1. Nylon 6/6	1.5	.015/.030	*11,200	175,000	2.1	220	4.5 varies	1.13/1.15
2. Nylon 6	1.6	.013/.025	*11,800	395,000	1.1	150	4.6	1.13
3. Acetal	0.2	.016/.030	*10,000	410,000	1.4/2.3	255	5.8	1.42
4. Polycarbonate 30% G/F, 15% PTFE	0.06	.0035	*17,500	1,200,000	2	290	1.50	1.55
5. Polyester (thermoplastic)	0.08	.020	*8,000 •12,000	340,000	1.2	130	5.3	1.3
6. Polyphenylene sulfide 30% G/F 15% PTFE	0.03	.002	*19,000	1,300,000	1.10	500	1.50	1.69
7. Polyester elastomer	0.3	.012	*3,780 •5,500	—	—	122	10.00	1.25
8. Phenolic (molded)	0.45	.007	•7,000	340,000	.29	270	3.75	1.42

*These are average values for comparison purpose only.
Source: Clifford E. Adams, Plastic Gearing, Marcel Dekker Inc., N.Y. 1986. Reference 1.

Table 18-3 Physical Properties of DELRIN Acetal Resin and ZYTEL Nylon Resin

Properties – Units	ASTM	"DELRIN" 500	"DELRIN" 100	"ZYTEL" 101 .2% Moisture	"ZYTEL" 101 2.5% Moisture
Yield Strength, psi	D638*	10,000		11,800	8,500
Shear Strength, psi	D732*	9,510		9,600	—
Impact Strength (Izod)	D256*	1.4	2.3	0.9	2.0
Elongation at Yield, %	D638*	15	75	5	25
Modulus of Elasticity, psi	D790*	410,000		410,000	175,000
Hardness, Rockwell	D785*	M 94, R 120		M79 R118	M 94, R 120, etc.
Coefficient of Linear Thermal Expansion, in./in.°F	D696	4.5 x 10^{-5}		4.5 x 10^{-5}	—
Water Absorption 24 hrs. %	D570	0.25		1.5	—
Saturation, %	D570	0.9		8.0	
Specific Gravity	D792	1.425		1.14	1.14

*Test conducted at 73°F
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Table 18-4 Properties of Nylatron GSM Nylon

Property	Units	ASTM No.	Value
Specific Gravity	–	D 792	1.15 - 1.17
Tensile Strength, 73°F	psi	D 638	11,000 - 14,000
Elongation, 73°F	%	D 638	10 - 60
Modulus of Elasticity, 73°F	psi	D 638	350,000 - 450,000
Compressive Strength @ 0.1% Offset	psi	D 695	9,000
@ 1.0% Offset			12,000
Shear Strength, 73°F	psi	D 732	10,500 - 11,500
Tensile Impact, 73°F	lb.ft./in.^{2}	–	80 - 130
Deformation Under Load 122°F, 2000psi	%	D 621	0.5 - 1.0
Hardness (Rockwell), 73°F	–	D-785	R112 - 120
Coefficient of Friction (Dry vs Steel) Dynamic	–	–	.15 - .35
Heat Distortion Temp. 66 psi	°F	D-648	400 - 425
264psi	°F	D-648	200 - 425
Melting Point	°F	D-789	430 ±10
Flammability	–	D-635	Self-extinguishing
Coefficient of Linear Thermal Expansion	in./in.°F	D-696	5.0 x 10^{-5}
Water Absorption 24 Hours	%	D-570	.6 - 1.2
Saturation	%	D-570	5.5 - 6.5

Resistant to: Common Solvents, Hydrocarbons, Esters, Ketones, Alkalis, Diluted Acids
Not Resistant to: Phenol, Formic Acid, Concentrated Mineral Acid
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Table 18-5 Typical Thermal Properties of "CELCON" Acetal Copolymer

Property	ASTM Test Method	Units	M Series	GC-25A
Flow, Softening and Use Temperature				
Flow Temperature	D 569	°F	345	—
Melting Point	—	°F	329	331
Vicat Softening Point	D 1525	°F	324	324
Unmolding Temperature[1]	—	°F	320	—
Thermal Deflection and Deformation				
Deflection Temperature	D 648			
@264 psi		°F	230	322
@66 psi		°F	316	
Deformation under Load (2000 psi @ 122°F)	D 621	%	1.0	0.6
Miscellaneous				
Thermal Conductivity	—	BTU / hr./ ft^2 /°F/in.	1.6	—
Specific Heat	—	BTU / lb./°F	0.35	—
Coefficient of Linear Thermal Expansion (Range: -30°C to + 30°C)	D 696	in./in.°F		
Flow direction			4.7 x 10^{-5}	2.2 x 10^{-5}
Traverse direction			4.7 x 10^{-5}	4.7 x 10^{-5}
Flammability	D 635	in./min.	1.1	—
Average Mold Shrinkage[2]	—	in. / in.		
Flow direction			0.022	0.004
Transverse direction			0.018	0.018

[1]Unmolding temperature is the temperature at which a plastic part loses its structural integrity (under its own weight) after a half-hour exposure.
[2]Data Bulletin C3A, "Injection Molding Celcon," gives information of factors which influence mold shrinkage.
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Table 18-6 Typical Physical / Mechanical Properties of CELCON® Acetal Copolymer

Property English Units (Metric Units)	ASTM Test Method	Nominal Specimen Size	Temp.	M-Series Values	GC-25A Values	Temp.	M-Series Values	GC-25A Values
Specific Gravity	D 792			1.41	1.59		1.41	1.59
Density lbs/in³ (g/cm³)				0.0507	0.057			
Specific Volume in³/lbs (cm³/g)				19.7	17.54		0.71	0.63
Tensile Strength at Yield lbs/in² (kgf/cm²)	D 638 Speed B	Type I 1/8"	-40 °F 73 °F 160 °F	13,700 8,800 5,000	16,000 (at break)	-40 °C 23 °C 70 °C	965 620 350	1120 (at break)
Elongation at Break %	D 638 Speed B	Type I 1/8" Thick	-40 °F 73 °F 160 °F	M25/30 M90/20 M270/15 M25/75 M90/60 M270/40 250	2 – 3	-40 °C 23 °C	M25/30 M90/20 M270/15 M25/75 M90/60 M270/40 250	2 – 3
Tensile Modulus lbs/in² (kgf/cm²)	D 638	Type I 1/8" Thick		410,000	1.2 x 10^6	70 °C	28,800	84,500
Flexural Modulus lbs/in² (kgf/cm²)	D 790	5" x 1/2" x 1/8" Thick	73 °F 160 °F 220 °F	375,000 180,000 100,000	1.05x10^6 0.7x10^6 0.5x10^6	23 °C 70 °C 105 °C	26,400 12,700 7,000	74,000 50,000 35,000
Flexural Stress at 5% Deformation lbs/in² (kgf/cm²)	D 790	5" x 1/2" x 1/8" Thick		13,000			915	
Compressive Stress at 1% Deflection lbs/in² (kgf/cm²) at 10% Deflection lbs/in² (kgf/cm²)	D 695	1" x 1/2" x 1/2"		4,500 16,000			320 1,100	
Izod Impact Strength (Notched) lb.ft./in. notch (kgf • cm/cm notch)	D 256	2-1/2" x 1/2" x 1/8" machined notch	-40 °F 73 °F	M25/1.2 M90/1.0 M270/0.8 M25/1.5 M90/1.3 M270/1.0	1.1	-40 °C 23 °C	M25/6.5 M90/5.5 M270/4.4 M25/8.0 M90/7.0 M270/5.5	6.0
Tensile Impact Strength lb.ft./in.² (kgf • cm/cm²)	D 1822	L–Specimen 1/8" Thick		M25/90 M90/70 M270/60	50		M25/190 M90/150 M270/130	110
Rockwell Hardness M Scale	D 785	2" x 1/8" Disc		80			80	
Shear Strength lbs/in² (kgf/cm²)	D 732	2" x 1/8" Disc	73 °F 120 °F 160 °F	7,700 6,700 5,700	8,300	23 °C 50 °C 70 °C	540 470 400	584
Water Absorption 24 – hr. Immersion %	D 570	2" x 1/8" Disc		0.22	0.29		0.22	0.29
Equilibrium, 50% R.H. %				0.16			0.16	
Equilibrium, Immersion				0.80			0.80	
Taper Abrasion 1000 g Load CS–17 Wheel	D 1044	4" x 4"		14mg per 1000 cycles			14mg per 1000 cycles	
Coefficient of Dynamic Friction • against steel, brass and aluminum • against Celcon	D 1894	3" x 4"		0.15 0.35			0.15 0.35	

Many of the properties of thermoplastics are dependent upon processing conditions, and the test results presented are typical values only. These test results were obtained under standardized test conditions, and with the exception of specific gravity, should not be used as a basis for engineering design. Values were obtained from specimens injection molded in unpigmented material. In common with other thermoplastics, incorporation into Celcon of color pigments or additional U.V. stabilizers may affect some test results. Celcon GC25A test results are obtained from material predried for 3 hours at 240 °F (116 °C) before molding. All values generated at 50% r.h. & 73 °F (23 °C) unless indicated otherwise. Reprinted with the permission of Celanese Plastics and Specialties Co.; see Reference 3.

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Table 18-7 Mechanical Properties of Nylon MC901 and Duracon M90

Properties	Testing Method ASTM	Unit	Nylon MC901	Duracon M90
Tensile Strength	D 638	kgf/cm^2	800 – 980	620
Elongation	D 638	%	10 – 50	60
Modules of Elasticity (Tensile)	D 638	kgf/cm^2	30 – 35	28.8
Yield Point (Compression)	D 695	kgf/cm^2	940 – 1050	—
5% Deformation Point	D 695	kgf/cm^2	940 – 970	—
Modules of Elasticity (Compress)	D 695	kgf/cm^2	33 – 36	—
Shearing Strength	D 732	kgf/cm^2	735 – 805	540
Rockwell Hardness	D 785	R scale	115 – 120	980
Bending Strength	D 790	kgf/cm^2	980 – 1120	980
Density (23°C)	D 792	g/cm^3	1.15 – 1.17	1.41
Poisson's Ratio	—	—	0.40	0.35

Table 18-8 Thermal Properties of Nylon MC901 and Duracon M90

Properties	Testing Method ASTM	Unit	Nylon MC901	Duracon M90
Thermal Conductivity	C 177	10^{-1} kcal/mhr°C	2	2
Coeff. of Linear Thermal Expansion	D 696	10^{-5} cm/cm/°C	9	9 – 13
Specifical Heat (20°C)	D 648	cal/°Cgrf	0.4	0.35
Thermal Deformation Temperature (18.5 kgf/cm^2)	D 648	°C	160 – 200	110
Thermal Deformation Temperature (4.6 kgf/cm^2)	D 621	°C	200 – 215	158
Antithermal Temperature (Long Term)		°C	120 – 150	—
Deformation Rate Under Load (140 kgf/cm^2, 50°C)		%	0.65	—
Melting Point		°C	220 – 223	165

Table 18-9 Water and Moisture Absorption Property of Nylon MC901 and Duracon M90

Conditions	Testing Method ASTM	Unit	Nylon MC901	Duracon M90
Rate of Water Absorption (at room temp. in water, 24 hrs.)	D 570	%	0.5 – 1.0	0.22
Saturation Absorption Value (in water)		%	5.5 – 7.0	0.80
Saturation Absorption Value (in air, room temp.)		%	2.5 – 3.5	0.16

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Table 18-10 Modulus of Elasticity, Coefficients of Thermal Expansion and Density of Materials

Material	Modulus of Elasticity (flexural) (lb/in.^{2})	Coefficient of Thermal Expansion (per ^{0}F)	Temperature Range of Coefficient (^{0}F)	Density (lb/in.^{3})
Ferrous Metals				
Cast Irons:				
Malleable	25 to 28 x 10^6	6.6 x 10^{-6}	68 to 750	.265
Gray cast	9 to 23 x 10^6	6.0 x 10^{-6}	32 to 212	.260
Ductile	23 to 25 x 10^6	8.2 x 10^{-6}	68 to 750	.259
Steels:				
Cast Steel	29 to 30 x 10^6	8.2 x 10^{-6}	68 to 1000	.283
Plain carbon	29 to 30 x 10^6	8.3 x 10^{-6}	68 to 1000	.286
Low alloy,cast and wrought	30 x 10^6	8.0 x 10^{-6}	0 to 1000	.280
High alloy	30 x 10^6	8 to 9 x 10^{-6}	68 to 1000	.284
Nitriding, wrought	29 to 30 x 10^6	6.5 x 10^{-6}	32 to 900	.286
AISI 4140	29 x 10^6	6.2 x 10^{-6}	32 to 212	.284
Stainless:				
AISI 300 series	28 x 10^6	9.6 x 10^{-6}	32 to 212	.287
AISI 400 series	29 x 10^6	5.6 x 10^{-6}	32 to 212	.280
Nonferrous Metals:				
Aluminum alloys, wrought	10 to 10.6 x 10^6	12.6 x 10^{-6}	68 to 212	.098
Aluminum, sand–cast	10.5 x 10^6	11.9 to 12.7 x 10^{-6}	68 to 212	.097
Aluminum, die–cast	10.3 x 10^6	11.4 to 12.2 x 10^{-6}	68 to 212	.096
Beryllium copper	18 x 10^6	9.3 x 10^{-6}	68 to 212	.297
Brasses	16 to 17 x 10^6	11.2 x 10^{-6}	68 to 572	.306
Bronzes	17 to 18 x 10^6	9.8 x 10^{-6}	68 to 572	.317
Copper, wrought	17 x 10^6	9.8 x 10^{-6}	68 to 750	.323
Magnesium alloys, wrought	6.5 x 10^6	14.5 x 10^{-6}	68 to 212	.065
Magnesium, die-cast	6.5 x 106	14 x 10^{-6}	68 to 212	.065
Monel	26 x 10^6	7.8 x 10^{-6}	32 to 212	.319
Nickel and alloys	19 to 30 x 10^6	7.6 x 10^{-6}	68 to 212	.302
Nickel, low-expansion alloys	24 x 10^6	1.2 to 5 x 10^{-6}	–200 to 400	.292
Titanium, unalloyed	15 to 16 x 10^6	5.8 x 10^{-6}	68 to 1650	.163
Titanium alloys, wrought	13 to 17.5 x 10^6	5.0 to 7 x 10^{-6}	68 to 572	.166
Zinc, die-cast	2 to 5 x 10^6	5.2 x 10^{-6}	68 to 212	.24
Powder Metals:				
Iron (unalloyed)	12 to 25 x 10^6	—	—	.21 to .27
Iron–carbon	13 x 10^6	7 x 10^{-6}	68 to 750	.22
Iron–copper–carbon	13 to 15 x 10^6	7 x 10^{-6}	68 to 750	.22
AISI 4630	18 to 23 x 10^6	—	—	.25
Stainless steels:				
AISI 300 series	15 to 20 x 10^6	—	—	.24
AISI 400 series	14 to 20 x 10^6	—	—	.23
Brass	10 x 10^6	—	—	.26
Bronze	8 to 13 x 10^6	10 x 10^{-6}	68 to 750	.28
Nonmetallics:				
Acrylic	3.5 to 4.5 x 10^5	3.0 to 4 x 10^{-5}	0 to 100	.043
Delrin (acetal resin)	4.1 x 10^5	5.5 x 10^{-5}	85 to 220	.051
Fluorocarbon resin (TFE)	4.0 to 6.5 x 10^4	5.5 x 10^{-5}	–22 to 86	.078
Nylon	1.6 to 4.5 x 10^5	4.5 to 5.5 x 10^{-5}	–22 to 86	.041
Phenolic laminate:				
Paper base	1.1 to 1.8 x 10^5	0.9 to 1.4 x 10^{-5}	–22 to 86	.048
Cotton base	0.8 to 1.3 x 10^5	0.7 to 1.5 x 10^{-5}	–22 to 86	.048
Linen base	0.8 to 1.1 x 10^5	0.8 to 1.4 x 10^{-5}	–22 to 86	.049
Polystyrene (general purpose)	4.0 to 5 x 10^5	3.3 to 4.4 x 10^{-5}	–22 to 86	.038

Source: Michalec, G.W., Precision Gearing, Wiley 1966

Table 18-11 Wear Characteristics of Plastics

Material	Steel	Brass	Polyurethane	Polycarbonate	MoS_2-Filled Nylon	Nylon 6/10	Nylon 6/6	Polystyrene	ABS	Acetal
Acetal	F	P	G	F	G	G	G	F	F	G
ABS	P	P	G	G	G	G	G	P	F	
Polystyrene	P	P	F	F	G	F	F	F		
Nylon 6-6	E	F	E	F	E	G	G			
Nylon 6-10	E	F	E	F	E	G				
MoS2-Filled Nylon	E	G	E	F	E					
Polycarbonate	G	F	G	G						
Polyurethane	E	F	G							
Brass	G	P								
Steel	F									

Key
E — Excellent
G — Good
F — Fair
P — Poor

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Table 18-12 Poisson's Ratio ν for Unfilled Thermoplastics

Polymer	ν
Acetal	0.35
Nylon 6/6	0.39
Modified PPO	0.38
Polycarbonate	0.36
Polystyrene	0.33
PVC	0.38
TFE (Tetrafluorethylene)	0.46
FEP (Fluorinated Ethylene Propylene)	0.48

Source: Clifford E. Adams, Plastic Gearing,
Marcel Dekker Inc., New York 1986. Reference 1.

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Table 18-13 Material Ranking by Water Absorption Rate

Material	Rate of Change %
Polytetrafluoroethylene	0.0
Polyethylene: medium density	< 0.01
high density	< 0.01
high molecular weight	< 0.01
low density	< 0.015
Polyphenylene sulfides (40% glass filled)	0.01
Polyester: thermosetting and alkyds	
low shrink	0.01 – 0.25
glass – preformed chopping roving	0.01 – 1.0
Polyester: linear aromatic	0.02
Polyphenylene sulfide: unfilled	0.02
Polyester: thermoplastic (18% glass)	0.02 – 0.07
Polyurethane: cast liquid methane	0.02 – 1.5
Polyester synthetic: fiber filled – alkyd	0.05 – 0.20
glass filled – alkyd	0.05 – 0.25
mineral filled – alkyd	0.05 – 0.50
glass–woven cloth	0.05 – 0.50
glass–premix, chopped	0.06 – 0.28
Nylon 12 (30% glass)	0.07
Polycarbonate (10–40% glass)	0.07 – 0.20
Styrene–acrylonitrile copolymer (20–33% glass filled)	0.08 – 0.22
Polyester thermoplastic:	0.09
thermoplastic PTMT (20% asbestos)	0.10
glass sheet molding	0.10 – 0.15
Polycarbonate <10%glass	0.12
Phenolic cast: mineral filled	0.12 – 0.36
Polyester alkyd: asbestos filled	0.14
Polycarbonate: unfilled	0.15 – 0.18
Polyester cast: rigid	0.15 – 0.60
Acetal: TFE	0.20
Nylon 6/12 (30–35% glass)	0.20
6/10 (30–35% glass)	0.20
Polyester alkyd vinyl ester thermoset	0.20
Styrene–acrylonitrile copolymer: unfilled	0.20 – 0.30
Polycarbonate ABS alloy	0.20 – 0.35
Phenolic cast: unfilled	0.20 – 0.40
Acetal copolymer	0.22
homopolymer	0.25
Nylon 12 (unmodified)	0.25
Acetal (20% glass)	0.25 – 0.29
Poly (ancide–imide)	0.28
Acetal (25% glass)	0.29
Nylon 11 (unmodified)	0.30
Polyester elastomer	0.30 – 0.60
Polyamide	0.32
Nylon: 6/12 (unmodified)	0.40
6/10 (unmodified)	0.40
Polyester-thermosetting and alkyds (cast flexible)	0.50 – 2.50
Nylon 6 (cast)	0.60 – 1.20
Polyurethane elastomer thermoplastic	0.70 – 0.90
Nylon 6/6: MoS2	0.80 – 1.10
30 – 35% glass	0.90
unmodified	1.10 – 1.50
nucleated	1.10 – 1.50
Nylon 6 (30 – 35% glass)	1.30
unmodified	1.30 – 1.90
nucleated	1.30 – 1.90
Nylon 6/6 – 6 (copolymer)	1.50 – 1.20

Source: Clifford E. Adams, Plastic Gearing, Marcel Dekker, Inc., New York, 1986. Reference 1.

Table 18-14 Safe Stress

Plastic	Safe stress, psi	
	Unfilled	Glass-reinforced
ABS Resins	3000	6000
Acetal	5000	7000
Nylon	6000	12000
Polycarbonate	6000	9000
Polyester	3500	8000
Polyurethane	2500	

Source: Clifford E. Adams, Plastic Gearing,
Marcel Dekker Inc.,New York 1986. Reference 1.

It is important to stress the resistance to chemical corrosion of some plastic materials. These properties of some of materials used in the products presented in this catalog are further explored.

Nylon MC901

Nylon MC901 has almost the same level of antichemical corrosion property as Nylon resins. In general, it has a better antiorganic solvent property, but has a weaker antiacid property. The properties are as follows:

- For many nonorganic acids, even at low concentration at normal temperature, it should not be used without further tests.
- For nonorganic alkali at room temperature, it can be used to a certain level of concentration.
- For the solutions of nonorganic salts, we may apply them to a fairly high level of temperature and concentration.
- MC901 has better antiacid ability and stability in organic acids than in nonorganic acids, except for formic acid.
- MC901 is stable at room temperature in organic compounds of ester series and ketone series.
- It is also stable in mineral oil, vegetable oil and animal oil, at room temperature.

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Duracon M90

This plastic has outstanding antiorganic properties. However, it has the disadvantage of having limited suitable adhesives. Its main properties are:

- Good resistance against nonorganic chemicals, but will be corroded by strong acids such as nitric, sulfuric and chloric acids.
- Household chemicals, such as synthetic detergents, have almost no effect on M90.
- M90 does not deteriorate even under long term operation in high temperature lubricating oil, except for some additives in high grade lubricants.
- With grease, M90 behaves the same as with oil lubricants.

Gear designers interested in using this material should be aware of properties regarding individual chemicals. Plastic manufacturers' technical information manuals should be consulted prior to making gear design decisions.

18.3 Choice Of Pressure Angles And Modules

Pressure angles of 14.5°, 20° and 25° are used in plastic gears. The 20° pressure angle is usually preferred due to its stronger tooth shape and reduced undercutting compared to the 14.5° pressure angle system. The 25° pressure angle has the highest load-carrying ability, but is more sensitive to center distance variation and hence runs less quietly. The choice is dependent on the application.

The determination of the appropriate module or diametral pitch is a compromise between a number of different design requirements. A larger module is associated with larger and stronger teeth. For a given pitch diameter, however, this also means a smaller number of teeth with a correspondingly greater likelihood of undercut at very low number of teeth. Larger teeth are generally associated with more sliding than smaller teeth.

On the other hand, smaller modules, which are associated with smaller teeth, tend to provide greater load sharing due to the compliance of plastic gears. However, a limiting condition would eventually be reached when mechanical interference occurs as a result of too much compliance. Smaller teeth are also more sensitive to tooth errors and may be more highly stressed.

A good procedure is probably to size the pinion first, since it is the more highly loaded member. It should be proportioned to support the required loads, but should not be over designed.

18.4 Strength Of Plastic Spur Gears

In the following text, main consideration will be given to Nylon MC901 and Duracon M90. However, the basic equations used are applicable to all other plastic materials if the appropriate values for the factors are applied.

18.4.1 Bending Strength of Spur Gears

Nylon MC901

The allowable tangential force F (kgf) at the pitch circle of a Nylon MC901 spur gear can be obtained from the Lewis formula.

$$F = myb\,\sigma_b \ \text{(kgf)} \qquad \textbf{(18-1)}$$

where:

- m = Module (mm)
- y = Form factor at pitch point (see **Table 18-15**)
- b = Teeth width (mm)
- σ_b = Allowable bending stress (kgf/mm^2) (see **Figure 18-1**)

Duracon M90

The allowable tangential force F (kgf) at pitch circle of a Duracon M90 spur gear can also be obtained from the Lewis formula.

$$F = myb\,\sigma_b \ \text{(kgf)} \qquad \textbf{(18-2)}$$

where:

- m = Module (mm)
- y = Form factor at pitch point (see **Table 18-15**)
- b = Teeth width (mm)
- σ_b = Allowable bending stress (kgf/mm^2)

The allowable bending stress can be calculated by **Equation (18-3)**:

$$\sigma_b = \sigma_b' \frac{K_V K_T K_L K_M}{C_S} \qquad \textbf{(18-3)}$$

where:

- σ_b' = Maximum allowable bending stress under ideal condition (kgf/mm^2) (see **Figure 18-2**)
- C_S = Working factor (see **Table 18-17**)
- K_V = Speed factor (see **Figure 18-3**)
- K_T = Temperature factor (see **Figure 18-4**)
- K_L = Lubrication factor (see **Table 18-18**)
- K_M = Material factor (see **Table 19-19**)

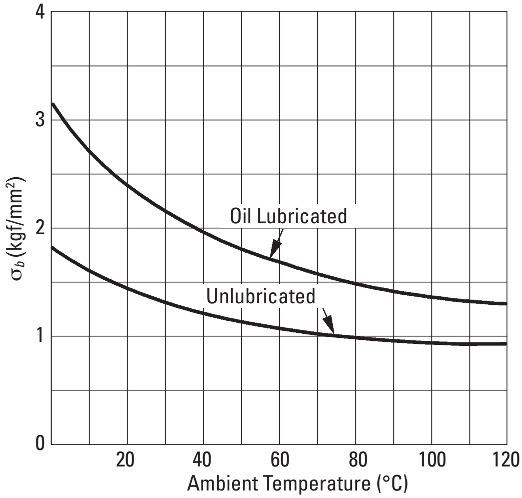

Fig. 18-1 Allowable Bending Stress, σ_b (kgf/mm^2)

Table 18-15 Form Factor, *y*

Number of Teeth	Form Factor		
	14.5°	**20° Standard Tooth**	**20° Stub Tooth**
12	0.355	0.415	0.496
14	0.399	0.468	0.540
16	0.430	0.503	0.578
18	0.458	0.522	0.603
20	0.480	0.544	0.628
22	0.496	0.559	0.648
24	0.509	0.572	0.664
26	0.522	0.588	0.678
28	0.535	0.597	0.688
30	0.540	0.606	0.698
34	0.553	0.628	0.714
38	0.565	0.651	0.729
40	0.569	0.657	0.733
50	0.588	0.694	0.757
60	0.604	0.713	0.774
75	0.613	0.735	0.792
100	0.622	0.757	0.808
150	0.635	0.779	0.830
300	0.650	0.801	0.855
Rack	0.660	0.823	0.881

Table 18-16 Speed Factor, K_V

Lubrication	Tangential Speed (m/sec)	Factor K_V
Lubricated	Under 12	1.0
	Over 12	0.85
Unlubricated	Under 5	1.0
	Over 5	0.7

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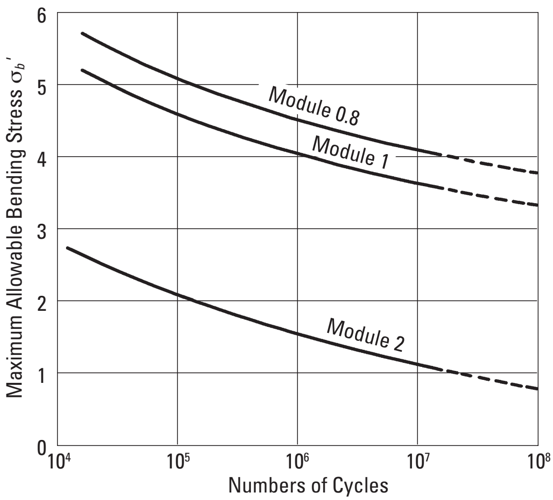

Fig. 18-2 Maximum Allowable Bending Stress under Ideal Condition, σ_b' (kgf/mm^2)

Table 18-17 Working Factor, C_S

Types of Load	Daily Operating Hours			
	24 hrs. / day	8-10 hrs. / day	3 hrs. / day	0.5 hrs. / day
Uniform Load	1.25	1.00	0.80	0.50
Light Impact	1.50	1.25	1.00	0.80
Medium impact	1.75	1.50	1.25	1.00
Heavy Impact	2.00	1.75	1.50	1.25

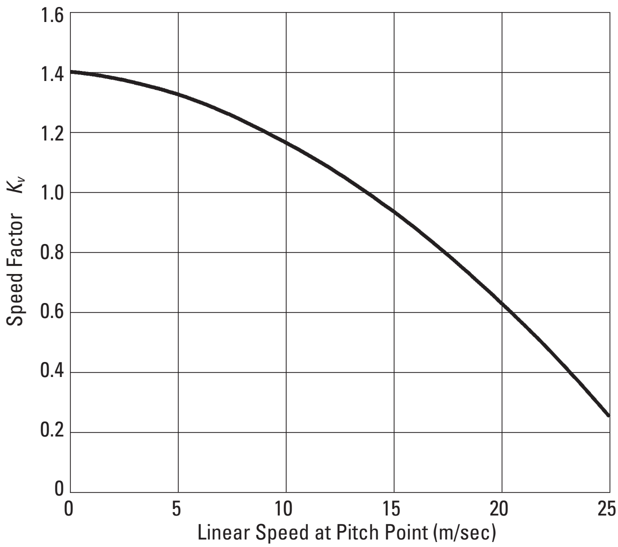

Fig. 18-3 Speed Factor, K_V

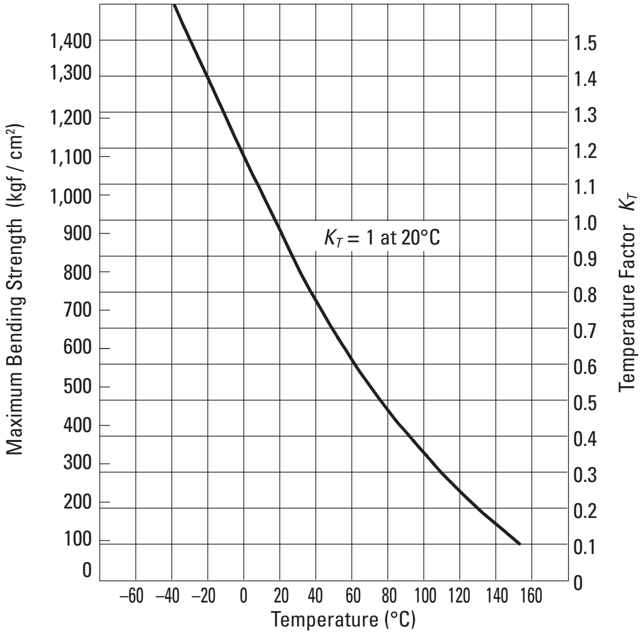

Fig. 18-4 Temperature Factor, K_T

Table 18-18 Lubrication Factor, K_L

Lubrication	K_L
Initial Grease Lubrication	1
Continuous Oil Lubrication	1.5 – 3.0

Table 18-19 Material Factor, K_M

Material Combination	K_M
Duracon vs. Metal	1
Duracon vs. Duracon	0.75

Application Notes

In designing plastic gears, the effects of heat and moisture must be given careful consideration. The related problems are:

1. Backlash

Plastic gears have larger coefficients of thermal expansion. Also, they have an affinity to absorb moisture and swell. Good design requires allowance for a greater amount of backlash than for metal gears.

2. Lubrication

Most plastic gears do not require lubrication. However, temperature rise due to meshing may be controlled by the cooling effect of a lubricant as well as by reduction of friction. Often, in the case of high-speed rotational speeds, lubrication is critical.

3. Plastic gear with metal mate

If one of the gears of a mated pair is metal, there will be a heat sink that combats a high temperature rise. The effectiveness depends upon the particular metal, amount of metal mass, and rotational speed.

18.4.2 Surface Strength of Plastic Spur Gears

Duracon M90

Duracon gears have less friction and wear when in an oil lubrication condition. However, the calculation of strength must take into consideration a no-lubrication condition. The surface strength using Hertz contact stress, S_c, is calculated by **Equation (18-4)**.

$$S_c = \sqrt{\frac{F}{bd_1} \frac{u+1}{u}} \cdot \sqrt{\frac{1.4}{\left(\frac{1}{E_1} + \frac{1}{E_2}\right) \sin 2\alpha}} \quad (\text{kgf/mm}^2) \qquad \textbf{(18-4)}$$

where:

F = Tangential force on surface (kgf)
b = Tooth width (mm)
d_1 = Pitch diameter of pinion (mm)
u = Gear ratio = z_2/z_1
E = Modulus of elasticity of material (kgf/mm^2) (see **Figure 18-5**)
α = Pressure angle

If the value of Hertz contact stress, S_c, is calculated by **Equation (18-4)** and the value falls below the curve of **Figure 18-6**, then it is directly applicable as a safe design. If the calculated value falls above the curve, the Duracon gear is unsafe.

Figure 18-6 is based upon data for a pair of Duracon gears: $m = 2$, $v = 12$ m/s, and operating at room temperature. For working conditions that are similar or better, the values in the figure can be used.

18.4.3 Bending Strength Of Plastic Bevel Gears

Nylon MC901

The allowable tangential force at the pitch circle is calculated by **Equation (18-5)**.

$$F = m \frac{R_a - b}{R_a} y b \sigma_b \quad (\text{kgf}) \qquad \textbf{(18-5)}$$

$$z_V = \frac{z}{\cos \delta} \qquad \textbf{(18-6)}$$

where:

m = module (mm)
R_a = Outer cone distance (mm)
b = Tooth width (mm)
y = Form factor at pitch point, which is obtained from **Table 18-15** by computing the number of teeth of equivalent spur gear via **Equation (18-6)**.
σ_b = Allowable bending stress
z_V = Number of teeth of equivalent spur gear
δ = Pitch cone angle (degree)

Other variables may be calculated the same way as for spur gears.

Duracon M90

The allowable tangential force F (kgf) on pitch circle of Duracon M90 bevel gears can be obtained from **Equation (18-7)**.

$$F = m \frac{R_a - b}{R_a} y b \sigma_b \quad (\text{kgf}) \qquad \textbf{(18-7)}$$

and y = Form factor at pitch point, which is obtained from **Table 18-15** by first computing the number of teeth of equivalent spur gear using **Equation (18-6)**.

Other variables may be calculated the same way as for spur gears.

18.4.4 Bending Strength Of Plastic Worm Gears

Nylon MC901

Generally, the worm is much stronger than the worm gear. Therefore, it is necessary to calculate the strength of only the worm gear.

The allowable tangential force F (kgf) at the pitch circle of the worm gear is obtained from **Equation (18-8)**.

$$F = m_n y b \sigma_b \quad \text{(kgf)} \qquad \textbf{(18-8)}$$

$$z_V = \frac{z}{\cos^3 \gamma} \qquad \textbf{(18-9)}$$

where:
- m_n = Normal module (mm)
- y = Form factor at pitch point, which is obtained from **Table 18-15** by first computing the number of teeth of equivalent spur gear using **Equation (18-9)**.
- z_V = Number of teeth of equivalent spur gear
- γ = Lead angle

Worm meshes have relatively high sliding velocities, which induces a high temperature rise. This causes a sharp decrease in strength and abnormal friction wear. This is particularly true of an all plastic mesh. Therefore, sliding speeds must be contained within recommendations of **Table 18-20**.

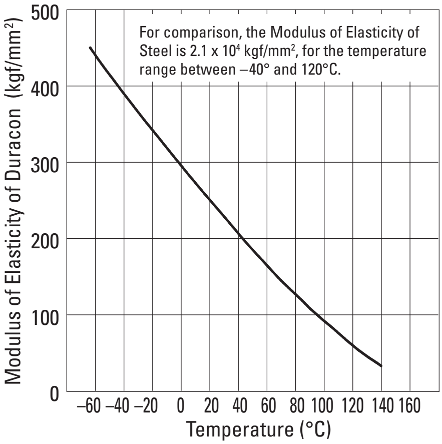

Fig. 18-5 Modulus of Elasticity in Bending of Duracon

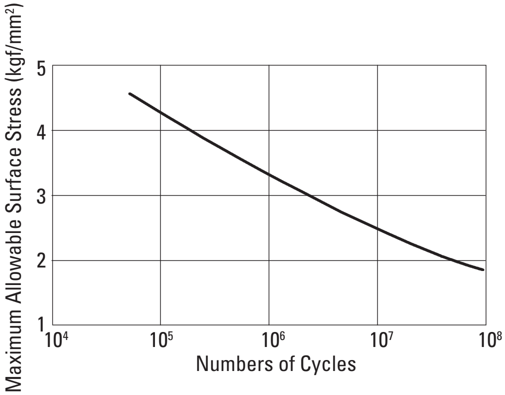

Fig. 18-6 Maximum Allowable Surface Stress (Spur Gears)

Table 18-20 Material Combinations and Limits of Sliding Speed

Material of Worm	Material of Worm Gear	Lubrication Condition	Sliding Speed
"MC" Nylon	"MC" Nylon	No Lubrication	Under 0.125 m/s
Steel	"MC" Nylon	No Lubrication	Under 1 m/s
Steel	"MC" Nylon	Initial Lubrication	Under 1.5 m/s
Steel	"MC" Nylon	Continuous Lubrication	Under 2.5 m/s

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Sliding speed $v_s = \dfrac{\pi\ d_1 n_1}{60000 \cos\gamma}$ (m/s)

Lubrication of plastic worms is vital, particularly under high load and continuous operation.

18.4.5 Strength Of Plastic Keyway

Fastening of a plastic gear to the shaft is often done by means of a key and keyway. Then, the critical thing is the stress level imposed upon the keyway sides. This is calculated by **Equation (18-10)**.

$$\sigma = \frac{200\,T}{d\,l h} \quad (\text{kgf/cm}^2) \qquad \textbf{(18-10)}$$

where:
- σ = Pressure on the keyway sides (kgf/cm^2)
- T = Transmitted torque (kgf • m)
- d = Diameter of shaft (cm)
- l = Effective length of keyway (cm)
- h = Depth of keyway (cm)

The maximum allowable surface pressure for MC901 is 200 kgf/cm^2, and this must not be exceeded. Also, the keyway's corner must have a suitable radius to avoid stress concentration. The distance from the root of the gear to the bottom of the keyway should be at least twice the tooth whole depth, *h*.

Keyways are not to be used when the following conditions exist:
- Excessive keyway stress
- High ambient temperature
- High impact
- Large outside diameter gears

When above conditions prevail, it is expedient to use a metallic hub in the gear. Then, a keyway may be cut in the metal hub.

A metallic hub can be fixed in the plastic gear by several methods:
- Press the metallic hub into the plastic gear, ensuring fastening with a knurl or screw.
- Screw fasten metal discs on each side of the plastic gear.
- Thermofuse the metal hub to the gear.

18.5 Effect Of Part Shrinkage On Plastic Gear Design

The nature of the part and the molding operation have a significant effect on the molded gear. From the design point of view, the most important effect is the shrinkage of the gear relative to the size of the mold cavity.

Gear shrinkage depends upon mold proportions, gear geometry, material, ambient temperature and time. Shrinkage is usually expressed in millimeters per millimeter. For example, if a plastic gear with a shrinkage rate of 0.022 mm/mm has a pitch diameter of 50 mm while in the mold, the pitch diameter after molding will be reduced by (50)(0.022) or 1.1 mm, and becomes 48.9 mm after it leaves the mold.

Depending upon the material and the molding process, shrinkage rates ranging from about 0.001 mm/mm to 0.030 mm/mm occur in plastic gears (see **Table 18-1** and **Figure 18-7**). Sometimes shrinkage rates are expressed as a percentage. For example, a shrinkage rate of 0.0025 mm/mm can be stated as a 0.25% shrinkage rate.

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The effect of shrinkage must be anticipated in the design of the mold and requires expert knowledge. Accurate and specific treatment of this phenomenon is a result of years of experience in building molds for gears; hence, details go beyond the scope of this presentation.

In general, the final size of a molded gear is a result of the following factors:

1. Plastic material being molded.
2. Injection pressure.
3. Injection temperature.
4. Injection hold time.
5. Mold cure time and mold temperature.
6. Configuration of part (presence of web, insert, spokes, ribs, etc.).
7. Location, number and size of gates.
8. Treatment of part after molding.

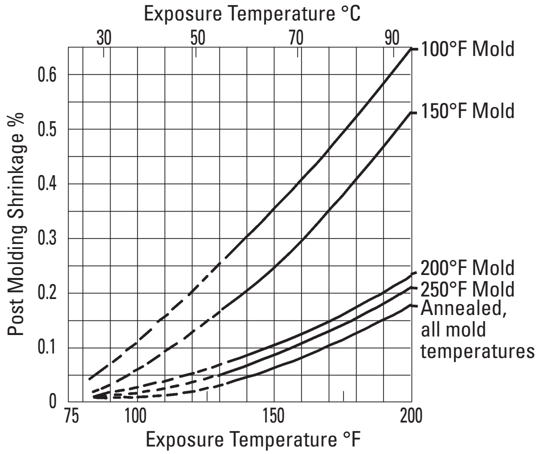

Fig. 18-7 Shrinkage for Delrin in Air
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From the above, it becomes obvious that with the same mold – by changing molding parameters – parts of different sizes can be produced.

The form of the gear tooth itself changes as a result of shrinkage, irrespective of it shrinking away from the mold, as shown in **Figure 18-8**. The resulting gear will be too thin at the top and too thick at the base. The pressure angle will have increased, resulting in the possibility of binding, as well as greater wear.

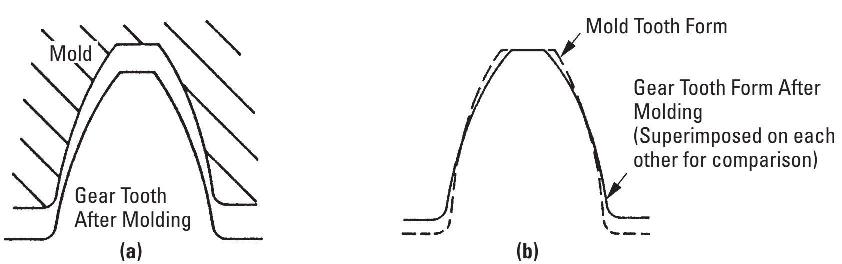

Fig. 18-8 Change of Tooth Profile

In order to obtain an idea of the effect of part shrinkage subsequent to molding, the following equations are presented where the primes refer to quantities after the shrinkage occurred:

$$\cos\alpha' = \frac{\cos\alpha}{1 + s^*} \qquad \textbf{(18-11)}$$

$$m' = (1 - s^*)m \qquad \textbf{(18-12)}$$

$$d' = zm' \qquad \textbf{(18-13)}$$

$$p' = \pi m' \qquad \textbf{(18-14)}$$

where:
s^* = shrinkage rate (mm/mm)
m = module
α = pressure angle
d = pitch diameter (mm)
p' = circular pitch (mm)
z = number of teeth

It follows that a hob generating the electrode for a cavity which will produce a post shrinkage standard gear would need to be of a nonstandard configuration.

Let us assume that an electrode is cut for a 20° pressure angle, module 1, 64 tooth gear which will be made of acetal ($s^* = 0.022$) and will have 64 mm pitch diameter after molding.

$$\cos\alpha = \cos\alpha'(1 + s^*) = 0.93969262\ (1 + 0.022) = 0.96036$$

therefore, $\alpha = 16°11'$ pressure angle

$$m = \frac{m'}{1 - s^*} = \frac{1}{1 - 0.022} = 1.0225$$

The pitch diameter of the electrode, therefore, will be:

$$d = zm = 64 \times 1.0225 = 65.44 \text{ mm}$$

For the sake of simplicity, we are ignoring the correction which has to be made to compensate for the electrode gap which results in the cavity being larger than the electrode.

The shrinking process can give rise to residual stresses within the gear, especially if it has sections of different thicknesses. For this reason, a hubless gear is less likely to be warped than a gear with a hub.

If necessary, a gear can be annealed after molding in order to relieve residual stresses. However, since this adds another operation in the manufacturing of the gear, annealing should be considered only under the following circumstances:

1. If maximum dimensional stability is essential.
2. If the stresses in the gear would otherwise exceed the design limit.
3. If close tolerances and high-temperature operation makes annealing necessary.

Annealing adds a small amount of lubricant within the gear surface region. If the prior gear lubrication is marginal, this can be helpful.

18.6 Proper Use Of Plastic Gears

18.6.1 Backlash

Due to the thermal expansion of plastic gears, which is significantly greater than that of metal gears, and the effects of tolerances, one should make sure that meshing gears do not bind in the course of service. Several means are available for introducing backlash into the system. Perhaps the simplest is to enlarge center distance. Care must be taken, however, to ensure that the contact ratio remains adequate.

It is possible also to thin out the tooth profile during manufacturing, but this adds to the manufacturing cost and requires careful consideration of the tooth geometry.

To some extent, the flexibility of the bearings and clearances can compensate for thermal expansion. If a small change in center distance is necessary and feasible, it probably represents the best and least expensive compromise.

18.6.2 Environment and Tolerances

In any discussion of tolerances for plastic gears, it is necessary to distinguish between manufacturing tolerances and dimensional changes due to environmental conditions.

As far as manufacturing is concerned, plastic gears can be made to high accuracy, if desired. For injection molded gears, Total Composite Error can readily be held within a range of roughly 0.075 – 0.125 mm, with a corresponding Tooth-to-Tooth Composite Error of about 0.025 – 0.050 mm. Higher accuracies can be obtained if the more expensive filled materials, mold design, tooling and quality control are used.

In addition to thermal expansion changes, there are permanent dimensional changes as the result of moisture absorption. Also, there are dimensional changes due to compliance under load. The coefficient of thermal expansion of plastics is on the order of four to ten times those of metals (see **Tables 18-3** and **18-10**). In addition, most plastics are hygroscopic (i.e., absorb moisture) and dimensional changes on the order of 0.1% or more can develop in the course of time, if the humidity is sufficient. As a result, one should attempt to make sure that a tolerance which is specified is not smaller than the inevitable dimensional changes which arise as a result of environmental conditions. At the same time, the greater compliance of plastic gears, as compared to metal gears, suggests that the necessity for close tolerances need not always be as high as those required for metal gears.

18.6.3 Avoiding Stress Concentration

In order to minimize stress concentration and maximize the life of a plastic gear, the root fillet radius should be as large as possible, consistent with conjugate gear action. Sudden changes in cross section and sharp corners should be avoided, especially in view of the possibility of additional residual stresses which may have occurred in the course of the molding operation.

18.6.4 Metal Inserts

Injection molded metal inserts are used in plastic gears for a variety of reasons:

1. To avoid an extra finishing operation.
2. To achieve greater dimensional stability, because the metal will shrink less and is not sensitive to moisture; it is, also, a better heat sink.
3. To provide greater load-carrying capacity.
4. To provide increased rigidity.
5. To permit repeated assembly and disassembly.
6. To provide a more precise bore to shaft fit.

Inserts can be molded into the part or subsequently assembled. In the case of subsequent insertion of inserts, stress concentrations may be present which may lead to cracking of the parts. The interference limits for press fits must be obeyed depending on the material used; also, proper minimum wall thicknesses around the inserts must be left. The insertion of inserts may be accomplished by ultrasonically driving in the insert. In this case, the material actually melts into the knurling at the insert periphery.

Inserts are usually produced by screw machines and made of aluminum or brass. It is advantageous to attempt to match the coefficient of thermal expansion of the plastic to the materials used for inserts. This will reduce the residual stresses in the plastic part of the gear during contraction while cooling after molding.

When metal inserts are used, generous radii and fillets in the plastic gear are recommended to avoid stress concentration. It is also possible to use other types of metal inserts, such as self-threading, self-tapping screws, press fits and knurled inserts. One advantage of the first two of these is that they permit repeated assembly and disassembly without part failure or fatigue.

18.6.5 Attachment Of Plastic Gears to Shafts

Several methods of attaching gears to shafts are in common use. These include splines, keys, integral shafts, set screws, and plain and knurled press fits. **Table 18-21** lists some of the basic characteristics of each of these fastening methods.

Table 18-21 Characteristics of Various Shaft Attachment Methods

Nature of Gear-Shaft Connection	Torque Capacity	Cost	Disassembly	Comments
Set Screw	Limited	Low	Not good unless threaded metal insert is used	Questionable reliability, particularly under vibration or reversing drive
Press fit	Limited	Low	Not possible	Residual stresses need to be considered
Knurled Shaft Connection	Fair	Low	Not possible	A permanent assembly
Spline	Good	High	Good	Suited for close tolerance
Key	Good	Reasonably Low	Good	Requires good fits
Integral Shaft	Good	Low	Not Possible	Bending load on shaft needs to be watched

18.6.6 Lubrication

Depending on the application, plastic gears can operate with continuous lubrication, initial lubrication, or no lubrication. According to L.D. Martin ("Injection Molded Plastic Gears", Plastic Design and Processing, 1968; Part 1, August, pp 38-45; Part 2, September, pp. 33-35):

1. All gears function more effectively with lubrication and will have a longer service life.
2. A light spindle oil (SAE 10) is generally recommended as are the usual lubricants; these include silicone and hydrocarbon oils, and in some cases cold water is acceptable as well.

3. Under certain conditions, dry lubricants such as molybdenum disulfide, can be used to reduce tooth friction.

Ample experience and evidence exist substantiating that plastic gears can operate with a metal mate without the need of a lubricant, as long as the stress levels are not exceeded. It is also true that in the case of a moderate stress level, relative to the materials rating, plastic gears can be meshed together without a lubricant. However, as the stress level is increased, there is a tendency for a localized plastic-to-plastic welding to occur, which increases friction and wear. The level of this problem varies with the particular type of plastic.

A key advantage of plastic gearing is that, for many applications, running dry is adequate. When a situation of stress and shock level is uncertain, using the proper lubricant will provide a safety margin and certainly will cause no harm. The chief consideration should be in choosing a lubricant's chemical compatibility with the particular plastic. Least likely to encounter problems with typical gear oils and greases are: nylons, Delrins (acetals), phenolics, polyethylene and polypropylene. Materials requiring caution are: polystyrene, polycarbonates, polyvinyl chloride and ABS resins.

An alternate to external lubrication is to use plastics fortified with a solid state lubricant. Molybdenum disulfide in nylon and acetal are commonly used. Also, graphite, colloidal carbon and silicone are used as fillers.

In no event should there be need of an elaborate sophisticated lubrication system such as for metal gearing. If such a system is contemplated, then the choice of plastic gearing is in question. Simplicity is the plastic gear's inherent feature.

18.6.7 Molded vs. Cut Plastic Gears

Although not nearly as common as the injection molding process, both thermosetting and thermoplastic plastic gears can be readily machined. The machining of plastic gears can be considered for high precision parts with close tolerances and for the development of prototypes for which the investment in a mold may not be justified.

Standard stock gears of reasonable precision are produced by using blanks molded with brass inserts, which are subsequently hobbed to close tolerances.

When to use molded gears vs. cut plastic gears is usually determined on the basis of production quantity, body features that may favor molding, quality level and unit cost. Often, the initial prototype quantity will be machine cut, and investment in molding tools is deferred until the product and market is assured. However, with some plastics this approach can encounter problems.

The performance of molded vs. cut plastic gears is not always identical. Differences occur due to subtle causes. Bar stock and molding stock may not be precisely the same. Molding temperature can have an effect. Also, surface finishes will be different for cut vs. molded gears. And finally, there is the impact of shrinkage with molding which may not have been adequately compensated.

18.6.8 Elimination of Gear Noise

Incomplete conjugate action and/or excessive backlash are usually the source of noise. Plastic molded gears are generally less accurate than their metal counterparts. Furthermore, due to the presence of a larger Total Composite Error, there is more backlash built into the gear train.

To avoid noise, more resilient material, such as urethane, can be used. **Figure 18-9** shows several gears made of urethane which, in mesh with Delrin gears, produce a practically noiseless gear train. The face width of the urethane gears must be increased correspondingly to compensate for lower load carrying ability of this material.

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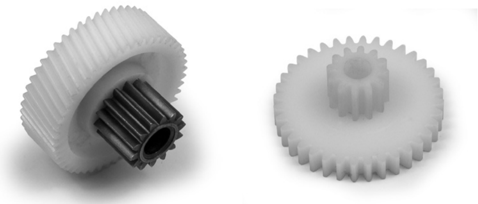


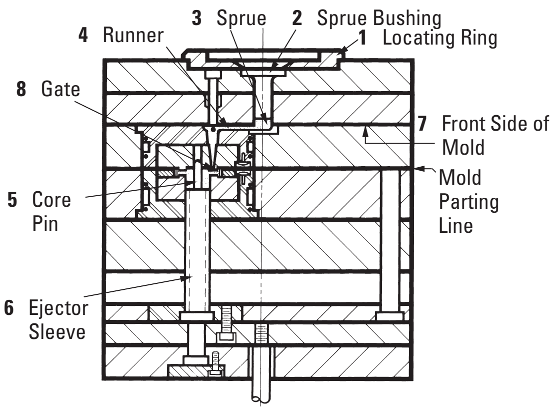
Fig. 18-9 Gears Made of Urethane

18.7 Mold Construction

Depending on the quantity of gears to be produced, a decision has to be made to make one single cavity or a multiplicity of identical cavities. If more than one cavity is involved, these are used as "family molds" inserted in mold bases which can accommodate a number of cavities for identical or different parts.

Since special terminology will be used, we shall first describe the elements shown in **Figure 18-10**.

1. **Locating Ring** is the element which assures the proper location of the mold on the platen with respect to the nozzle which injects the molten plastic.
2. **Sprue Bushing** is the element which mates with the nozzle. It has a spherical or flat receptacle which accurately mates with the surface of the nozzle.
3. **Sprue** is the channel in the sprue bushing through which the molten plastic is injected.
4. **Runner** is the channel which distributes material to different cavities within the same mold base.
5. **Core Pin** is the element which, by its presence, restricts the flow of plastic; hence, a hole or void will be created in the molded part.
6. **Ejector Sleeves** are operated by the molding machine. These have a relative motion with respect to the cavity in the direction which will cause ejection of the part from the mold.
7. **Front Side** is considered the side on which the sprue bushing and the nozzle are located.
8. **Gate** is the orifice through which the molten plastic enters the cavity.
9. **Vent** (not visible due to its small size) is a minuscule opening through which the air can be evacuated from the cavity as the molten plastic fills it. The vent is configured to let air escape, but does not fill up with plastic.



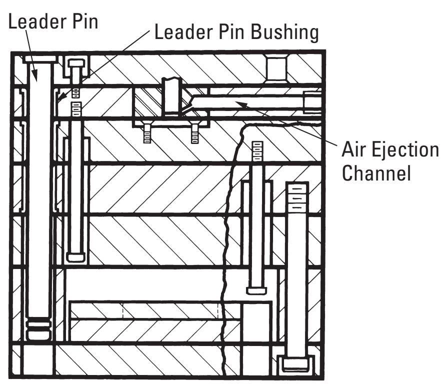


Fig. 18-10 Mold Nomenclature

The location of the gate on the gear is extremely important. If a side gate is used, as shown in **Figure 18-11**, the material is injected in one spot and from there it flows to fill out the cavity. This creates a weld line opposite to the gate. Since the plastic material is less fluid at that point in time, it will be of limited strength where the weld is located.

Furthermore, the shrinkage of the material in the direction of the flow will be different from that perpendicular to the flow. As a result, a side-gated gear or rotating part will be somewhat elliptical rather than round.

In order to eliminate this problem, "diaphragm gating" can be used, which will cause the injection of material in all directions at the same time **(Figure 18-12)**. The disadvantage of this method is the presence of a burr at the hub and no means of support of the core pin because of the presence of the sprue.

The best, but most elaborate, way is "multiple pin gating" **(Figure 18-13)**. In this case, the plastic is injected at several places symmetrically located. This will assure reasonable viscosity of plastic when the material welds, as well as create uniform shrinkage in all directions. The problem is the elaborate nature of the mold arrangement – so called 3-plate molds, in **Figure 18-14** – accompanied by high costs. If precision is a requirement, this way of molding is a must, particularly if the gears are of a larger diameter.

To compare the complexity of a 3-plate mold with a 2-plate mold, which is used for edge gating, **Figure 18-15** can serve as an illustration.

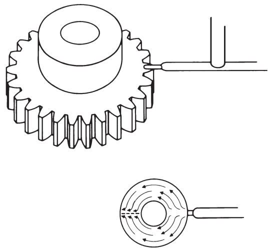


Fig. 18-11 Side Gating

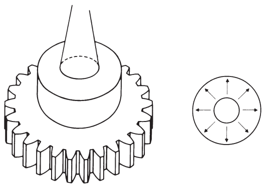


Fig. 18-12 Diaphragm Gating

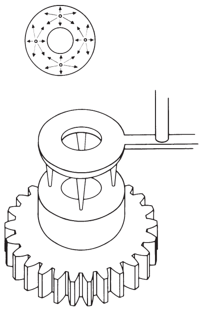
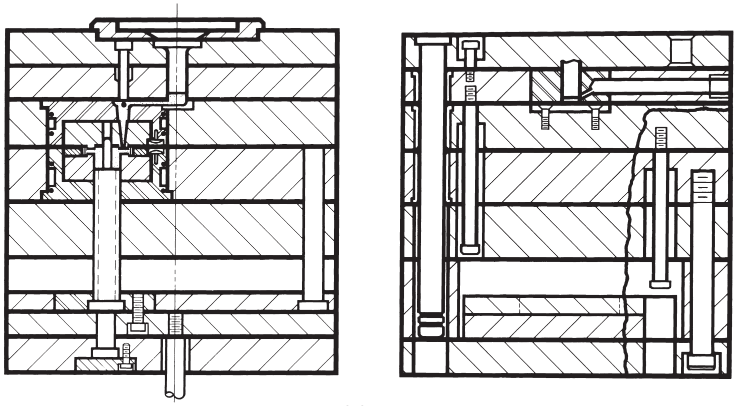
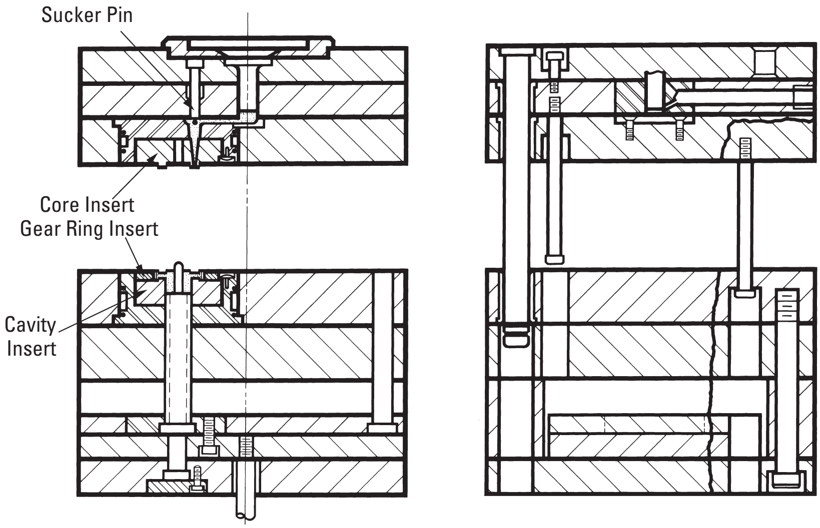


Fig. 18-13 Multiple Pin Gating



(a) Mold Closed



(b) Gates Separated from Molded Parts

Fig. 18-14 Three-Plate Mold

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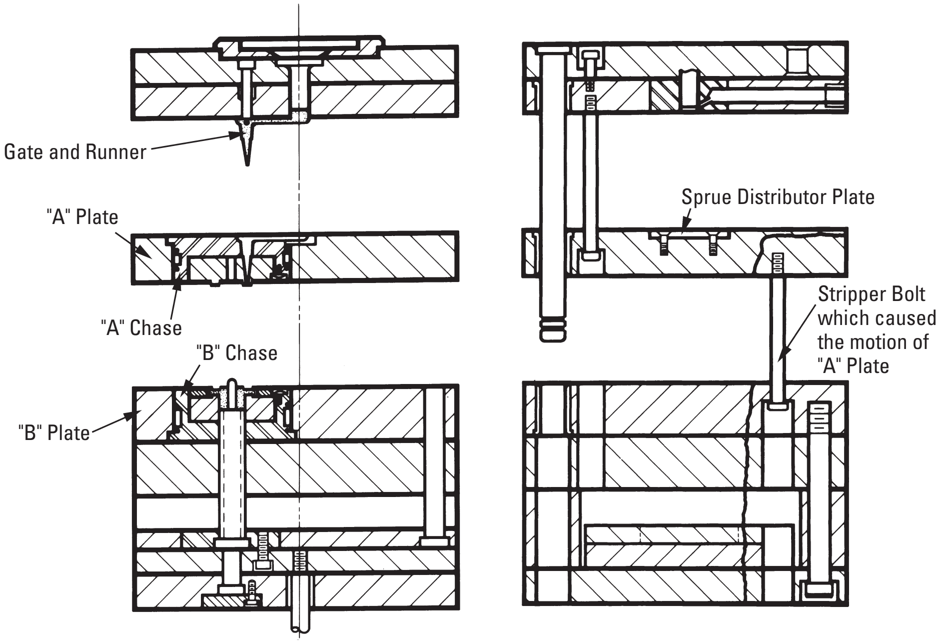
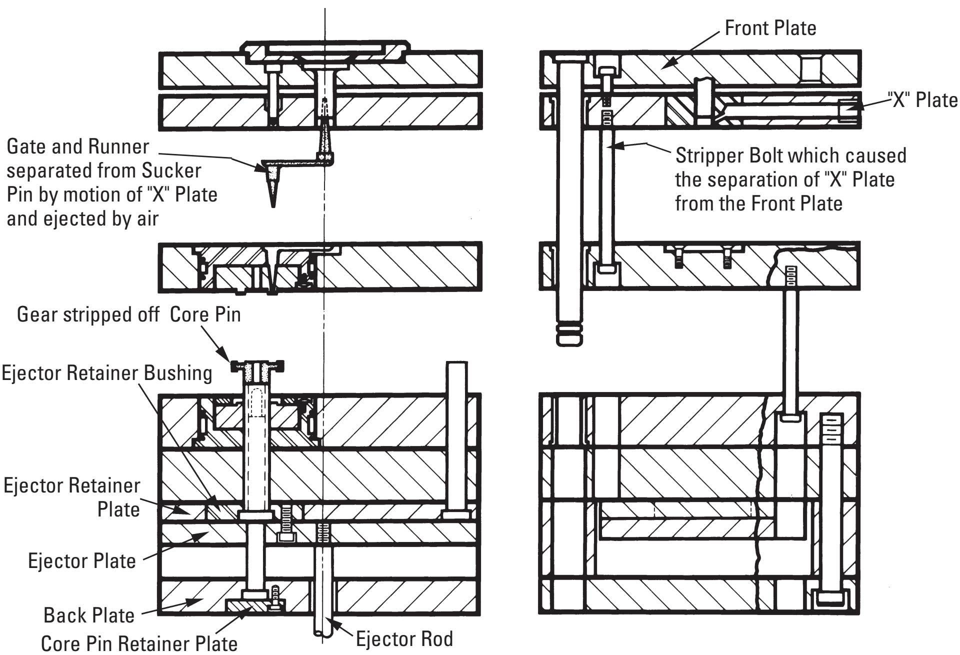
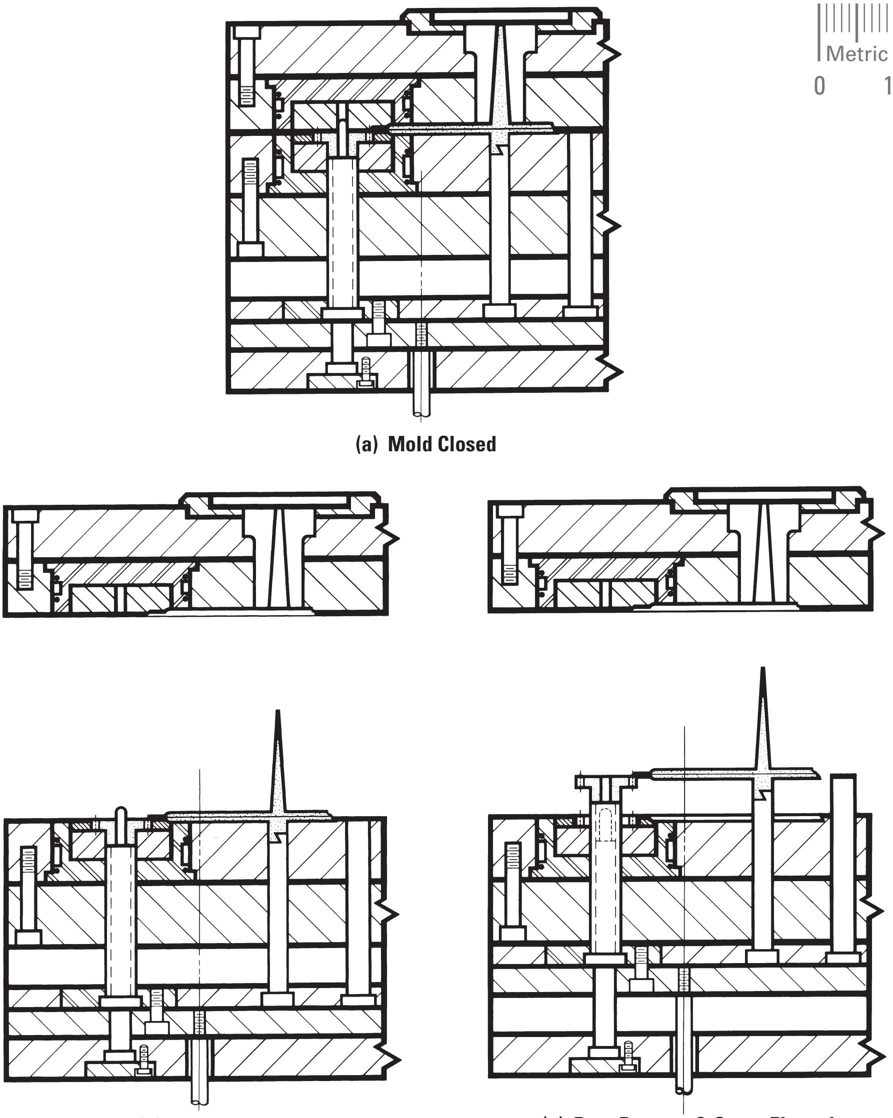


(c) Gate and Runner Exposed



(d) Mold Open

Fig. 18-14 (Cont.) Three-Plate Mold



(a) Mold Closed

(b) Mold Open

(c) Part, Runners & Sprue Ejected

Fig. 18-15 Two-Plate Mold

SECTION 19 FEATURES OF TOOTH SURFACE CONTACT

Tooth surface contact is critical to noise, vibration, efficiency, strength, wear and life. To obtain good contact, the designer must give proper consideration to the following features:

- **Modifying the Tooth Shape**
  Improve tooth contact by crowning or relieving.
- **Using Higher Precision Gear**
  Specify higher accuracy by design. Also, specify that the manufacturing process is to include grinding or lapping.
- **Controlling the Accuracy of the Gear Assembly**
  Specify adequate shaft parallelism and perpendicularity of the gear housing (box or structure).

Surface contact quality of spur and helical gears can be reasonably controlled and verified through piece part inspection. However, for the most part, bevel and worm gears cannot be equally well inspected. Consequently, final inspection of bevel and worm mesh tooth contact in assembly provides a quality criterion for control. Then, as required, gears can be axially adjusted to achieve desired contact.

JIS B 1741 classifies surface contact into three levels, as presented in **Table 19-1.**

The percentage in **Table 19-1**considers only the effective width and height of teeth.

Table 19-1 Levels of Gear Surface Contact

Level	Types of Gear	Levels of Surface Contact	
		Tooth Width Direction	Tooth Height Direction
A	Cylindrical Gears	More than 70%	More than 40%
	Bevel Gears	More than 50%	
	Worm Gears		
B	Cylindrical Gears	More than 50%	More than 30%
	Bevel Gears	More than 35%	
	Worm Gears		
C	Cylindrical Gears	More than 35%	More than 20%
	Bevel Gears	More than 25%	
	Worm Gears	More than 20%	

19.1 Surface Contact Of Spur And Helical Meshes

A check of contact is, typically, only done to verify the accuracy of the installation, rather than the individual gears. The usual method is to blue dye the gear teeth and operate for a short time. This reveals the contact area for inspection and evaluation.

19.2 Surface Contact Of A Bevel Gear

It is important to check the surface contact of a bevel gear both during manufacturing and again in final assembly. The method is to apply a colored dye and observe the contact area after running. Usually some load is applied, either the actual or applied braking, to realize a realistic contact condition. Ideal contact favors the toe end under no or light load, as shown in **Figure 19-1**; and, as load is increased to full load, contact shifts to the central part of the tooth width.

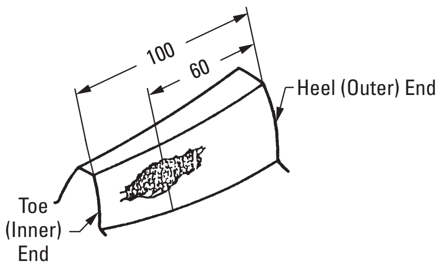


Fig. 19-1 The Contact Trace on Central Front End

Even when a gear is ideally manufactured, it may reveal poor surface contact due to lack of precision in housing or improper mounting position, or both. Usual major faults are:

1. Shafts are not intersecting, but are skew (offset error).
2. Shaft angle error of gear box.
3. Mounting distance error.

Errors 1 and 2 can be corrected only by reprocessing the housing/mounting. Error 3 can be corrected by adjusting the gears in an axial direction. All three errors may be the cause of improper backlash.

19.2.1 The Offset Error of Shaft Alignment

If a gear box has an offset error, then it will produce crossed end contact, as shown in **Figure 19-2**. This error often appears as if error is in the gear tooth orientation.

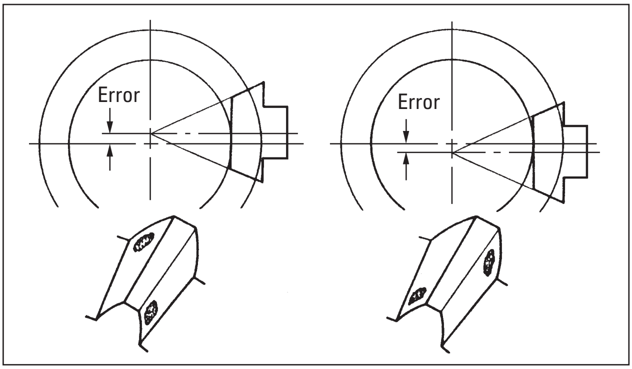


Fig. 19-2 Poor Contact Due to Offset Error of Shafts

19.2.2 The Shaft Angle Error of Gear Box

As **Figure 19-3** shows, the contact trace will move toward the toe end if the shaft angle error is positive; the contact trace will move toward the heel end if the shaft angle error is negative.

19.2.3 Mounting Distance Error

When the mounting distance of the pinion is a positive error, the contact of the pinion will move towards the tooth root, while the contact of the mating gear will move toward the top of the tooth. This is the same situation as if the pressure angle of the pinion is smaller than that of the gear. On the other hand, if the mounting distance of the pinion has a negative error, the contact of the pinion will move toward the top and that of the gear will move toward the root. This is similar to the pressure angle of the pinion being larger than that of the gear. These errors may be diminished by axial adjustment with a backing shim. The various contact patterns due to mounting distance errors are shown in **Figure 19-4.**

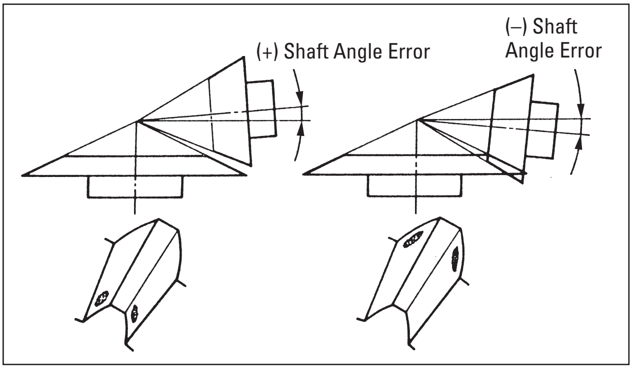


Fig. 19-3 Poor Contact Due to Shaft Angle Error

Mounting distance error will cause a change of backlash; positive error will increase backlash; and negative, decrease. Since the mounting distance error of the pinion affects the surface contact greatly, it is customary to adjust the gear rather than the pinion in its axial direction.

19.3 Surface Contact Of Worm And Worm Gear

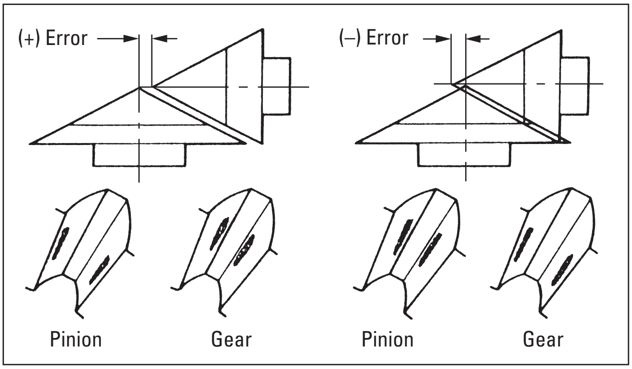


Fig. 19-4 Poor Contact Due to Error in Mounting Distance

There is no specific Japanese standard concerning worm gearing, except for some specifications regarding surface contact in JIS B 1741.

Therefore, it is the general practice to test the tooth contact and backlash with a tester. **Figure 19-5** shows the ideal contact for a worm gear mesh.

From **Figure 19-5**, we realize that the ideal portion of contact inclines to the receding side. The approaching side has a smaller contact trace than the receding side. Because the clearance in the approaching side is larger than in the receding side, the oil film is established much easier in the approaching side. However, an excellent worm gear in conjunction with a defective gear box will decrease the level of tooth contact and the performance.

There are three major factors, besides the gear itself, which may influence the surface contact:

1. Shaft Angle Error.
2. Center Distance Error.
3. Mounting Distance Error of Worm Gear.

Errors number 1 and number 2 can only be corrected by remaking the housing. Error number 3 may be decreased by adjusting the worm gear along the axial direction. These three errors introduce varying degrees of backlash.

19.3.1. Shaft Angle Error

If the gear box has a shaft angle error, then it will produce crossed contact as shown in **Figure 19-6**.

A helix angle error will also produce a similar crossed contact.

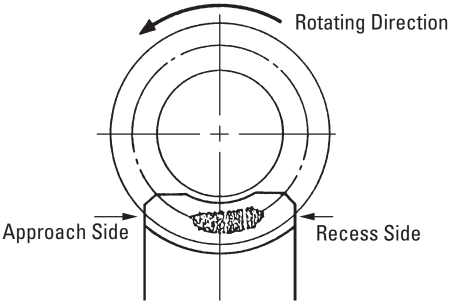


Fig. 19-5 Ideal Surface Contact of Worm Gear

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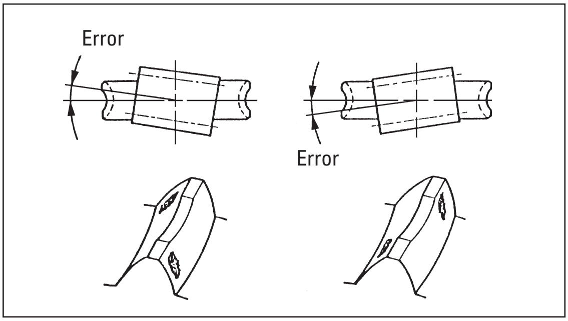

Fig. 19-6 Poor Contact Due to Shaft Angle Error

19.3.2 Center Distance Error

Even when exaggerated center distance errors exist, as shown in **Figure 19-7**, the results are crossed end contacts. Such errors not only cause bad contact but also greatly influence backlash.

A positive center distance error causes increased backlash. A negative error will decrease backlash and may result in a tight mesh, or even make it impossible to assemble.

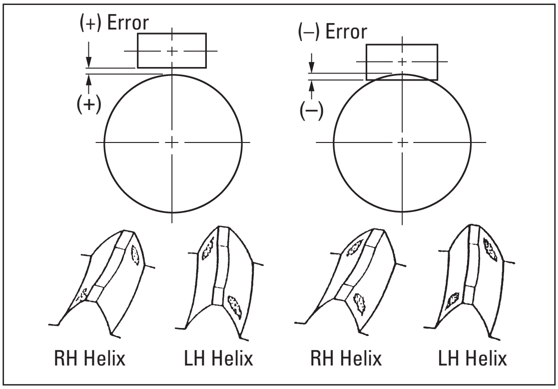

Fig. 19-7 Poor Contact Due to Center Distance Error

19.3.3 Mounting Distance Error

Figure 19-8 shows the resulting poor contact from mounting distance error of the worm gear. From the figure, we can see the contact shifts toward the worm gear tooth's edge. The direction of shift in the contact area matches the direction of worm gear mounting error. This error affects backlash, which tends to decrease as the error increases. The error can be diminished by microadjustment of the worm gear in the axial direction.

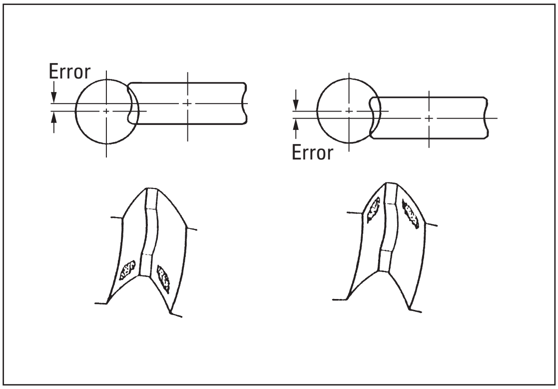

Fig. 19-8 Poor Contact Due to Mounting Distance Error

SECTION 20 LUBRICATION OF GEARS

The purpose of lubricating gears is as follows:

1. Promote sliding between teeth to reduce the coefficient of friction (μ).
2. Limit the temperature rise caused by rolling and sliding friction.

To avoid difficulties such as tooth wear and premature failure, the correct lubricant must be chosen.

20.1 Methods Of Lubrication

There are three gear lubrication methods in general use:

1. Grease lubrication.
2. Splash lubrication (oil bath method).
3. Forced oil circulation lubrication.

There is no single best lubricant and method. Choice depends upon tangential speed (m/s) and rotating speed (rpm). At low speed, grease lubrication is a good choice. For medium and high speeds, splash lubrication and forced circulation lubrication are more appropriate, but there are exceptions. Sometimes, for maintenance reasons, a grease lubricant is used even with high speed. **Table 20-1** presents lubricants, methods and their applicable ranges of speed.

Table 20-1(A) Ranges of Tangential Speed (m/s) for Spur and Bevel Gears

No.	Lubrication	Range of Tangential Speed (m/s): 0 5 10 15 20 25
1	Grease Lubrication	
2	Splash Lubrication	
3	Forced Circulation Lubrication	

Table 20-1(B) Ranges of Sliding Speed (m/s) for Worm Gears

No.	Lubrication	Range of Sliding Speed (m/s): 0 5 10 15 20 25
1	Grease Lubrication	
2	Splash Lubrication	
3	Forced Circulation Lubrication	

The following is a brief discussion of the three lubrication methods.

20.1.1 Grease Lubrication

Grease lubrication is suitable for any gear system that is open or enclosed, so long as it runs at low speed. There are three major points regarding grease:

1. Choosing a lubricant with suitable viscosity.
   A lubricant with good fluidity is especially effective in an enclosed system.
2. Not suitable for use under high load and continuous operation.
   The cooling effect of grease is not as good as lubricating oil. So it may become a problem with temperature rise under high load and continuous operating conditions.
3. Proper quantity of grease.
   There must be sufficient grease to do the job. However, too much grease can be harmful, particularly in an enclosed system. Excess grease will cause agitation, viscous drag and result in power loss.

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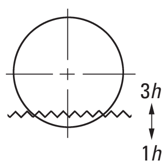
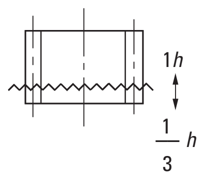
20.1.2 Splash Lubrication

Splash lubrication is used with an enclosed system. The rotating gears splash lubricant onto the gear system and bearings. It needs at least 3 m/s tangential speed to be effective. However, splash lubrication has several problems, two of them being oil level and temperature limitation.

1. Oil level:
   There will be excessive agitation loss if the oil level is too high. On the other hand, there will not be effective lubrication or ability to cool the gears if the level is too low. **Table 20-2** shows guide lines for proper oil level. Also, the oil level during operation must be monitored, as contrasted with the static level, in that the oil level will drop when the gears are in motion. This problem may be countered by raising the static level of lubricant or installing an oil pan.
2. Temperature limitation:
   The temperature of a gear system may rise because of friction loss due to gears, bearings and lubricant agitation. Rising temperature may cause one or more of the following problems:
   - Lower viscosity of lubricant.
   - Accelerated degradation of lubricant.
   - Deformation of housing, gears and shafts.
   - Decreased backlash.

   New high-performance lubricants can withstand up to 80 to 90°C. This temperature can be regarded as the limit. If the lubricant's temperature is expected to exceed this limit, cooling fins should be added to the gear box, or a cooling fan incorporated into the system.

Table 20-2 Adequate Oil Level

Types of Gears	Spur Gears and Helical Gears	
Gear Orientation	**Horizontal Shaft**	**Vertical Shaft**
Oil level Level 0		

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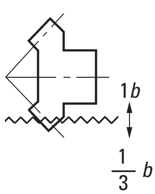
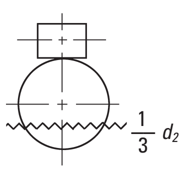
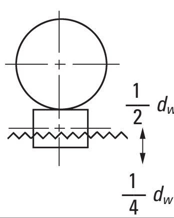
Types of Gears	Bevel Gears	Worm Gears	
Gear Orientation	**Horizontal Shaft**	**Worm Above**	**Worm Below**
Oil level Level 0			

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h = Full depth, b = Tooth width
d_2 = Pitch diameter of worm gear, d_w = Pitch diameter of worm

20.1.3 Forced-Circulation Lubrication

Forced-circulation lubrication applies lubricant to the contact portion of the teeth by means of an oil pump. There are drop, spray and oil mist methods of application.

1. Drop method:
   An oil pump is used to suck-up the lubricant and then directly drop it on the contact portion of the gears via a delivery pipe.
2. Spray method:
   An oil pump is used to spray the lubricant directly on the contact area of the gears.
3. Oil mist method:
   Lubricant is mixed with compressed air to form an oil mist that is sprayed against the contact region of the gears. It is especially suitable for high-speed gearing.

   Oil tank, pump, filter, piping and other devices are needed in the forced-lubrication system. Therefore, it is used only for special high-speed or large gear box applications. By filtering and cooling the circulating lubricant, the right viscosity and cleanliness can be maintained. This is considered to be the best way to lubricate gears.

20.2 Gear Lubricants

An oil film must be formed at the contact surface of the teeth to minimize friction and to prevent dry metal-to-metal contact. The lubricant should have the properties listed in **Table 20-3**.

Table 20-3 The Properties that Lubricant Should Possess

No.	Properties	Description
1	Correct and Proper Viscosity	Lubricant should maintain a proper viscosity to form a stable oil film at the specified temperature and speed of operation.
2	Antiscoring Property	Lubricant should have the property to prevent the scoring failure of tooth surface while under high load.
3	Oxidization and Heat Stability	A good lubricant should not oxidize easily and must perform in moist and high-temperature environment for long duration.
4	Water Antiaffinity Property	Moisture tends to condense due to temperature change, when the gears are stopped. The lubricant should have the property of isolating moisture and water from lubricant.
5	Antifoam Property	If the lubricant foams under agitation, it will not provide a good oil film. Anti-foam property is a vital requirement.
6	Anticorrosion Property	Lubrication should be neutral and stable to prevent corrosion from rust that may mix into the oil.

20.2.1 Viscosity of Lubricant

The correct viscosity is the most important consideration in choosing a proper lubricant. The viscosity grade of industrial lubricant is regulated in JIS K 2001. **Table 20-4** expresses ISO viscosity grade of industrial lubricants.

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Table 20-4 ISO Viscosity Grade of Industrial Lubricant (JIS K 2001)

ISO Viscosity Grade	Kinematic Viscosity Center Value 10^{-6} m²/s (cSt) (40°C)	Kinematic Viscosity Range 10^{-6} m²/s (cSt) (40°C)
ISO VG 2	2.2	More than 1.98 and less than 2.42
ISO VG 3	3.2	More than 2.88 and less than 3.52
ISO VG 5	4.6	More than 4.14 and less than 5.06
ISO VG 7	6.8	More than 6.12 and less than 7.48
ISO VG 10	10	More than 9.00 and less than 11.0
ISO VG 15	15	More than 13.5 and less than 16.5
ISO VG 22	22	More than 19.8 and less than 24.2
ISO VG 32	32	More than 28.8 and less than 35.2
ISO VG 46	46	More than 41.4 and less than 50.6
ISO VG 68	68	More than 61.2 and less than 74.8
ISO VG 100	100	More than 90.0 and less than 110
ISO VG 150	150	More than 135 and less than 165
ISO VG 220	220	More than 198 and less than 242
ISO VG 320	320	More than 288 and less than 352
ISO VG 460	460	More than 414 and less than 506
ISO VG 680	680	More than 612 and less than 748
ISO VG 1000	1000	More than 900 and less than 1100
ISO VG 1500	1500	More than 1350 and less than 1650

JIS K 2219 regulates the gear oil for industrial and automobile use. **Table 20-5** shows the classes and viscosities for industrial gear oils.

Table 20-5 Industrial Gear Oil

Types of Industrial Gear Oil		Usage
Class One	ISO VG 32 ISO VG 46 ISO VG 68 ISO VG 100 ISO VG 150 ISO VG 220 ISO VG 320 ISO VG 460	Mainly used in a general and lightly loaded enclosed gear system
Class Two	ISO VG 68 ISO VG 100 ISO VG 150 ISO VG 220 ISO VG 320 ISO VG 460 ISO VG 680	Mainly used in a general medium to heavily loaded enclosed gear system

JIS K 2220 regulates the specification of grease which is based on NLGI viscosity ranges. These are shown in **Table 20-6**.

Table 20-6 NLGI Viscosity Grades

NLGI No.	ASTM Worked Penetration at 25°C	State	Application
No. 000	445 … 475	Semiliquid	For Central Lubrication System (No. 000 – No. 0)
No. 00	400 … 430	Semiliquid	
No. 0	335 … 385	Very soft paste	For Automobile Chassis (No. 0 – No. 2)
No. 1	310 … 340	Soft paste	For Ball & Roller Bearing, General Use (No. 1 – No. 3)
No. 2	265 … 295	Medium firm paste	For Automobile Wheel Bearing (No. 2 – No. 4)
No. 3	220 … 250	Semihard paste	
No. 4	175 … 205	Hard paste	For Sleeve Bearing (Pillow Block) (No. 4 – No. 6)
No. 5	130 … 165	Very hard paste	
No. 6	85 … 115	Very hard paste	

Besides JIS viscosity classifications, **Table 20-7** contains AGMA viscosity grades and their equivalent ISO viscosity grades.

Table 20-7 AGMA Viscosity Grades

AGMA No. of Gear Oil		ISO Viscosity Grades
R & O Type	EP Type	
1		VG 46
2	2 EP	VG 68
3	3 EP	VG 100
4	4 EP	VG 150
5	5 EP	VG 220
6	6 EP	VG 320
7 7 comp	7 EP	VG 460
8 8 comp	8 EP	VG 680
8A comp		VG 1000
9	9 EP	VG 1500

20.2.2 Selection Of Lubricant

It is practical to select a lubricant by following the catalog or technical manual of the manufacturer. **Table 20-8** is the application guide from AGMA 250.03 "Lubrication of Industrial Enclosed Gear Drives".

Table 20-9 is the application guide chart for worm gears from AGMA 250.03.

Table 20-10 expresses the reference value of viscosity of lubricant used in the equations for the strength of worm gears in JGMA 405-01.

Table 20-8 Recommended Lubricants by AGMA

Gear Type		Size of Gear Equipment (mm)		Ambient temperature °C –10 … 16	Ambient temperature °C 10 … 52
				AGMA No.	
Parallel Shaft System	Single Stage Reduction	Center Distance (Output Side)	Less than 200	2 to 3	3 to 4
			200 … 500	2 to 3	4 to 5
			More than 500	3 to 4	4 to 5
	Double Stage Reduction		Less than 200	2 to 3	3 to 4
			200 … 500	3 to 4	4 to 5
			More than 500	3 to 4	4 to 5
	Triple Stage Reduction		Less than 200	2 to 3	3 to 4
			200 … 500	3 to 4	4 to 5
			More than 500	4 to 5	5 to 6
Planetary Gear System		Outside Diameter of Gear Casing	Less than 400	2 to 3	3 to 4
			More than 400	3 to 4	4 to 5
Straight and Spiral Bevel Gearing		Cone Distance	Less than 300	2 to 3	4 to 5
			More than 300	3 to 4	5 to 6
Gearmotor				2 to 3	4 to 5
High-speed Gear Equipment				1	2

Table 20-9 Recommended Lubricants for Worm Gears by AGMA

Types of Worm	Center Distance mm	Rotating Speed of Worm rpm	Ambient Temperature, °C –10…16	Ambient Temperature, °C 10…52	Rotating Speed of Worm rpm	Ambient Temperature, °C –10…16	Ambient Temperature, °C 10…52
Cylindrical Type	≤150 150…300 300…460 460…600 600 <	≤ 700 ≤ 450 ≤ 300 ≤ 250 ≤ 200	7 Comp	8 Comp	700 < 450 < 300 < 250 < 200 <	7 Comp	8 Comp
Throated Type	≤ 150 150…300 300…460 460…600 600 <	≤ 700 ≤ 450 ≤ 300 ≤ 250 ≤ 200	8 Comp	8A Comp	700 < 450 < 300 < 250 < 200 <	8 Comp	

Table 20-10 Reference Values of Viscosity Unit: cSt / 37.8°C

Operating Temperature		Sliding Speed m/s		
Maximum Running	Starting Temperature	Less than 2.5	2.5 … 5	More than 5
0°C … 10°C	–10°C … 0°C	110 … 130	110 … 130	110 … 130
0°C … 10°C	More than 0°C	110 … 150	110 … 150	110 … 150
10°C … 30°C	More than 0°C	200 … 245	150 … 200	150 … 200
30°C … 55°C	More than 0°C	350 … 510	245 … 350	200 … 245
55°C … 80°C	More than 0°C	510 … 780	350 … 510	245 … 350
80°C … 100°C	More than 0°C	900 … 1100	510 … 780	350 … 510

SECTION 21 GEAR NOISE

There are several causes of noise. The noise and vibration in rotating gears, especially at high loads and high speeds, need to be addressed. Following are ways to reduce the noise. These points should be considered in the design stage of gear systems.

1. Use High-Precision Gears

- Reduce the pitch error, tooth profile error, runout error and lead error.
- Grind teeth to improve the accuracy as well as the surface finish.

2. Use Better Surface Finish on Gears

- Grinding, lapping and honing the tooth surface, or running in gears in oil for a period of time can also improve the smoothness of tooth surface and reduce the noise.

3. Ensure a Correct Tooth Contact

- Crowning and relieving can prevent end contact.
- Proper tooth profile modification is also effective.
- Eliminate impact on tooth surface.

4. Have A Proper Amount of Backlash

- A smaller backlash will help reduce pulsating transmission.
- A bigger backlash, in general, causes less problems.

5. Increase the Contact Ratio

- Bigger contact ratio lowers the noise. Decreasing pressure angle and/or increasing tooth depth can produce a larger contact ratio.
- Enlarging overlap ratio will reduce the noise. Because of this relationship, a helical gear is quieter than the spur gear and a spiral bevel gear is quieter than the straight bevel gear.

6. Use Small Gears

- Adopt smaller module gears and smaller outside diameter gears.

7. Use High-Rigidity Gears

- Increasing face width can give a higher rigidity that will help in reducing noise.
- Reinforce housing and shafts to increase rigidity.

8. Use High-Vibration-Damping Material

- Plastic gears will be quiet in light load, low speed operation.
- Cast iron gears have lower noise than steel gears.

9. Apply Suitable Lubrication

- Lubricate gears sufficiently.
- High-viscosity lubricant will have the tendency to reduce the noise.

10. Lower Load and Speed

- Lowering rpm and load as far as possible will reduce gear noise.

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